

## IDENTIFICATION OF DYNAMIC PARAMETERS IN THE CONTEXT OF DIAGNOSTICS OF STEEL TANK WITH DIFFERENT LEVEL OF LIQUID

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### Summary

The paper presents an experimental study to identify the dynamic parameters of a water-filled tank model. The experimental modal analysis approach involving the excitation at one point of the structure and acquisition of vibrations at selected points distributed over the tank surface allowed to determine the resonance frequencies, mode shapes and damping ratios. The main aim of the research was to examine the influence of the level of liquid on the modal parameters. The experimental study showed that the level of water in the tank had an important impact on the values of natural frequencies and damping ratios.

Keywords: liquid storage tank, experimental investigations, experimental modal analysis, dynamic parameters, impact tests

### IDENTYFIKACJA WŁASNOŚCI DYNAMICZNYCH W KONTEKŚCIE DIAGNOSTYKI ZBIORNIKA STALOWEGO PRZY ZMIENNYM POZIOMIE NAPEŁNIENIA CIECZĄ

### Streszczenie

Praca przedstawia badania doświadczalne przeprowadzone w celu identyfikacji parametrów dynamicznych modelu stalowego zbiornika do przechowywania cieczy. Jako techniki badawczej użyto eksperymentalnej analizy modalnej wywołując drgania obciążeniem impulsowym przyłożonym w jednym punkcie konstrukcji i mierząc przyspieszenia drgań w wybranych punktach rozłożonych na powierzchni zbiornika. W badaniach zidentyfikowano częstotliwości drgań własnych, postacie drgań własnych oraz współczynniki tłumienia. Głównym celem pracy była analiza wpływu poziomu cieczy na parametry modalne zbiornika. W pracy wykazano, że stopień napełnienia cieczą znacząco wpływa na częstotliwości drgań własnych oraz współczynniki tłumienia.

Słowa kluczowe: zbiornik na ciecz, badania doświadczalne, eksperymentalna analiza modalna, parametry dynamiczne, test wzbudzenia impulsowego

## 1. INTRODUCTION

Liquid storage tanks are commonly used for storing fuel, oil, liquefied gas or water. During their service lifetime they undergo various types of loads. A special attention is aimed at the dynamic response to wind or seismic loading in order to assure the reliability and safety of tank structures. Since the dynamic response of a structure depends on its dynamic parameters, any diagnostic procedure should begin with determination of resonance frequencies, mode shapes of vibration and damping ratios.

Dynamic identification of tank structures was the subject of many previous studies. Evaluation of dynamic parameters of a model of a tank with different levels of liquid was presented by Burkacki and Jankowski [1]. They studied the resistance of the cylindrical steel tank for mining tremors and earthquakes. The experiment was carried out on two

models of tanks with the application of a shake table. The investigation allowed determining the dominant values of resonance frequencies and damping ratios. The study of a tank tested on a shake table was also presented by Sweedan and El Damatty [2]. They evaluated modal parameters of the conical tank partially filled with water. Amiri and Sabbagh-Yazdi [3] performed numerical analyses to determine the dynamic properties of cylindrical tanks. In their paper, the effect of a roof on the natural frequencies and mode shapes was presented. Virella et al. [4] showed numerical results of analyses of the influence of the hydrostatic pressure on the natural frequencies and mode shapes. Curadelli et al. [5, 6] studied the dynamic response of a spherical container partially filled with liquid to horizontal base motion as well as damage detection through changes in dynamic parameters.

The dynamic behaviour of a silo structure filled with sand was investigated by Wilde et al. [7]. Identification of mode shapes was performed on the cylindrical silo with various levels of sand.

The dynamic parameters of tanks are affected by their shape. In previous studies, emphasis was primarily focused on spherical and conical tanks. In this paper, a cylindrical laboratory model of a steel tank excited in a controlled manner using a modal hammer is tested. The study is devoted to detailed experimental investigations for identification of modal parameters of the water-filled tank model. The main aim of the research is to examine the influence of the level of liquid on the modal parameters.

## 2. EXPERIMENTAL INVESTIGATIONS

### 2.1. Model description

The object of the investigation (Fig. 1) was a laboratory model of a steel tank [8, 9]. It consisted of a shell which was welded to a base. Diameters of the elliptical shell were  $D_1 = 293$  mm and  $D_2 = 303$  mm. The height of the shell was  $H = 180$  mm and the wall thickness was  $t_s = 3.4$  mm. The base in the form of octagonal plate has the thickness of  $t_b = 5$  mm. Characteristics of material were: mass density  $\rho = 7850$  kg/m<sup>3</sup>, Young's modulus  $E = 190$  GPa and Poisson ratio  $\nu = 0.25$ . The liquid contained in the model of a tank was the water.

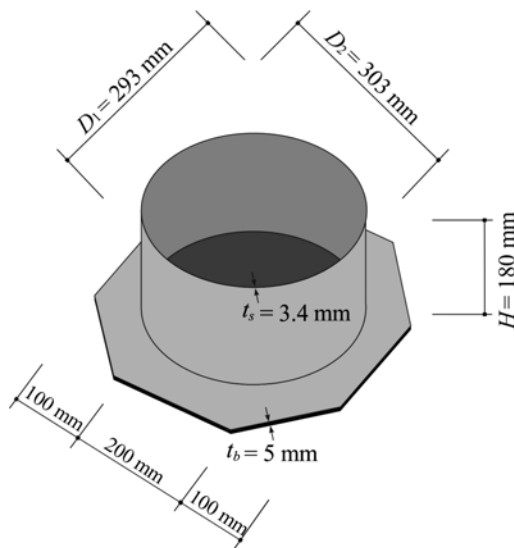


Fig. 1. Geometry of laboratory model of steel tank

### 2.2. Experimental set-up

The experimental set-up for identification of dynamic parameters is presented in Fig. 2. The tank was subjected to a dynamic pulse load applied perpendicular to the shell surface by means of the modal hammer PCB 086C03. The hammer had a built-in sensor that allowed measurement of the

applied force. The green circle in Fig. 2a indicates the position of the impact point.

The vibration responses were recorded using triaxial accelerometers PCB models 356A16 and 356B18. They were attached outside the shell. The acceleration time histories were measured in the directions perpendicular to the tank surface. The data were collected by the 40-channel LMS SCADAS system. The signals were acquired with a sampling frequency equal to 12.8 kHz. The length of the measured signals was 6 s.

During experiments two spatial configurations of measurement points were considered. They were considered as test #1 and test #2. In test #1, there were 6 measurement points distributed along the height of the model with a spatial step of 30 mm (Fig. 2a, Fig. 3). In test #2, measurements were taken with the use of 24 accelerometers. Measurement points were distributed along two circumferential cross-sections, situated at the height of 180 mm and 90 mm from the base of the tank (Fig. 2b, Fig. 3). The weight of the empty tank was about 10 kg, while the weight of the tank completely filled with water was about 22.5 kg. The total weight of accelerometers in test #1 was 1.44% of the empty tank weight while in test #2 was 6.96% of the empty tank weight.

At first measurements of vibrations were conducted for the empty tank. Then the tank was partially filled with the water until complete filling and measurements were repeated for 18 gradually increasing water levels from 10 mm to 180 mm.

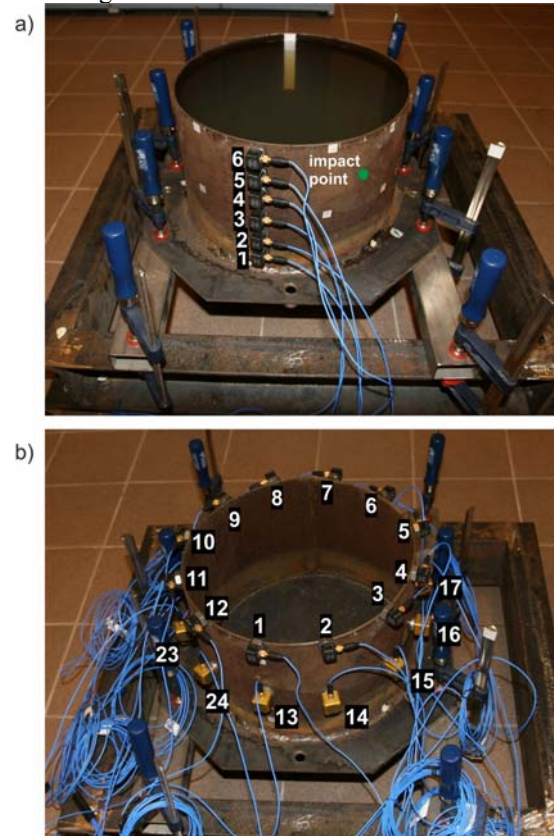


Fig. 2. Experimental set-up: a) test #1; b) test #2

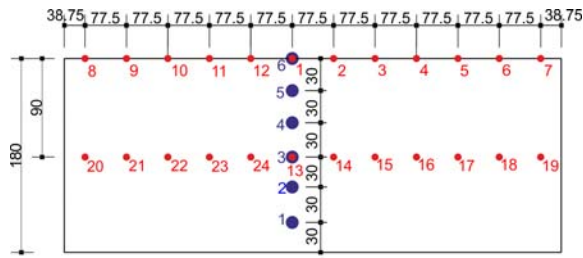


Fig. 3. Distribution and numbering of measurement points in test #1 (blue points) and test #2 (red points)

### 3. IDENTIFICATION OF MODAL PARAMETERS

Any discrete dynamic system can be described by the equation of motion [9–12]:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{p}(t), \quad (1)$$

with the initial conditions:

$$\mathbf{u}(0) = \mathbf{u}_0, \quad \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0, \quad (2)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{K}$  are the mass, damping and stiffness matrices, respectively. The size of system matrices is  $N \times N$ , where  $N$  denotes the number of degrees of freedom.

Using the Laplace transform, the equation of motion (1) with the initial conditions (2) can be converted into algebraic equations in complex variable  $s$ :

$$(\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K})\mathbf{U}(s) + \mathbf{M}s\mathbf{U}(0) + \mathbf{C}\mathbf{U}(0) = \mathbf{P}(s). \quad (3)$$

Assuming zero initial conditions, the solution in the frequency domain  $s = i\omega$  can be written as:

$$\mathbf{H}(\omega)\mathbf{P}(\omega) = \mathbf{U}(\omega), \quad (4)$$

where the transfer function matrix is defined as:

$$\mathbf{H}(\omega) = [\mathbf{M}(i\omega)^2 + \mathbf{C}(i\omega) + \mathbf{K}]^{-1}. \quad (5)$$

The matrix  $\mathbf{H}(\omega)$  contains the individual frequency response functions (FRFs)  $H_{jk}(\omega)$  which are determined by impacting at point  $k$  and measuring the response at point  $j$ . The FRF in the form of the relationship between the displacement of vibrations and the excitation force transformed to the frequency domain is known as the receptance:

$$H_{jk}(\omega) = \frac{U_j(\omega)}{P_k(\omega)}. \quad (6)$$

Because in many practical applications vibrations are measured as accelerations, the dynamic properties of a system can be expressed as the accelerance:

$$A_{jk}(\omega) = \frac{\ddot{U}_j(\omega)}{P_k(\omega)}, \quad (7)$$

where the relationship between the receptance and the accelerance is:

$$A_{jk}(\omega) = -\omega^2 H_{jk}(\omega). \quad (8)$$

Thus the receptance can be expressed in terms of accelerations as:

$$H_{jk}(\omega) = -\frac{1}{\omega^2} \frac{\ddot{U}_j(\omega)}{P_k(\omega)}. \quad (9)$$

For vibration signals acquired at all measurement points and the modal hammer signal collected at one established point, one column of the transfer function matrix  $\mathbf{H}(\omega)$  can be obtained. Any column of the transfer matrix contains information about modal parameters. The imaginary part of the FRF provides information not only about the direction but also about the amplitude of the mode shapes [9–11]. The individual resonance frequencies  $\omega_n$  can be obtained based on the absolute value of the frequency response functions as frequency peaks. Damping ratios can be calculated by means of the half-power bandwidth method. From the relation:

$$|H(\omega_1)| = |H(\omega_2)| = \frac{|H(\omega_n)|}{\sqrt{2}}, \quad (10)$$

the damping ratio can be calculated as:

$$\xi = \frac{\omega_2 - \omega_1}{2\omega_n}. \quad (11)$$

### 4. RESULTS AND DISCUSSION

Results of experimental modal analysis of the steel tank are presented in this section. In the considered frequency range from 0 Hz to 900 Hz, five resonance frequencies, mode shapes and damping parameters were identified with respect to changing water levels.

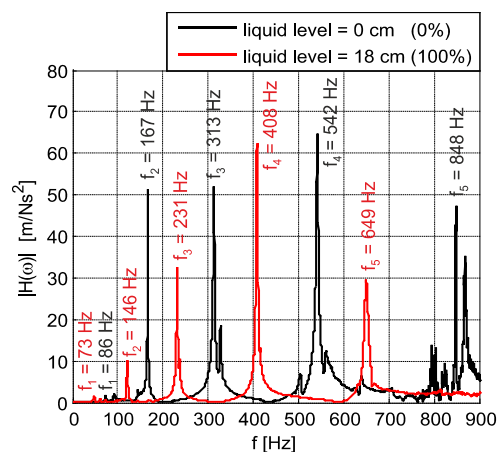


Fig. 4. Frequency response functions for the empty tank and for the full tank calculated based on the acceleration registered in point #6 (test #1)

Figure 4 illustrates the absolute value of the frequency response functions for both the empty tank and the tank completely filled with water. These FRFs were calculated based on the

acceleration registered in point #6 (for test #1). For the empty tank, the first five resonance frequencies were:  $f_1 = 86$  Hz,  $f_2 = 167$  Hz,  $f_3 = 313$  Hz,  $f_4 = 542$  Hz,  $f_5 = 848$  Hz. The increase of the level of water caused a gradual decrease in the natural frequency. Resulting resonance frequencies for the tank completely filled with water were:  $f_1 = 73$  Hz,  $f_2 = 146$  Hz,  $f_3 = 231$  Hz,  $f_4 = 408$  Hz,  $f_5 = 649$  Hz. The drop in the values of the resonant frequencies of the completely full tank in relation to the empty tank was: 15%, 12.6%, 26.2%, 24.7% and 23.5% for the first five frequencies respectively.

Figure 5 shows the relationship between resonance frequencies and liquid level as well as the

relationship between damping ratios and the liquid level for the first five vibration modes. A gradual decrease in the values of resonance frequencies with the increase of the water level can be observed for all vibration modes. The changes in the damping ratios were less regular. The damping ratios were maintained at a constant level up to 50% filling of the tank. The gradual increase of the damping ratio can be observed for the first, second and fifth modes, for liquid levels from 50% to 100%. This phenomenon is especially visible for the second vibration mode, where damping ratio grew about 3.5 times from the empty tank to the tank completely filled with water.

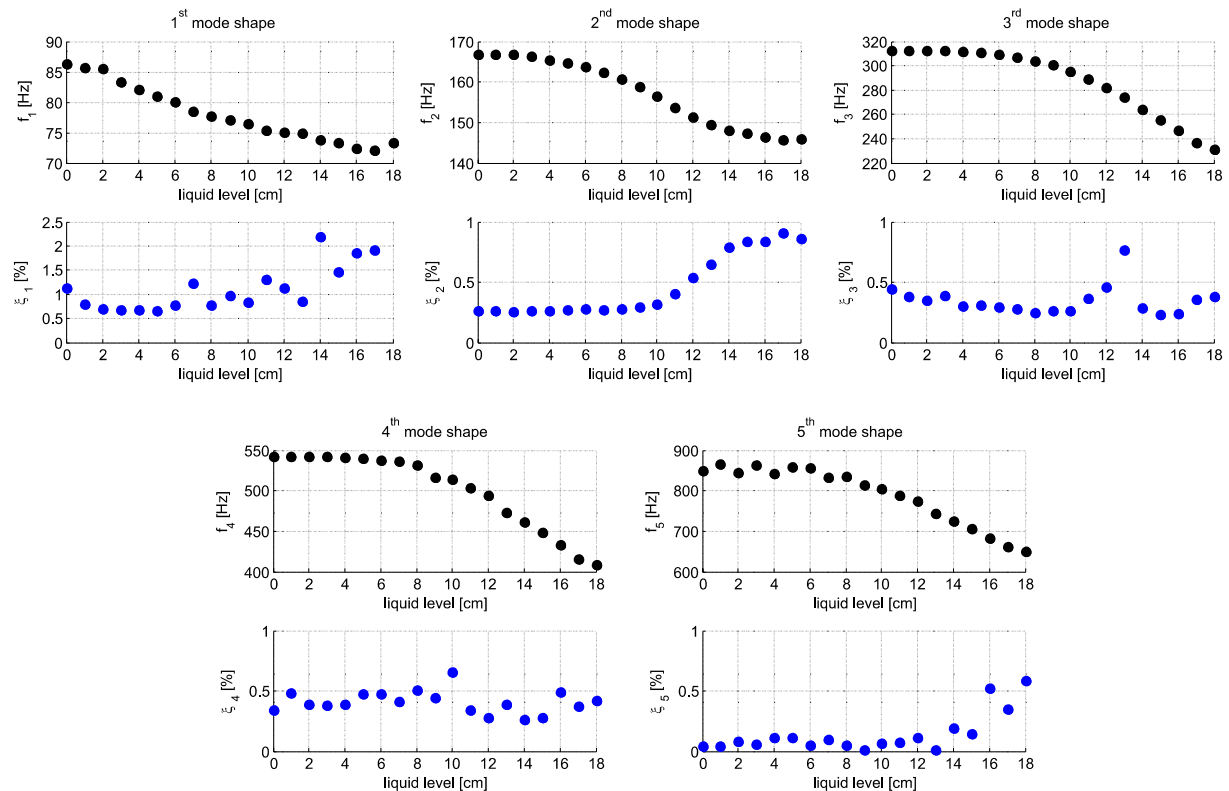


Fig. 5. Resonance frequencies and damping ratios of the first five modes with respect to the liquid level (test #1)

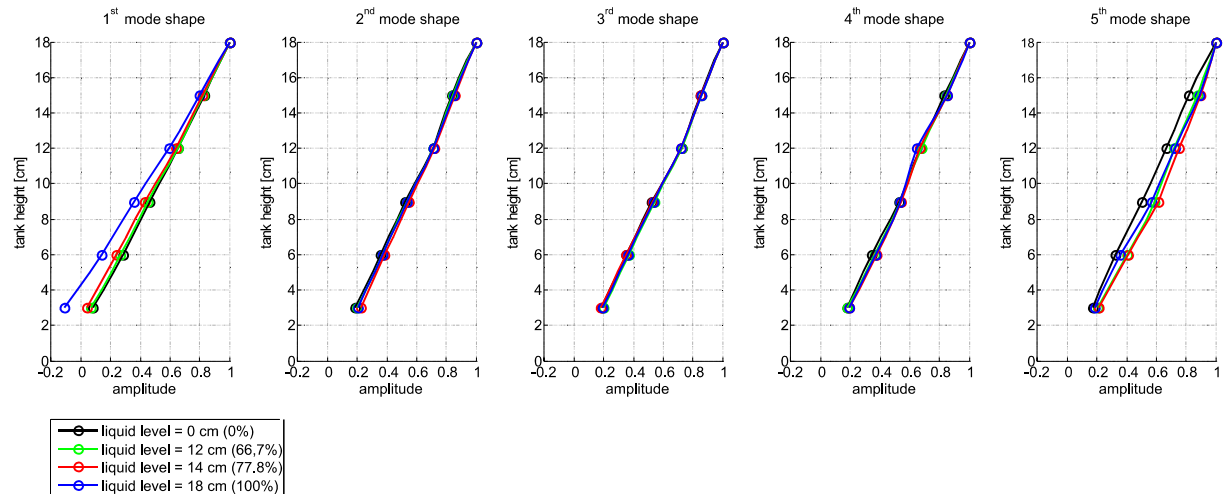


Fig. 6. Mode shapes corresponding to the first five natural frequencies with respect to the liquid level (test #1)



Cylindrical shell structures can vibrate in two types of modes, namely circumferential modes and vertical modes. Mode shapes identified in test #1 are presented in Fig. 6 for selected liquid levels: 0%, 66.7%, 77.8% and 100%. The results show a very high degree of similarity between all five forms of mode shapes. Moreover differences due to the level of liquid in the tank were not noticeable. Small discrepancies were noticed only for the first and the fifth mode shapes (Fig. 6).

Mode shapes of two circumferential cross-sections identified in test #2 are given in Fig. 7a. They were determined by the frequency response

functions  $\mathbf{H}(\omega)$  based on the accelerations registered in 24 points. Results for the empty tank and the tank completely filled with water were compared. The differences in the mode shapes between these two cases of liquid levels were not noticeable. The shapes of modes of the upper and lower rings were similar, wherein the amplitude for the upper ring was larger. The dominant vibration types for each identified forms were ovaling mode shapes. The main displacements occurred as a result of circumferential motion. Visualisations of mode shapes in 3D for the empty tank are depicted in Fig. 7b.

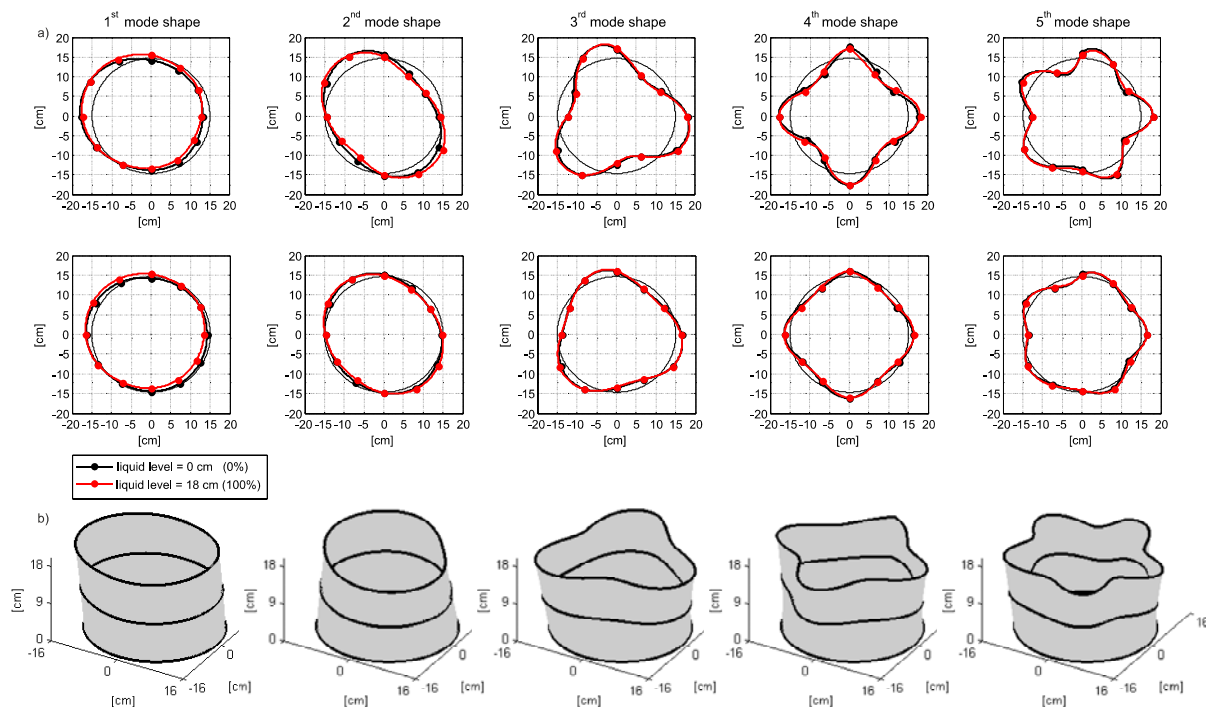


Fig. 7. Mode shapes corresponding to the first five resonance frequencies (test #2): a) mode shapes of two measured rings for the empty tank and for the full tank; b) 3D views of mode shapes for the empty tank

## 5. CONCLUSIONS

The paper presents the experimental study to identify the dynamic parameters of the water-filled tank model. The experimental modal analysis approach involving the excitation at one point of the model and acquisition of vibrations at selected points distributed over the tank surface allowed to determine the resonance frequencies, mode shapes and damping ratios.

The main aim of the research was to examine the influence of the level of liquid on the modal parameters. In the analysed frequency range, five resonance frequencies, mode shapes and damping parameters were identified with respect to the water level changing from 0% to 100% with the step of 5.56%. The identified resonance frequencies gradually decreased with the increasing level of

liquid in the tank for all considering vibration modes. The difference in the value of the resonance frequency between the empty tank and completely filled with water was as high as 26.2%. Damping ratios were maintained at a constant level up to 50% filling of the tank. The gradual increase of the damping ratio was observed for the first, second and fifth modes, for the liquid levels above 50%. The level of liquid in the tank water did not have considerable effect on the identified mode shapes.

## BIBLIOGRAPHY

- [1] Burkacki D., Jankowski R. *Badania eksperymentalne parametrów dynamicznych modeli zbiorników stalowych na stole sejsmicznym*. Zeszyty Naukowe Politechniki Rzeszowskiej, Budownictwo i Inżynieria Środowiska, nr 283 (2012), pp. 341–348.

- [2] Sweedan A.M.I., El Damatty A.A. *Experimental identification of the vibration modes of liquid-filled conical tanks and validation of a numerical model*. Earthquake engineering and structural dynamics, vol. 32 (2003), pp.1407–1430.
- [3] Amiri M., Sabbagh-Yazdi S.R. *Influence of roof on dynamic characteristics of dome roof tanks partially filled with liquid*. Thin-Walled Structures, vol. 50 (2012), pp. 56–67.
- [4] Virella J.C., Godoy L.A., Suárez L.E. *Fundamental modes of tank-liquid systems under horizontal motions*. Engineering Structures, vol. 28 (2006), pp. 1450–1461.
- [5] Curadelli O., Ambrosini D., Mirasso A., Amani M. *Resonant frequencies in an elevated spherical container partially filled with water: FEM and measurement*. Journal of Fluids and Structures, vol. 26 (2010), pp. 148–159.
- [6] Curadelli O., Ambrosini D. *Damage detection in elevated spherical containers partially filled with liquid*. Engineering Structures, vol. 33 (2011), pp. 2708–2715.
- [7] Wilde K., Rucka M., Tejchman J. *Silo music – mechanism of dynamic flow and structure interaction*. Powder Technology, vol. 186 (2008), pp. 113–129.
- [8] Rucka M., Wilde K. *Neuro-wavelet damage detection technique in beam, plate and shell structures with experimental validation*. Journal of Theoretical and Applied Mechanics, vol. 48 (2010), pp. 579–604.
- [9] Rucka M. *Wavelet analysis in detection and localization of damage in engineering structures*. Doctoral thesis, Gdańsk University of Technology, Gdańsk, 2005.
- [10] Rucka M., Wilde K., *Dynamika budowli z przykładami w środowisku MATLAB®*. Wydawnictwo Politechniki Gdańskiej, Gdańsk, 2014.
- [11] Maia N.M.M., Silva J.M.M. *Theoretical and experimental modal analysis*. Research Studies Press Ltd., Baldock, 1997.
- [12] Chopra A.K. *Dynamics of structures*. Prentice Hall, Upper Saddle River, 2011.



monitoring.

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