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Homoclinic solutions for nonautonomous second order Hamiltonian systems with a coercive potential ☆

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Abstract

We shall be concerned with the existence of homoclinic solutions for the second order Hamiltonian system $\ddot{q} - V_q(t,q) = f(t)$, where $t \in \mathbb{R}$ and $q \in \mathbb{R}^n$. A potential $V \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$ is T-periodic in t, coercive in q and the integral of $V(\cdot, 0)$ over $[0, T]$ is equal to 0. A function $f : \mathbb{R} \to \mathbb{R}^n$ is continuous, bounded, square integrable and $f \neq 0$. We will show that there exists a solution q_0 such that $q_0(t) \to 0$ and $\dot{q}_0(t) \to 0$, as $t \to \pm\infty$. Although $q \equiv 0$ is not a solution of our system, we are to call q_0 a homoclinic solution. It is obtained as a limit of $2kT$-periodic orbits of a sequence of the second order differential equations.

Keywords: Coercive functional; Homoclinic solutions; Hamiltonian systems; Palais–Smale condition

1. Introduction

Let us consider the second order Hamiltonian system

$$\ddot{q} - V_q(t,q) = f(t), \tag{HS}$$

where $t \in \mathbb{R}$, $q \in \mathbb{R}^n$ and $V : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}^n$ satisfy the following conditions:

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(A_1) V is C^1-smooth, T-periodic with respect to t, $T > 0$,
(A_2) there is a constant $b > 0$ such that for all $(t,q) \in \mathbb{R} \times \mathbb{R}^n$

$$V(t,q) \geqslant V(t,0) + b|q|^2,$$

(A_3) $\int_0^T V(t,0)\,dt = 0$,
(A_4) $f \neq 0$ is a continuous and bounded function such that $\int_{\mathbb{R}} |f(t)|^2\,dt < \infty$.

Here and subsequently, $|\cdot| : \mathbb{R}^n \to [0,\infty)$ is the norm induced by the standard inner product $(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ given by

$$(x,y) = \sum_{i=1}^{n} x_i y_i,$$

where $x = (x_1,\dots,x_n)$, $y = (y_1,\dots,y_n)$.

The existence of connecting orbits (homoclinic and heteroclinic orbits) is one of the most important problems in the theory of Hamiltonian systems. It has been intensively studying by many mathematicians. Let us only mention here [1,3,4,7,10,12,13]. A lot of papers are concerned with solutions homoclinic to 0. See for instance [2,6,8,11,14]. In our case, $q \equiv 0$ is not a solution of (HS). Thus our Hamiltonian system does not possess a solution homoclinic to 0, in the classical meaning. However, we can still ask about the existence of solutions emanating from 0 and terminating at 0.

Definition 1.1. We will say that a solution $q : \mathbb{R} \to \mathbb{R}^n$ of (HS) is homoclinic to $x \in \mathbb{R}^n$, if $q(t) \to x$, as $t \to \pm\infty$.

In this paper we will study the existence of solutions homoclinic to $x = 0$. Under the comparatively general assumptions (A_1)–(A_4), we will show that the Hamiltonian system (HS) has a homoclinic solution with an additional regularity property. Our main result states as follows.

Theorem 1.1. *If the conditions* (A_1)–(A_4) *are satisfied then the system* (HS) *possesses a homoclinic solution* $q \in W^{1,2}(\mathbb{R},\mathbb{R}^n)$ *such that* $\dot{q}(t) \to 0$*, as* $t \to \pm\infty$.

At the end of this section, we give the main idea of the proof.

For each $k \in \mathbb{N}$, let $E_k := W^{1,2}_{2kT}(\mathbb{R},\mathbb{R}^n)$, the Hilbert space of $2kT$-periodic functions from $\mathbb{R}$ into $\mathbb{R}^n$ under the norm

$$\|q\|_{E_k} := \left(\int_{-kT}^{kT} \left(\left|\dot{q}(t)\right|^2 + \left|q(t)\right|^2 \right) dt \right)^{\frac{1}{2}}.$$

In order to receive a homoclinic solution of (HS), we consider a sequence of systems of differential equations:

$$\ddot{q} - V_q(t,q) = f_k(t), \tag{HS_k}$$

where for every $k \in \mathbb{N}$, $f_k : \mathbb{R} \to \mathbb{R}^n$ is a $2kT$-periodic extension of the restriction of f to the interval $[-kT, kT)$. Let us remark that f_k has not to be continuous at points $kT + 2kTj$, $j \in \mathbb{Z}$.

Our homoclinic solution is a limit in C^1_{loc}-topology of a certain sequence of functions $q_k \in E_k$. Each q_k is a $2kT$-periodic solution of (HS_k) obtained via a standard minimizing argument (see Theorem 2.2).

The presented method generalizes that of [11]. Paul Rabinowitz obtained a homoclinic solution of a Hamiltonian system $\ddot{q} + V_q(t, q) = 0$ as a limit of its periodic solutions. We adapt his method to the system (HS) by introducing an approximative sequence of differential equations (HS_k).

2. Proof of Theorem 1.1

For each $k \in \mathbb{N}$, let $L^2_{2kT}(\mathbb{R}, \mathbb{R}^n)$ denote the Hilbert space of $2kT$-periodic functions on $\mathbb{R}$ with values in $\mathbb{R}^n$ under the norm

$$\|q\|_{L^2_{2kT}} := \left(\int_{-kT}^{kT} |q(t)|^2\, dt \right)^{\frac{1}{2}}.$$

Let $I_k : E_k \to \mathbb{R}$ be defined by

$$I_k(q) := \int_{-kT}^{kT} \left(\frac{1}{2} |\dot{q}(t)|^2 + V\big(t, q(t)\big) + \big(f_k(t), q(t)\big) \right) dt. \tag{1}$$

Then $I_k \in C^1(E_k, \mathbb{R})$ and one can easily check that

$$I'_k(q)v = \int_{-kT}^{kT} \Big[\big(\dot{q}(t), \dot{v}(t)\big) + \big(V_q\big(t, q(t)\big), v(t)\big) + \big(f_k(t), v(t)\big) \Big]\, dt. \tag{2}$$

Furthermore, critical points of I_k are classical $2kT$-periodic solutions of (HS_k).

We have divided the proof of Theorem 1.1 into a sequence of lemmas.

Lemma 2.1. *If V and f satisfy* (A_1)–(A_4) *then for every $k \in \mathbb{N}$ the system* (HS_k) *possesses a $2kT$-periodic solution.*

We will obtain a critical point of I_k by the use of a standard minimizing argument, i.e. the following

Theorem 2.2. *(See [9, Theorem 4.4].) Let E be a Banach space, $I : E \to \mathbb{R}$ a functional bounded from below and differentiable on E. If I satisfies the Palais–Smale condition then I has a minimum on E.*

Let us remind that I satisfies the Palais–Smale condition if every sequence $\{u_j\}_{j\in\mathbb{N}}$ in E such that $\{I(u_j)\}_{j\in\mathbb{N}}$ is bounded in $\mathbb{R}$ and $I'(u_j) \to 0$ in E^*, as $j \to \infty$, contains a convergent subsequence.

Proof of Lemma 2.1. Set

$$\beta := \min\{1, 2b\}, \qquad M := \left(\int_{\mathbb{R}} |f(t)|^2\, dt \right)^{\frac{1}{2}}.$$

From (A_4), M is finite. Moreover, we have

$$\|f_k\|_{L^2_{2kT}} \leqslant M. \tag{3}$$

Applying (A_2) and (A_3), for $q \in E_k$, we receive

$$\begin{aligned} I_k(q) &\geqslant \int_{-kT}^{kT} \left(\frac{1}{2}\big|\dot{q}(t)\big|^2 + V(t,0) + b\big|q(t)\big|^2 + \big(f_k(t), q(t)\big)\right) dt \\ &= \int_{-kT}^{kT} \left(\frac{1}{2}\big|\dot{q}(t)\big|^2 + b\big|q(t)\big|^2 + \big(f_k(t), q(t)\big)\right) dt \\ &\geqslant \frac{\beta}{2}\|q\|_{E_k}^2 + \int_{-kT}^{kT} \big(f_k(t), q(t)\big)\, dt \\ &\geqslant \frac{\beta}{2}\|q\|_{E_k}^2 - \|f_k\|_{L^2_{2kT}}\|q\|_{E_k}. \end{aligned}$$

From this and (3) we get

$$I_k(q) \geqslant \frac{\beta}{2}\|q\|_{E_k}^2 - M\|q\|_{E_k}. \tag{4}$$

Consequently, I_k is a functional bounded from below.

We now show that I_k satisfies the Palais–Smale condition. Assume that $\{u_j\}_{j\in\mathbb{N}}$ in E_k is a sequence such that $\{I_k(u_j)\}_{j\in\mathbb{N}}$ is bounded and $I_k'(u_j) \to 0$, as $j \to \infty$. Then there is $C_k > 0$ such that

$$\big|I_k(u_j)\big| \leqslant C_k \tag{5}$$

for each $j \in \mathbb{N}$. Combining (5) with (4) we receive

$$\beta\|u_j\|_{E_k}^2 - 2M\|u_j\|_{E_k} - 2C_k \leqslant 0. \tag{6}$$

Since $\beta > 0$, by (6) we conclude that $\{u_j\}_{j\in\mathbb{N}}$ is a bounded sequence in E_k. Therefore it possesses a weakly convergent subsequence. Without loss of generality, we can assume that there is $u \in E_k$ such that $u_j \rightharpoonup u$, as $j \to \infty$, which implies $u_j \to u$ uniformly on $[-kT, kT]$. Thus $\|u_j - u\|_{L^2_{2kT}} \to 0$, $I_k'(u)(u_j - u) \to 0$ and

$$\int_{-kT}^{kT} \big(V_q\big(t, u_j(t)\big) - V_q\big(t, u(t)\big), u_j(t) - u(t)\big)\, dt \to 0,$$

as $j \to \infty$. Moreover, since $I_k'(u_j) \to 0$, as $j \to \infty$, we have

$$\big|I_k'(u_j)(u_j - u)\big| \leqslant \big\|I_k'(u_j)\big\|_{E_k^*}\|u_j - u\|_{E_k} \to 0.$$

Finally, using (2) we get

$$\begin{aligned} \|\dot{u}_j - \dot{u}\|_{L^2_{2kT}}^2 &= \big(I_k'(u_j) - I_k'(u)\big)(u_j - u) \\ &\quad - \int_{-kT}^{kT} \big(V_q\big(t, u_j(t)\big) - V_q\big(t, u(t)\big), u_j(t) - u(t)\big)\, dt. \end{aligned}$$

Hence $\|\dot{u}_j - \dot{u}\|_{L^2_{2kT}} \to 0$, and, in consequence, $\|u_j - u\|_{E_k} \to 0$, as $j \to \infty$.

By Theorem 2.2 we conclude that for every $k \in \mathbb{N}$ there exists $q_k \in E_k$ such that

$$I_k(q_k) = \inf_{q \in E_k} I_k(q), \qquad I_k'(q_k) = 0. \tag{7}$$

Set

$$\varrho := \frac{M + \sqrt{M^2 + 2\beta}}{\beta} > 0.$$

Let us notice that ϱ is independent of k. By (A_3), for every $k \in \mathbb{N}$, we have $I_k(0) = 0$. Therefore $I_k(q_k) \leqslant 0$. Furthermore, from (4) it follows that for every $k \in \mathbb{N}$, if $\|q\|_{E_k} \geqslant \varrho$ then $I_k(q) \geqslant 1$. Hence, for each $k \in \mathbb{N}$,

$$\|q_k\|_{E_k} < \varrho. \qquad \square \tag{8}$$

Remark. From the inequality (4) we conclude that for each $k \in \mathbb{N}$, I_k is coercive. Applying some elementary arguments we are able to prove that every I_k is weakly lower semicontinuous. In this way we also get a critical point of I_k (see [5, Theorem 1] and [9, Theorem 1.1]).

For every $p \in \mathbb{N} \cup \{0\}$, let $C^p_{\mathrm{loc}}(\mathbb{R}, \mathbb{R}^n)$ denote the space of C^p-smooth functions on $\mathbb{R}$ with values in $\mathbb{R}^n$ under the topology of almost uniformly convergence of functions and all derivatives up to the order p.

Lemma 2.3. *Let* $\{q_k\}_{k \in \mathbb{N}}$ *be the sequence defined by* (7). *Then there exists a subsequence* $\{q_{k_j}\}_{j \in \mathbb{N}}$ *convergent to a certain* q_0 *in* $C^1_{\mathrm{loc}}(\mathbb{R}, \mathbb{R}^n)$.

To prove this lemma we need the estimation made by Rabinowitz in [11].

Let $L^\infty_{2kT}(\mathbb{R}, \mathbb{R}^n)$ be a space of $2kT$-periodic essentially bounded measurable functions from $\mathbb{R}$ into $\mathbb{R}^n$ under the norm

$$\|q\|_{L^\infty_{2kT}} := \operatorname{ess\,sup}\{|q(t)|: \ t \in [-kT, kT]\}.$$

Fact 2.4. *(See (2.18) in [11].) There exists* $C > 0$ *such that for each* $k \in \mathbb{N}$ *and for each* $q \in E_k$,

$$\|q\|_{L^\infty_{2kT}} \leqslant C\|q\|_{E_k}. \tag{9}$$

Proof of Lemma 2.3. First, we will show that $\{q_k\}_{k \in \mathbb{N}}$, $\{\dot{q}_k\}_{k \in \mathbb{N}}$ and $\{\ddot{q}_k\}_{k \in \mathbb{N}}$ are equibounded sequences.

Combining (8) with (9), for each $k \in \mathbb{N}$, we get

$$\|q_k\|_{L^\infty_{2kT}} \leqslant C\|q_k\|_{E_k} < C\varrho. \tag{10}$$

Since q_k is a $2kT$-periodic solution of (HS_k), for every $t \in [-kT, kT)$

$$\ddot{q}_k(t) = V_q(t, q_k(t)) + f_k(t).$$

From this and (A_4)

$$\begin{aligned} |\ddot{q}_k(t)| &\leqslant |V_q(t, q_k(t))| + |f_k(t)| = |V_q(t, q_k(t))| + |f(t)| \\ &\leqslant |V_q(t, q_k(t))| + \sup_{t \in R} |f(t)| \end{aligned}$$

for $k \in \mathbb{N}$ and $t \in [-kT, kT)$. By (10) and (A_1) we conclude that there exists a constant $M_1 > 0$ independent of k such that

$$\|\ddot{q}_k\|_{L^\infty_{2kT}} \leqslant M_1. \tag{11}$$

Finally, for each $k \in \mathbb{N}$ and $t \in \mathbb{R}$, there is $t_k \in [t-1, t]$ such that

$$\dot{q}_k(t_k) = \int_{t-1}^{t} \dot{q}_k(s)\,ds = q_k(t) - q_k(t-1)$$

and

$$\dot{q}_k(t) = \int_{t_k}^{t} \ddot{q}_k(s)\,ds + \dot{q}_k(t_k).$$

Thus

$$\left|\dot{q}_k(t)\right| \leqslant \int_{t-1}^{t} \left|\ddot{q}_k(s)\right| ds + \left|q_k(t) - q_k(t-1)\right|.$$

Consequently, for each $k \in \mathbb{N}$,

$$\|\dot{q}_k\|_{L^\infty_{2kT}} \leqslant M_1 + 2C_\varrho \equiv M_2. \tag{12}$$

To finish the proof it is sufficient to remark that $\{\dot{q}_k\}_{k\in\mathbb{N}}$ and $\{q_k\}_{k\in\mathbb{N}}$ are equicontinuous. Indeed, for every $k \in \mathbb{N}$ and for all $t, s \in \mathbb{R}$, we have

$$\left|\dot{q}_k(t) - \dot{q}_k(s)\right| = \left|\int_s^t \ddot{q}_k(\tau)\,d\tau\right| \leqslant \left|\int_s^t \left|\ddot{q}_k(\tau)\right| d\tau\right| \leqslant M_1|t-s|.$$

Similarly,

$$\left|q_k(t) - q_k(s)\right| \leqslant M_2|t-s|.$$

Applying now the Arzelà–Ascoli lemma, we receive the claim.

Lemma 2.5. *Let $q_0 : \mathbb{R} \to \mathbb{R}^n$ be a function determined by Lemma* 2.3. *Then q_0 is a solution of* (HS) *such that $q_0(t) \to 0$ and $\dot{q}_0(t) \to 0$, as $t \to \pm\infty$.*

The proof of this lemma is based on two simple facts.

Fact 2.6. *Let $q : \mathbb{R} \to \mathbb{R}^n$ be a continuous map. If $\dot{q} : \mathbb{R} \to \mathbb{R}^n$ is continuous at t_0 then*

$$\lim_{t\to t_0} \frac{q(t) - q(t_0)}{t - t_0} = \dot{q}(t_0).$$

Fact 2.7. *Let $q : \mathbb{R} \to \mathbb{R}^n$ be a continuous map such that $\dot{q}$ is locally square integrable. Then*

$$\left|q(t)\right| \leqslant \sqrt{2}\left(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left(\left|q(s)\right|^2 + \left|\dot{q}(s)\right|^2\right) ds\right)^{\frac{1}{2}} \tag{13}$$

for every $t \in \mathbb{R}$.

The elementary proofs of these two facts can be found in [8, pp. 385–386]. Let us also remark that from (13) we immediately obtain (9). In particular, if $T > \frac{1}{2}$ then one can choose $C = \sqrt{2}$.

Proof of Lemma 2.5. First, we will show that q_0 satisfies (HS).

By Lemmas 2.1 and 2.3, we have $q_{k_j} \to q_0$ in $C^1_{\mathrm{loc}}(\mathbb{R}, \mathbb{R}^n)$, as $j \to \infty$, and

$$\ddot{q}_{k_j}(t) = V_q\big(t, q_{k_j}(t)\big) + f_{k_j}(t)$$

for every $j \in \mathbb{N}$ and $t \in [-k_jT, k_jT)$. Take $a, b \in \mathbb{R}$ such that $a < b$. There exists $j_0 \in \mathbb{N}$ such that for all $j > j_0$ and for every $t \in [a, b]$ we have

$$\ddot{q}_{k_j}(t) = V_q\big(t, q_{k_j}(t)\big) + f(t).$$

In consequence, for $j > j_0$, $\ddot{q}_{k_j}$ is continuous in $[a, b]$ and $\ddot{q}_{k_j}(t) \to V_q(t, q_0(t)) + f(t)$ uniformly on $[a, b]$. From Fact 2.6 it follows that $\ddot{q}_{k_j}$ is a classical derivative of $\dot{q}_{k_j}$ in (a, b) for each $j > j_0$. Moreover, since $\dot{q}_{k_j} \to \dot{q}_0$ uniformly on $[a, b]$, we get

$$V_q\big(t, q_0(t)\big) + f(t) = \ddot{q}_0(t)$$

for every $t \in (a, b)$. Since a and b are arbitrary, we conclude that q_0 satisfies (HS).

In the next step we will prove that $q_0(t) \to 0$, as $t \to \pm\infty$.

Remark that for every $l \in \mathbb{N}$ there is $j_0 \in \mathbb{N}$ such that for $j > j_0$ we have

$$\int_{-lT}^{lT} \big(\big|q_{k_j}(t)\big|^2 + \big|\dot{q}_{k_j}(t)\big|^2\big)\, dt \leqslant \|q_{k_j}\|^2_{E_{k_j}} \leqslant \varrho^2.$$

From this and Lemma 2.3 it follows that for each $l \in \mathbb{N}$,

$$\int_{-lT}^{lT} \big(\big|q_0(t)\big|^2 + \big|\dot{q}_0(t)\big|^2\big)\, dt \leqslant \varrho^2.$$

Letting $l \to \infty$, we obtain

$$\int_{-\infty}^{\infty} \big(\big|q_0(t)\big|^2 + \big|\dot{q}_0(t)\big|^2\big)\, dt \leqslant \varrho^2.$$

Hence

$$\int_{|t|\geqslant r} \big(\big|q_0(t)\big|^2 + \big|\dot{q}_0(t)\big|^2\big)\, dt \to 0, \tag{14}$$

as $r \to \infty$. By (13) and (14), we get $q_0(t) \to 0$, as $t \to \pm\infty$.

Finally, we will show that $\dot{q}_0(t) \to 0$, as $t \to \pm\infty$.

Applying (13), we receive

$$\big|\dot{q}_0(t)\big| \leqslant \sqrt{2}\left(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \big(\big|\dot{q}_0(s)\big|^2 + \big|\ddot{q}_0(s)\big|^2\big)\, ds\right)^{\frac{1}{2}}$$

for every $t \in \mathbb{R}$. From (14), we have

$$\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|\dot{q}_0(s)\right|^2 ds \to 0,$$

as $t \to \pm\infty$. Therefore, it suffices to observe that

$$\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|\ddot{q}_0(s)\right|^2 ds \to 0,$$

as $t \to \pm\infty$. Since q_0 is a solution of (HS), we have

$$\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|\ddot{q}_0(s)\right|^2 ds = \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|V_q\big(s, q_0(s)\big)\right|^2 ds + \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|f(s)\right|^2 ds$$
$$+ 2 \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \big(V_q\big(s, q_0(s)\big), f(s)\big)\, ds,$$

and so

$$\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|\ddot{q}_0(s)\right|^2 ds \leqslant \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|V_q\big(s, q_0(s)\big)\right|^2 ds + \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|f(s)\right|^2 ds$$
$$+ 2\left(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|V_q\big(s, q_0(s)\big)\right|^2 ds\right)^{\frac{1}{2}} \left(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|f(s)\right|^2 ds\right)^{\frac{1}{2}}.$$

(A_4) implies that

$$\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|f(s)\right|^2 ds \to 0,$$

as $t \to \pm\infty$.

Take $\varepsilon > 0$. By (A_2), $V_q(t, 0) = 0$ for each $t \in \mathbb{R}$. From (A_1), there is $\delta > 0$ such that for $t \in \mathbb{R}$ and $|q| < \delta$, $|V_q(t, q)| < \varepsilon$. Moreover, there is $r > 0$ such that if $|t| \geqslant r$, then $|q_0(t)| < \delta$. Hence, if $|t| \geqslant r + \frac{1}{2}$,

$$\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left|V_q\big(s, q_0(s)\big)\right|^2 ds < \varepsilon^2$$

which completes the proof. □

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