Postprint of: Tomaszewska A., Szymczak C., Identification of the Vistula Mounting tower model using measured modal data, Engineering Structures, Vol. 42 (2012), pp. 342-348, DOI: 10.1016/j.engstruct.2012.04.031
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Identification of the Vistula Mounting tower model using measured modal data

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Abstract

This paper discusses the problem of parametric identification of a historic masonry tower model. The tower tends to lean and its foundation stiffness is a concern to authorities. The authors identified a few modal characteristics of the tower, natural frequencies and mode shapes. Based on the first mode shape identified it is known that the structure behaves like a stiff solid on elastic foundation. Thus, a simple, five parameter plane model is taken into consideration. The unknown parameters are identified to be the solution to an optimisation problem, which involves using the sensitivity analysis and scatters of the modal identification. A hierarchical process is formulated, where two natural frequencies are assumed to be the input data. In this approach, the number of unknown parameters increases incrementally, and the process changes from even-posed to under-posed successively. Such approach allows one to control the final under-posed identification problem and leads to an increasingly better solution.

Keywords

Parametric identification; Mathematical modelling; Optimisation, Sensitivity analysis, Masonry tower.

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Introduction

This paper discusses the problem of parameters identification of the Vistula Mounting tower model. A mathematical model can be a source of information about structural behaviour, in addition to observations and *in situ* measurements. However, the model must be well-posed. In mathematical modelling several problems must be considered and some decisions should be taken. First of all, the type of the model, its size and the governing equations must be specified in order to properly represent the structure. Secondly, some variables describing the structure's behaviour must be quantifiable and measured. They will serve as the data for model validation. Finally, a method of model parameters identification must be chosen.

Nowadays, when measuring tools are highly developed, it is convenient to use modal characteristics of a structure as reference data for modelling. Those characteristics can describe both the global and the local behaviour of a structure. Thus a model can be validated on different levels. Experimental modal identification belongs to the class of inverse problems. The modal characteristics are determined through measured structural response to known (i.e. generated signals especially) or unknown (i.e. ambient) excitations. There are a lot of methods of experimental modal identification, many of which are described in the book [1]. A review of different experimental techniques for obtaining modal parameters of concrete structures is presented in [2]. In the book [3] a background for the peak picking method is given. A practical example of the modal model identification of a 200MW steam turbine foundation, working in a large power station is described in [4]. The model identification is based on measured response of the system to operating excitations. Another case is described in the paper [5], where application of different modal

identification techniques for collecting ambient response data from dynamic tests of cable-stayed bridge is discussed.

Parametric identification of a model is also an inverse problem. If response of a model, i.e. the modal characteristics, displacements, time histories, etc., can be compared to the response measured *in situ*, then the model parameters can be identified. For instance, the paper [6] describes an example, where concrete gravity dams were identified to predict dynamic characteristics of an existing concrete gravity dam with an empty reservoir. An experimental examination of a hybrid method for identifying both structural parameters and ground motion of earthquake-excited multi-storey buildings is described in [7]. Another case, concerning bridge finite element model updating, is presented in [8].

In this article a problem of modelling and parametric identification of a historic masonry tower in the Vistula Mounting Fortress in Gdansk (Fig.1) is described. The tower, in its original form, dates back to the 15th century. It was erected as a lighthouse and a defensive building. It was damaged several times in military conflicts. Nowadays it is 22.65 m high, and its external diameter is 7.7 m. The structure has seven floors. Concrete ceilings are reinforced. Its walls were built using masonry and were restored at different times. The average wall thickness is 1.25 m. The tower was founded on weak and layered subsoil. The foundations were made of boulders and lie just below the ground level. Conservation works in the Vistula Fortress have continued for the last 20 years. In May 2011 the Fortress was opened for visitors.

There are still, however, dilemmas concerning the tower. One of them concerns stiffness of the tower foundation because the building tends to lean. The author's task is to propose a relatively simple model of the tower and estimate foundation stiffness.

The overall behaviour of the tower can suggest a type of a suitable mathematical model.

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In order to determine it, dynamic measurements were taken and some modal characteristics have been identified. Signals measured at several structural points during ambient excitation have been utilized. It should be emphasised that only weak ambient excitations caused by the river and wind are possible to use in the modal identification process because of bad tower condition.

The type of a tower model was selected based on the form of the first mode shape. Since a considerable rotation - in comparison to the tower structural deformation - about the tower base is observable, a rigid solid body resting upon elastic foundation is considered to be a good approximation of the structure. Natural frequencies of the first and the second coplanar mode shapes, and two coordinates of the first mode shape are used as the data in the model parametric identification. In order to solve the problem of model identification, a least square error function was formulated as the objective function. To minimize it, an iterative procedure with implication of the sensitivity analysis was employed..

In this engineering case the authors wish to propose a hierarchical approach to solve the under-posed problem. Important elements of this task are scatters of the structure's measured modal characteristics. They are used to accurately define the optimisation problem.

2. Identification problem

2.1. Experimental modal identification

The difficulty of dynamic identification of masonry buildings is caused by low-energy vibrations of such structures (see [9]). The Peak Picking method (see [3]) was used for modal identification of the tower. The method is suitable for any signal, also for low-

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energy ones. It was used with success for other masonry tower examinations, for example for the historic masonry bell-tower, adjacent to the Cathedral of Monza in Italy [10]. The biggest advantage of the method, however, is the possibility of determining statistical errors of identified modal characteristics. This feature of the method was useful for this investigation.

The mode shapes errors arise from the fact that only estimates of the auto-spectra, which are basic functions in the peak picking method, can be calculated. Real values of the functions could be obtained for signals infinite in time and that is practically impossible. The estimates are affected by statistical errors, bias ε_b and random ε_r, which give a final error $\varepsilon = \varepsilon_b + \varepsilon_r$. They are presented in [3] and [11]. The formulae are:

$$\varepsilon_b\left[\hat{G}_{pp}(f)\right] \approx \frac{\Delta f^2}{24}\left[\frac{\left(G_{pp}(f)\right)''}{G_{pp}(f)}\right], \tag{1}$$

$$\varepsilon_r\left[\hat{G}_{pp}(f)\right] \approx \frac{1}{\sqrt{n_d}}, \tag{2}$$

where $\hat{G}_{pp}(f)$ is the estimate of auto-spectrum calculated for signal measured in a structural point *p*, Δf denotes the frequency resolution of the analyzed spectra, $\left(G_{pp}(f)\right)''$ is the second derivative of the $G_{pp}(f)$ function and n_d is a number of $p(t)$ signals analyzed.

Coordinates of a mode shape associated with the resonant frequency f_m are calculated according to the formula (3) (see [3]):

$$\phi_p(f_m) = \sqrt{\frac{\hat{G}_{pp}(f_m)}{\hat{G}_{rr}(f_m)}}, \tag{3}$$

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where $\phi_p(f_m)$ denotes the estimated mode shape coordinate at discretization point *p* and $\hat{G}_{rr}(f_m)$ is the auto-spectrum value for f_m, calculated for a signal $r(t)$, measured at the structural reference point *r*. Thus, according to the rules of error transfer the following statistical error of the mode shape coordinates is identified:

$$\varepsilon\left[\phi_p\right] = \frac{1}{2}\left(\varepsilon\left[\hat{G}_{pp}\right] + \varepsilon\left[\hat{G}_{rr}\right]\right) \tag{4}$$

The error of the measured natural frequencies has two components: the digitalisation error equal to the half of the spectrum resolution, and the random error calculated using dispersion of the measured resonant frequencies.

Acceleration of points selected across the tower were measured during ambient vibrations. Wind and water waves from the nearby situated river (Fig. 1) caused major environmental excitation. The measuring points were arranged along two opposite walls at the tower height on nine levels (Fig. 2). Accelerations in two horizontal directions, East-West (parallel to the wall surfaces) and North-South (perpendicular to the wall surfaces) were recorded at each point. Thus, 36 measuring points were set. A twelve-channel PULSE 3650C measuring system was used. Four series of measurements were carried out, in which four sensors were constantly in the same positions, in the reference points, situated at the level 11.05m above the ground level. The signals collected in the reference points were used for scaling measurements made in different series to the same order. Each measurement took 1024 seconds, 256 samples were collected per second, so each signal consisted of 262144 samples.

According to rules of the peak picking modal identification method, the auto- and cross-spectra of the time series, the coherence functions for various pairs of signals and the phase shifts between them were calculated. On that background only one resonant

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frequency of the tower was identified using signals measured across the North-South plane, whereas three were determined using time series measured in the East-West direction. An example cross-spectrum of signals measured at the tower top at two structural points situated on opposite walls is presented in Fig. 3. Nature of related mode shapes was also specified using the analysis of phase shifts between signals measured at different structural points. Additionally, coordinates of two first mode shapes in two perpendicular planes were determined. Hence, it is evident that $f_1^{N-S} = 1.416$ Hz and $f_1^{E-W} = 1.446$ Hz refer to the first two lateral mode shapes in two perpendicular directions: North-South and East-West, respectively. The associated mode shapes are presented in Fig.4. The coordinates of the mode shapes are calculated from the auto-spectra ratio, see formula (3), so relative values are obtained. The unitary value is found for a section, where the reference signals are recorded.

The subsequent frequencies identified in the East-West direction are $f_t = 4.485$ Hz, which relates to the torsional mode, and $f_2^{E-W} = 6.570$ Hz, connected with the second lateral mode shape in this plane. Unfortunately, the shapes of these modes cannot be determined because of absence of peaks for this frequency in most of auto-spectra. According to the formula (3), values of such peaks are used for a mode shape coordinates calculation. The frequencies f_1^{E-W}, f_t and f_2^{E-W} are marked in the cross-spectrum presented in Fig. 3.

In order to estimate the signal spectra, time histories were divided into 32 sections ($n_d = 32$). Thus, the error for all the modes is the same and amounts to $\varepsilon[\phi] = \varepsilon_r[\phi] = 0.177$, because $\varepsilon_b[\phi]$ is negligibly small as it is of the 0.001 order (see also [12]). The following values of the errors were obtained for the tower's natural

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frequencies $\varepsilon\left[f_1^{N-S} \right] = 0.00322$, $\varepsilon\left[f_1^{E-W} \right] = 0.00337$, $\varepsilon\left[f_t \right] = 0.00689$ and

$\varepsilon\left[f_2^{E-W} \right] = 0.00871$.

2.2. Optimisation scheme

Many different methodologies are used in identification of structural systems, many of which are described in [13]. In this task of parametric identification of the mathematical model, an optimisation problem was formulated. The square error function is assumed to be the objective function:

$$F(\boldsymbol{b}) = \sum_{i=1}^{i=S} \alpha_i \left(s_i(\boldsymbol{b}) - \hat{s}_i \right)^2, \tag{5}$$

where $\boldsymbol{b}$ denotes a vector of the design variables (the sought-after parameters of the model), $s_i(\boldsymbol{b})$ stands for the state variables of the mathematical model, $\hat{s}_i$ represents measured state variables of the tower and α_i is a weight coefficient determined for each state variable. In order to find the minimum of the objective function (5) an iterative procedure is proposed. The unknown state variables in the iteration of *k* can be formulated as:

$$s_i^k\left(\boldsymbol{b}^k + \delta\boldsymbol{b}^k\right) = s_i^k\left(\boldsymbol{b}^k\right) + \delta s_i^k\left(\boldsymbol{b}^k, \delta\boldsymbol{b}^k\right), \tag{6}$$

where $\delta s_i^k\left(\boldsymbol{b}^k, \delta\boldsymbol{b}^k\right)$ is the first variation of the state variable with respect to the design variable vector.

Finally, the optimization problem is reformulated as minimisation of the objective function in relation to the design variables vector variations:

$$\min_{\delta\boldsymbol{b}} F(\delta\boldsymbol{b}^k) = \min_{\delta\boldsymbol{b}} \sum_{i=1}^{i=S} \alpha_i \left(s_i^k(\boldsymbol{b}^k) + \delta s_i^k(\boldsymbol{b}^k, \delta\boldsymbol{b}^k) - \hat{s}_i \right)^2. \tag{7}$$

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Variations $\delta \boldsymbol{b}^k$, calculated at each stage k are used for updating the $\boldsymbol{b}$ vector: $\boldsymbol{b}^{k+1} = \boldsymbol{b}^k + \delta \boldsymbol{b}^k$. Calculations continue until the relative variations $\delta \boldsymbol{b}^k$ are smaller than the assumed accuracies. The mathematically complicated relation $\delta s_i(\boldsymbol{b}, \delta \boldsymbol{b})$ is substituted by approximation $\delta s_i = \left(\boldsymbol{w}_{sb}\right)^{\mathrm{T}} \delta \boldsymbol{b}$ determined by means of sensitivity analysis. The vector $\boldsymbol{w}_{sb}$ consists of first variations of the state variables with respect to variations of the design variables.

In this approach the radial natural frequencies squared λ_i and the coordinates of the first mode shape $\boldsymbol{\phi}_{n1}$ are the state variables. Thus, the objective function is formulated as follows:

$$\min_{\delta \boldsymbol{b}} F(\delta \boldsymbol{b}^k) = \min_{\delta \boldsymbol{b}} \left(\sum_{i=1}^{i=2} \alpha_j \left(\frac{\lambda_i^k(\boldsymbol{b}^k) - \hat{\lambda}_i}{\lambda_i^k(\boldsymbol{b}^k)} + \left(\bar{\boldsymbol{w}}_{\lambda b}^k\right)^{\mathrm{T}} \delta \bar{\boldsymbol{b}}^k \right)^2 + \sum_{n=1}^{n=2} \alpha_n \left(\frac{\phi_{n1}^k(\boldsymbol{b}^k) - \hat{\phi}_{n1}}{\phi_{n1}^k(\boldsymbol{b}^k)} + \bar{\boldsymbol{W}}_{\boldsymbol{\phi} b}^k \delta \bar{\boldsymbol{b}}^k \right)^2 \right), \tag{8}$$

where the vector $\bar{\boldsymbol{w}}_{\lambda b}$ and the matrix $\bar{\boldsymbol{W}}_{\boldsymbol{\phi} b}$ consist of the relative first variations of the radial natural frequency squared λ and of the mode shape $\boldsymbol{\phi}$ relative to the variations of the design variables, respectively. The coefficients are derived from the equation of motion for a discrete system. They are calculated according to the following formulae:

$$\bar{w}_{\lambda bj} = \frac{\partial \lambda_i}{\partial b_j} \frac{b_j}{\lambda_i} = \left[\boldsymbol{\phi}_i^{\mathrm{T}} \left(\frac{\partial \boldsymbol{K}}{\partial b_j} - \lambda_i \frac{\partial \boldsymbol{M}}{\partial b_j} \right) \boldsymbol{\phi}_i \right] \frac{b_j}{\lambda_i}, \tag{9}$$

$$\bar{\boldsymbol{W}}_{\boldsymbol{\phi} b} = \left\{ \bar{\boldsymbol{w}}_{\boldsymbol{\phi} b1} \ \ldots \ \bar{\boldsymbol{w}}_{\boldsymbol{\phi} bj} \ \ldots \ \bar{\boldsymbol{w}}_{\boldsymbol{\phi} bB} \right\}, \tag{10}$$

$$\bar{\boldsymbol{w}}_{\boldsymbol{\phi} bj} = \left\{ \frac{\partial \phi_{1i}}{\partial b_j} \frac{b_j}{\phi_{1i}}, \ldots, \frac{\partial \phi_{ni}}{\partial b_j} \frac{b_j}{\phi_{ni}}, \ldots, \frac{\partial \phi_{DOFi}}{\partial b_j} \frac{b_j}{\phi_{DOFi}} \right\}^{\mathrm{T}}, \tag{11}$$

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$$\boldsymbol{w}_{\boldsymbol{\phi}bj} = \left\{ \frac{\partial \boldsymbol{\phi}_{1i}}{\partial b_j}, ..., \frac{\partial \boldsymbol{\phi}_{ni}}{\partial b_j}, ..., \frac{\partial \boldsymbol{\phi}_{DOFi}}{\partial b_j} \right\}^{\mathrm{T}} = \sum_{m=1, m\neq i}^{m=DOF} \left(\frac{\boldsymbol{\phi}_m^{\mathrm{T}} \left[\frac{\partial \boldsymbol{K}}{\partial b_j} - \lambda_i \frac{\partial \boldsymbol{M}}{\partial b_j} \right] \boldsymbol{\phi}_i}{(\lambda_i - \lambda_m)} \boldsymbol{\phi}_m \right) - \frac{1}{2} \left(\boldsymbol{\phi}_i^{\mathrm{T}} \frac{\partial \boldsymbol{M}}{\partial b_j} \boldsymbol{\phi}_i \right) \boldsymbol{\phi}_i , \tag{12}$$

where $\lambda_i, \boldsymbol{\phi}_i$ denote the *i*th radial natural frequency squared and the *i*th mode shape, ϕ_{ni} is *n*th coordinate of the mode $\boldsymbol{\phi}_i \ (n = 1, ..., DOF)$, b_j represents the *j*th model parameter $(j = 1, ..., B)$, $\boldsymbol{K}$ and $\boldsymbol{M}$ stand for the stiffness matrix and the mass matrix, respectively.

2.3. The model and the identification setup

In order to make the calculations the governing equations of the mathematical model must be assumed. In case of the Vistula Mounting Fortress tower the type of model is determined based on two first mode shapes measured. The mode shapes (Fig. 4) confirm the hypothesis of a relatively weak foundation of the tower. They show that the tower leans almost like a stiff solid therefore a model of a rigid solid body resting on an elastic foundations can be a reasonable mathematical approximation of the building's behaviour. A small number of parameters is the advantage of this model. It is convenient because only a few modal characteristics of the tower are to be used as state variables in the model parametric identification. The benefit of models with smaller number of degrees of freedom is described for example in the paper [14], where a structural health monitoring procedure for the Guangzhou New TV Tower in China is presented.

A schematic diagram of the model is presented in Fig. 5. The plane model is the subject of interest. Therefore there are two dynamic degrees of freedom, namely: the displacement across the *x* axis and rotation φ, relative to the *y* axis. The foothold of the

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Cartesian coordinate system *xyz* is placed in the centre of gravity of the structure. The following equation of motion is valid:

$$\begin{bmatrix} m & 0 \\ 0 & J_y \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} k_x & -k_x z_c \\ -k_x z_c & k_\varphi + k_x z_c^2 \end{bmatrix} \begin{Bmatrix} x \\ \varphi \end{Bmatrix} = \mathbf{0}, \tag{13}$$

with the following five parameters: *m* mass of the tower, J_y the tower mass moment of inertia, z_c the tower's centre of gravity, and k_φ, k_x foundation stiffness coefficients. Those five parameters are to be determined based on measured state variables, the tower modal characteristics.

Initially only two state variables were used, namely the natural frequencies of the first and the second lateral mode shape measured across the East-West plane (the natural frequencies f_1^{E-W} and f_2^{E-W}). The natural frequencies can be quite easily measured compared to mode shapes, so in that respect they are convenient state variables. In order to solve this identification problem the authors applied a hierarchical approach.

In the hierarchical approach the task changes form the even-post to under-posed. At the first stage only two parameters, namely k_φ, k_x, are identified. Hence, we have an even-posed problem then. Three other parameters: m, J_y, z_c can be approximated based on known geometry of the structure and building materials, and they are assumed to be constant at this stage of calculations. At the second stage three parameters are identified - two are assumed as constant. At the third stage of calculations four parameters are sought-after and finally all five are identified at the fourth stage. The order of assigning assumed constants to unknown parameter groups determines the sensitivity analysis in the following manner. The higher the relative sensitivity coefficient of a given state variable to a given parameter variation the bigger the influence of that parameter on objective function minimisation. The parameter, which influences the objective function the most,

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is assigned to the unknowns set first, while the one of the smallest influence is the last one to be identified.

One should note that especially when dealing with under-posed problems, selecting the starting point for calculations is crucial to obtaining a reliable result. The starting point can be quite easily specified in case of such hierarchical approach. As for the even-posed problem, three parameters are roughly calculated based on the tower geometry and building materials used. Then the initial values of the other two unknowns can be calculated using the governing equations for the model assumed. In that way, when dealing with under-posed tasks, the starting point can be always verified drawing on solutions to a case with smaller number of parameters.

Having concluded the stage with two state variables, results obtained are tested for dependence on the number of state variables by considering four state variables. Two coordinates of the first lateral mode shape (related to f_1^{E-W}) in the East-West plane will be used as the additional data. Namely, ϕ_{t1}^{E-W} - displacement of the tower top and ϕ_{b1}^{E-W} - displacement at 5.1 m above the ground level.

The accuracy of results obtained in each case is verified by comparing the theoretical and the measured first mode shape.

2.4. The data

The following values of state variables were used in the optimisation: $\hat{\lambda}_1 = \left(2\pi f_1^{E-W}\right)^2 = 81.99\left(\text{rad/s}\right)^2$, $\hat{\lambda}_2 = \left(2\pi f_2^{E-W}\right)^2 = 1703.34\left(\text{rad/s}\right)^2$, $\hat{\phi}_{t1} = \phi_{t1}^{E-W} = 1.806\,[-]$, $\hat{\phi}_{b1} = \phi_{b1}^{E-W} = 0.453\,[-]$. Errors in measured state variables play a major role in the process. They are used to specify weighted coefficients of state variables

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and to calculate the final criterion formulation. The error values are given at p. 2.1.

Statistical errors of the measured natural frequencies are of the same order. Thus, in the first, hierarchical approach with two state variables, the weight coefficients are the same both for $\hat{\lambda}_1$ and $\hat{\lambda}_2$ at 1. In the second approach, with four state variables, due to major differences between statistical errors of the measured natural frequencies and the mode shapes, the weight coefficients for the four state variables are diverse. They are defined proportionally to the statistical errors of the modal identification (see p. 2.1), hence their sum is equal to 1. They are: $\alpha\left(\hat{\lambda}_1\right)=0.869$, $\alpha\left(\hat{\lambda}_2\right)=0.119$ and $\alpha\left(\hat{\phi}_{t1}\right)=\alpha\left(\hat{\phi}_{b1}\right)=0.00588$.

The final criterion for identification is defined by a relative difference between the measured and the calculated state variables. For each variable this difference must be smaller than its error obtained from the modal identification. The calculations continue as long as the difference for each state variable used decreases down to the assumed level. Therefore, the final criterion in case of two state variables is $\varepsilon\left(\lambda_i\right)=\frac{\hat{\lambda}_i-\lambda_i}{\hat{\lambda}_i}\le 10^{-4}$, $i=1,2$, while in case of four state variables there are defined as the following conditions: $\varepsilon\left(\phi_{n1}\right)=\frac{\hat{\phi}_{n1}-\phi_{n1}}{\hat{\phi}_{n1}}\le 0.177;\quad n=b,t$,

$\varepsilon\left(\lambda_1\right)=\frac{\hat{\lambda}_1-\lambda_1}{\hat{\lambda}_1}\le 0.00337$ and $\varepsilon\left(\lambda_2\right)=\frac{\hat{\lambda}_2-\lambda_2}{\hat{\lambda}_2}\le 0.00871$.

In the hierarchical approach the number of parameters increases incrementally. The order of adding constants to the sought-after parameters group determines the sensitivity analysis, as described in p. 2.3. The sensitivity coefficient obtained by correlating the two natural frequencies with given parameter variations is presented in Table 1. Then, the

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following vectors of design variables were studied in successive steps of the hierarchical approach:

$\boldsymbol{b1} = \{k_\varphi, k_x\}^{\mathrm{T}}$, $\boldsymbol{b2} = \{k_\varphi, k_x, z_c\}^{\mathrm{T}}$, $\boldsymbol{b3} = \{k_\varphi, k_x, z_c, J_y\}^{\mathrm{T}}$, $\boldsymbol{b4} = \{k_\varphi, k_x, z_c, J_y, m\}^{\mathrm{T}}$.

At each stage of iterative calculations an optimization problem was solved. The internal penalty function scheme is employed.

Convergence of the optimization process is not sufficient to take the obtained result as a reliable one. Especially when dealing with under-posed tasks the results strongly depend on the starting point of optimization - outputs vary for different starting values. Therefore there should be an additional condition to verify the results. In this investigation they are assessed by calculating the Normalized Modal Difference (*NMD*) [1] between the first mode shape calculated for the model and the measured mode shape of the tower.

$$NMD = \sqrt{\frac{1-MAC}{MAC}} \cdot 100\%, \qquad MAC\left(\hat{\boldsymbol{\phi}}, \boldsymbol{\phi}\right) = \frac{\left|\hat{\boldsymbol{\phi}}^{\mathrm{T}} \boldsymbol{\phi}\right|^2}{\left(\hat{\boldsymbol{\phi}}^{\mathrm{T}} \hat{\boldsymbol{\phi}}\right)\left(\boldsymbol{\phi}^{\mathrm{T}} \boldsymbol{\phi}\right)}. \tag{19}$$

MAC stands for the Modal Assurance Criterion. The smaller the *NMD* value the better the mode shape is matched.

2.5. Results

The identification results, along with the starting values and boundary conditions are presented in Table 2. A low, 4÷6%, *NMD* was obtained for each approach. That means the assumed model along with determined parameters is a good approximation of the real structure's behaviour. The *NMD* values calculated in each case can be assumed to be indicators of how effective an optimization is. Should the number of unknowns increase from two to four in the hierarchical approach the *NMD* values decrease. Consecutive values were 4.680, 4.539, 4.271% (see Tab.2). That means the results obtained were

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increasingly better. However, when the number of parameters increases to five, the *NMD* value increases to 6.326%. That case proves that there is an efficiency limit of the under-posed problems solving, depending on the number of unknowns. The case with five parameters was reattempted to be solved, based on four state variables and again a lower *NMD* value was reached of 4.666%. This test proves that the increment of state variables improves effectiveness of identification. The case with four state variables required more iteration steps (ten) than the task with two data (three) to reach the final criterion, because the criterion must be satisfied for more state variables at once.

Important elements of any optimization are the starting points and boundary conditions. If correctly selected the identification process should be stable. Starting points are determined based on preliminary calculations, as described in p. 2.3. Boundary conditions circumscribe results to physically possible ones. The starting values of z_c, J_y and m were similar in each case. On the other hand, the values of k_φ and k_x had to be changed in cases with four and five unknowns. In order to obtain a reliable result they had to be more comparable to expected final values. For example in a case with three unknowns the initial value of k_φ was 10^9 and the result obtained was $1.411 \cdot 10^{10}$, while in a case with four parameters the starting value had to be 10^{10} in order to obtain the resulting $1.391 \cdot 10^{10}$. Figure 6 shows convergence of the parameters k_φ and k_x for the case with four state variables.

In each case the calculated natural frequencies were almost the same as the measured quantities, according to small relative errors assumed acceptable as shown in Fig.7, where convergence of the state variables' relative errors is presented. A greater permitted error was assumed for mode shapes coordinates, however, the modes calculated were also similar to the measured ones (see Fig.4).

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3. Concluding remarks

This study has discussed the modelling and parametric identification of the historical tower in the Vistula Mounting Fortress. An iterative procedure has been proposed of parametric identification using the minimum square error function as an objective function for an optimisation problem. The procedure involves minimization of the objective function through employing the sensitivity analysis to approximate complex relations between parameter variations and the state variables variations .

An inverse under-posed problem has been reviewed. Five parameters of the tower model were identified based on two measured modal characteristics (natural frequencies). The result was verified with calculations based on four measured modal characters. A hierarchical process of the tower identification, depending on incrementally increasing number of parameters converges to increasingly better model of the structure. The presented example proved there is a limit in relation unknowns-data for the under-posed problems as far as the identification efficiency is considered. In the example, the identification efficacy measured as the difference between measured and analytical first mode shape, started to decrease in case of five unknown parameters and two state variables (the data). Using four state variables improved the obtained result. The numerical example shows an efficacy and a good convergence of the iterative procedure proposed.

Worthy of attention is the relationship between the modal identification of a structure and identification of its model. The measured resonant frequencies of a structure are convenient state variables, as they can be quite easily measured. The mode shapes, on the other hand, can suggest the type of model for a given structure, which would represent it

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correctly. Forms of the first two lateral mode shapes suggest that a model of a rigid solid body resting on elastic foundations is adequate.

The scatters of the measured modal characteristics are other valuable data. They help to formulate the optimisation problem accurately. In presented example they were a basis for weight coefficients of the state variables specification and for determining the criterion for concluded calculations. The numerical example for the tower of Vistula Mounting Fortress proves the proposed modelling and identification of the structure to be practically viable. It should be emphasised that only weak ambient excitations caused by the river and wind were used in the modal identification process.

Figure Captions

Fig. 1. Vistula Mounting Fortress with the tower in the centre

Fig. 2. Plan of the tower with positions and orientation of the accelerometers

Fig. 3. A cross-spectrum of signals measured across the East-West plane at the tower top at two structural points situated on opposite walls; three resonant frequencies identified in this plane are marked

Fig. 4. Two first lateral mode shapes of the tower, measured and theoretical: (a) the first one in the West-East plane, (b) the first one in the North-South plane; planes of vibrations are presented in the cross-section of the tower

Fig. 5. Rigid solid body model resting on elastic foundation

Fig.6. Convergence of the parameters k_φ and k_x for the case with four state variables

Fig. 7. Convergence of the state variables' relative errors $\varepsilon^k\left(\lambda_i\right), i=1,2$ and $\varepsilon^k\left(\phi_{n1}\right), n=b,t$

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Acknowledgements

This work has been supported by the Centre for Urban Construction and Rehabilitation CURE, founded by the 5-th EU Framework Programme, acting at the Gdansk University of Technology in Poland – this support is kindly acknowledged.

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Table 1

Table 1. Relative sensitivity coefficients ($\overline{w}_{\lambda b}$) of natural frequencies squared (λ_1 and λ_2) to a given parameter (b) variations

b	$\overline{w}_{\lambda 1 b}$	$\overline{w}_{\lambda 2 b}$
z_C	-1.272	1.274
J_y	-0.791	-0.209
m	-0.209	-0.791

Table 2

Table 2. Results of the model parameters identification

		Two state variables approach				Four state variables approach
	Identified parameters / The parameter name	k_φ, k_x	k_φ, k_x, z_c	k_φ, k_x, z_c J_y	k_φ, k_x, z_c J_y, m	k_φ, k_x, z_c J_y, m
Starting values	k_φ [Nm]	$1.00 \cdot 10^9$	$1.00 \cdot 10^9$	$1.00 \cdot 10^{10}$	$1.00 \cdot 10^{10}$	$1.00 \cdot 10^{10}$
	k_x [N/m]	$1.00 \cdot 10^9$	$1.00 \cdot 10^9$	$4.00 \cdot 10^8$	$4.00 \cdot 10^8$	$4.00 \cdot 10^8$
	z_c [m]	10.760	10.00	10.00	10.00	10.00
	J_y [kg·m^2]	$4.340 \cdot 10^7$	$4.340 \cdot 10^7$	$5.00 \cdot 10^7$	$4.30 \cdot 10^7$	$4.30 \cdot 10^7$
	m [kg]	$9.202 \cdot 10^5$	$9.202 \cdot 10^5$	$9.202 \cdot 10^5$	$9.200 \cdot 10^5$	$9.000 \cdot 10^5$
Boundary conditions	k_φ [Nm]	100 - ∞	100 - ∞	100 - ∞	100 - ∞	100 - ∞
	k_x [N/m]	100 - ∞	100 - ∞	100 - ∞	100 - ∞	100 - ∞
	z_c [m]	-	0 – 15.00	0 – 15.00	0 – 15.00	0 – 15.00
	J_y [kg·m^2]		-	10^6- ∞	10^6- ∞	10^6- ∞
	m [kg]			-	(9-10) $\cdot 10^5$	(9-10) $\cdot 10^5$
Identification result	k_φ [Nm]	$1.439 \cdot 10^{10}$	$1.411 \cdot 10^{10}$	$1.391 \cdot 10^{10}$	$1.399 \cdot 10^{10}$	$1.239 \cdot 10^{10}$
	k_x [N/m]	$3.877 \cdot 10^8$	$3.948 \cdot 10^8$	$4.279 \cdot 10^8$	$4.114 \cdot 10^8$	$4.264 \cdot 10^8$
	z_c [m]	starting value	10.647	10.898	10.441	9.921
	J_y [kg·m^2]		starting value	$4.980 \cdot 10^7$	$4.357 \cdot 10^7$	$4.115 \cdot 10^7$
	m [kg]			starting value	$9.418 \cdot 10^5$	$9.202 \cdot 10^5$
Number of iteration steps		3	4	4	3	10
NMD [%]		4.680	4.539	4.271	6.326	4.666

Figure 1

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Figure 2

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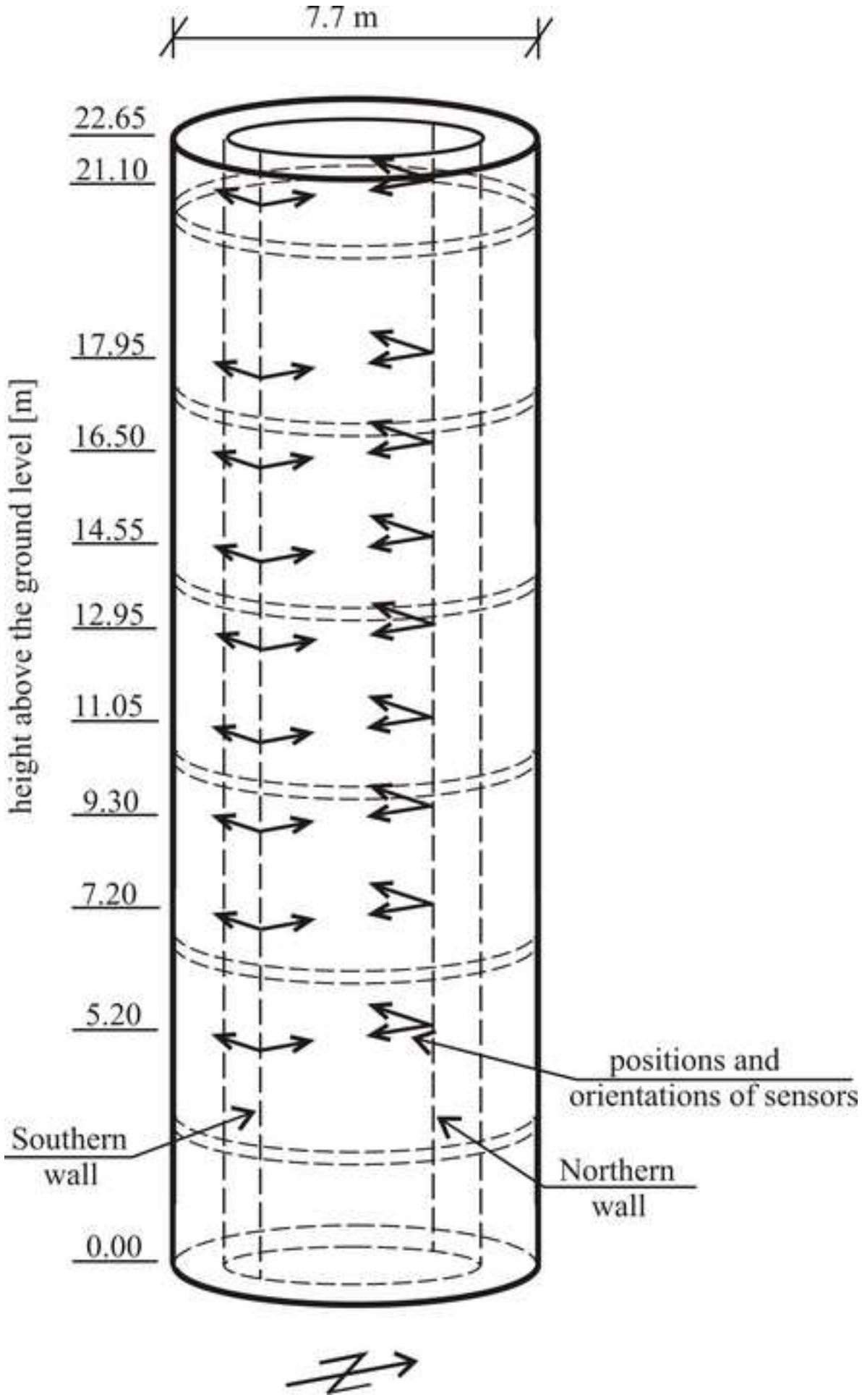

Figure 3

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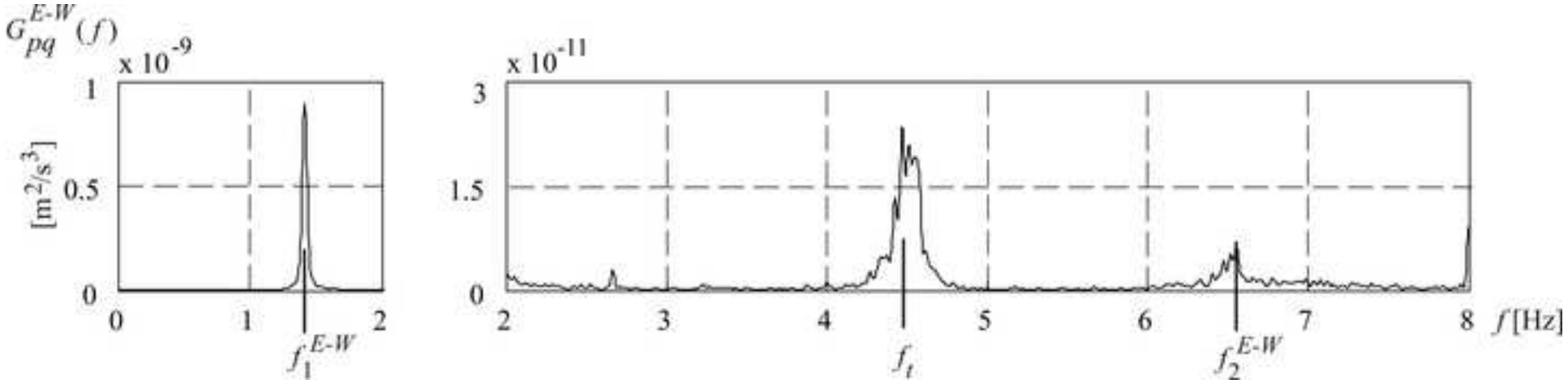

Figure 4a

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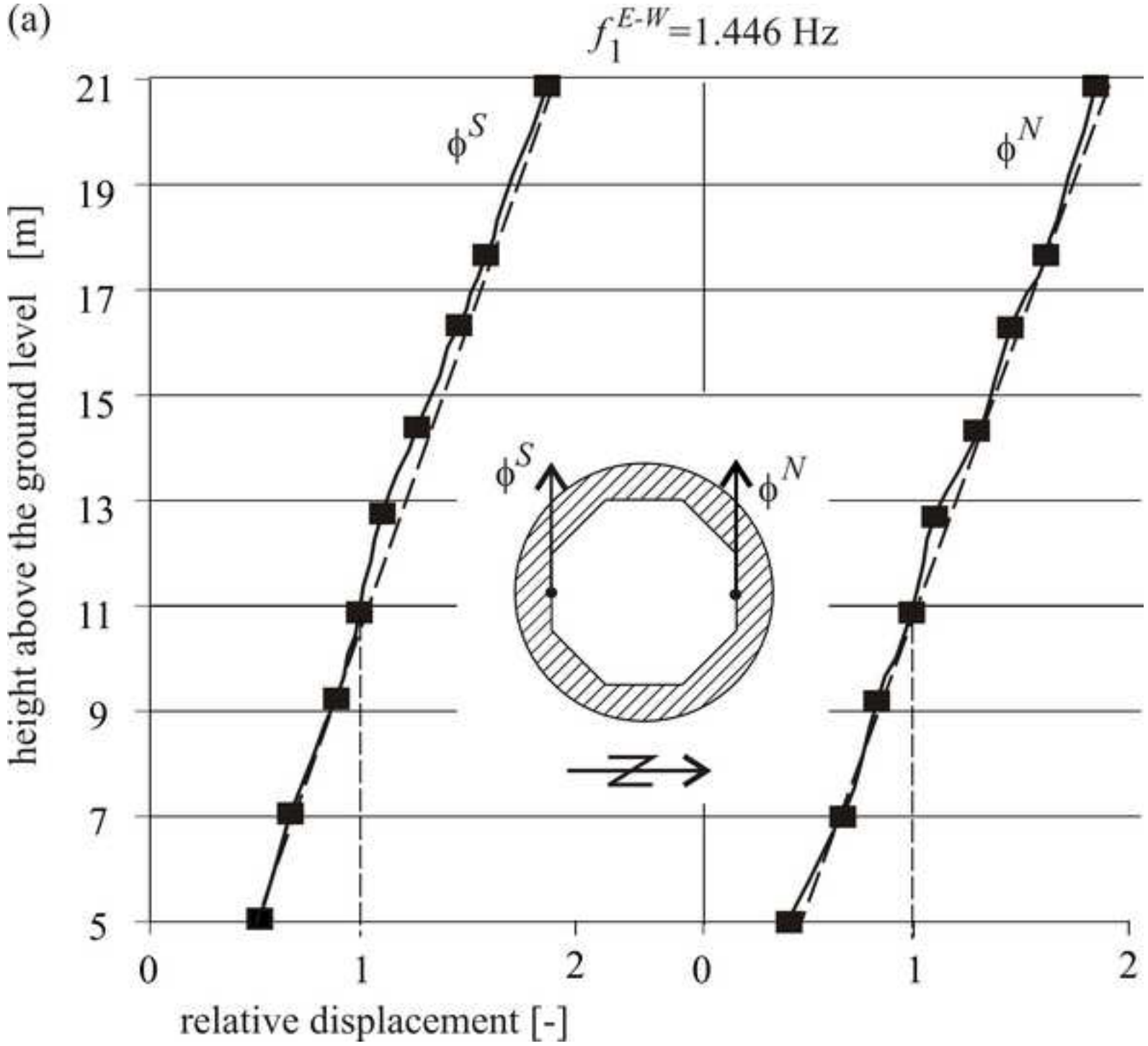

Figure 4b

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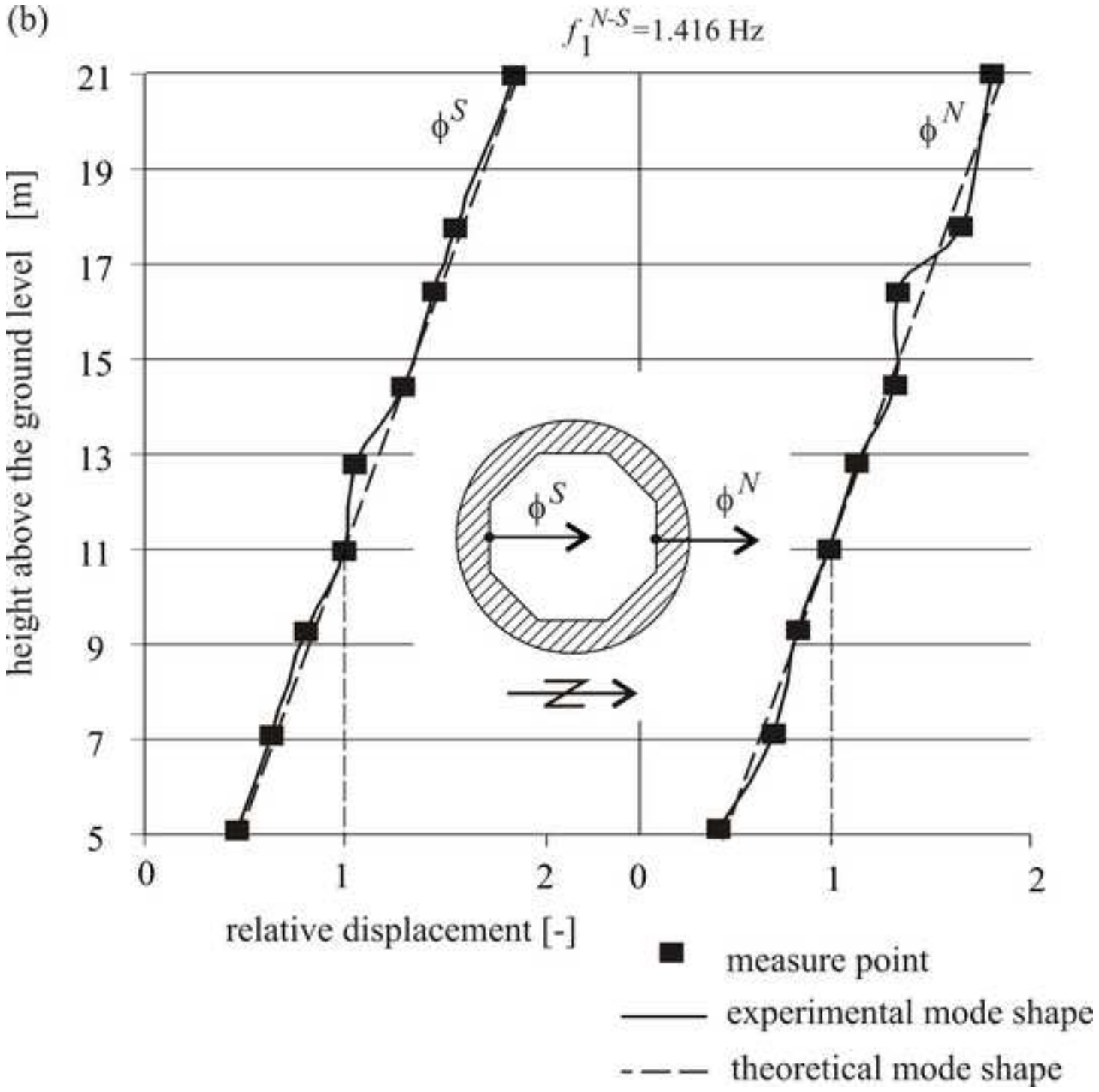

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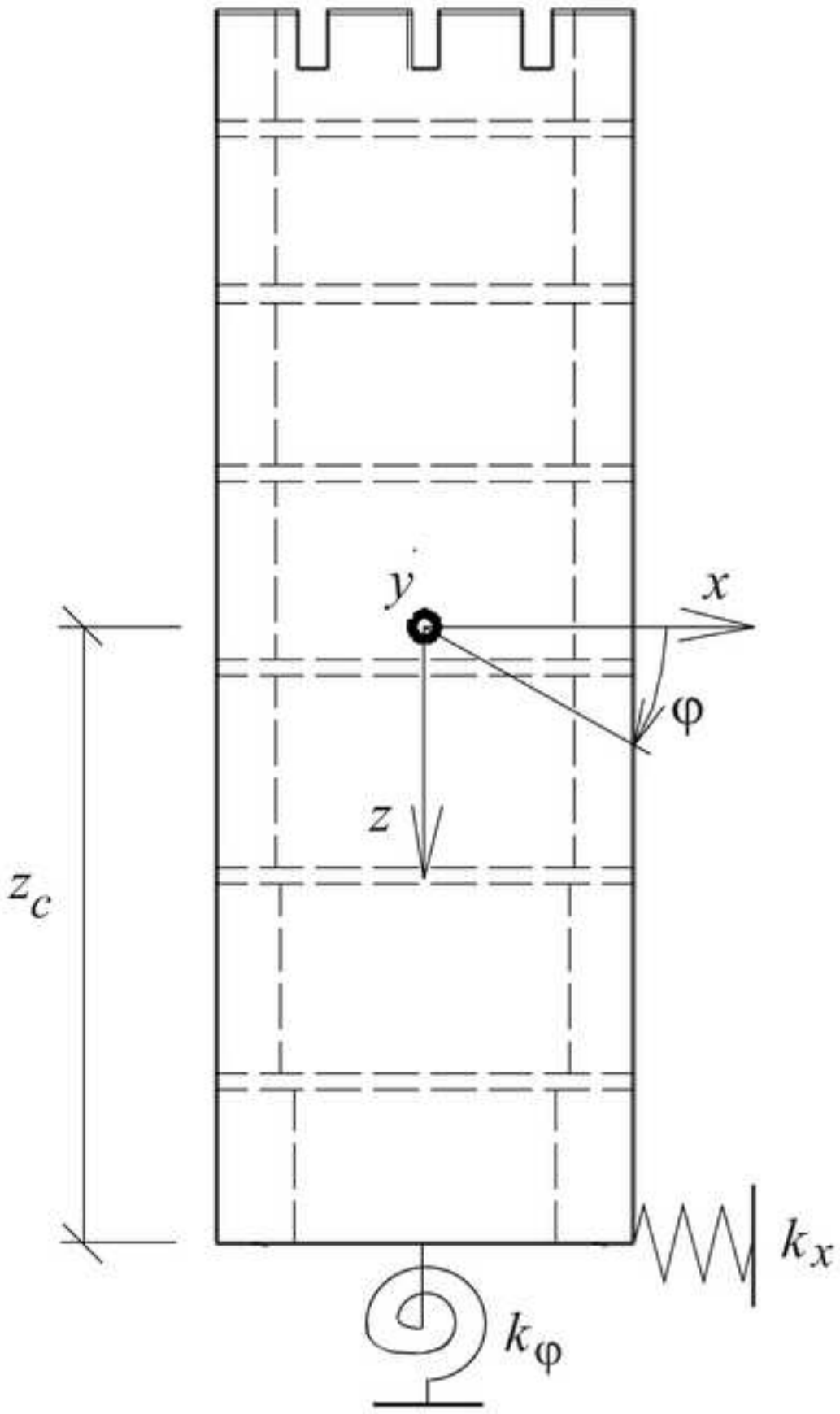

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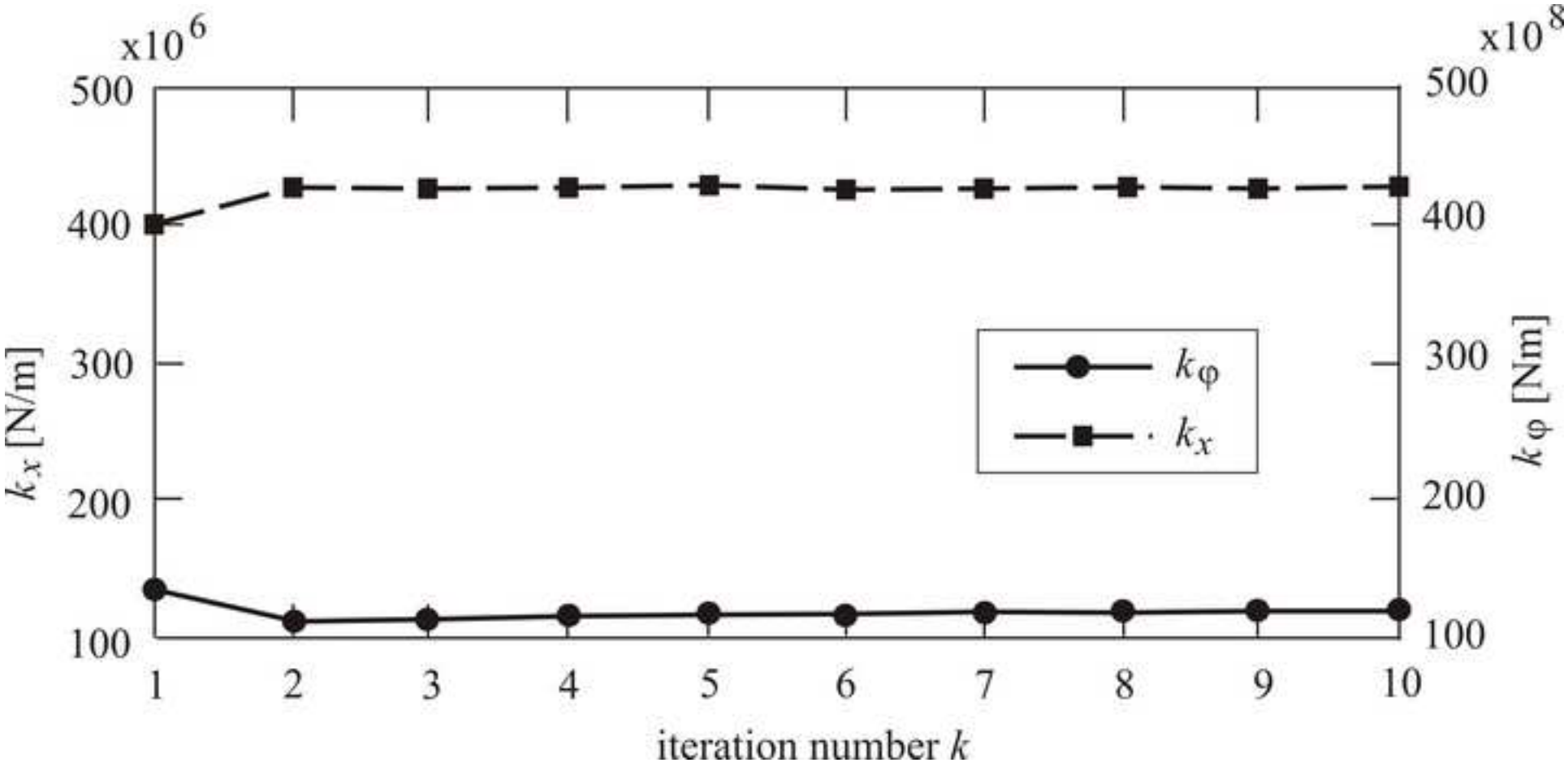
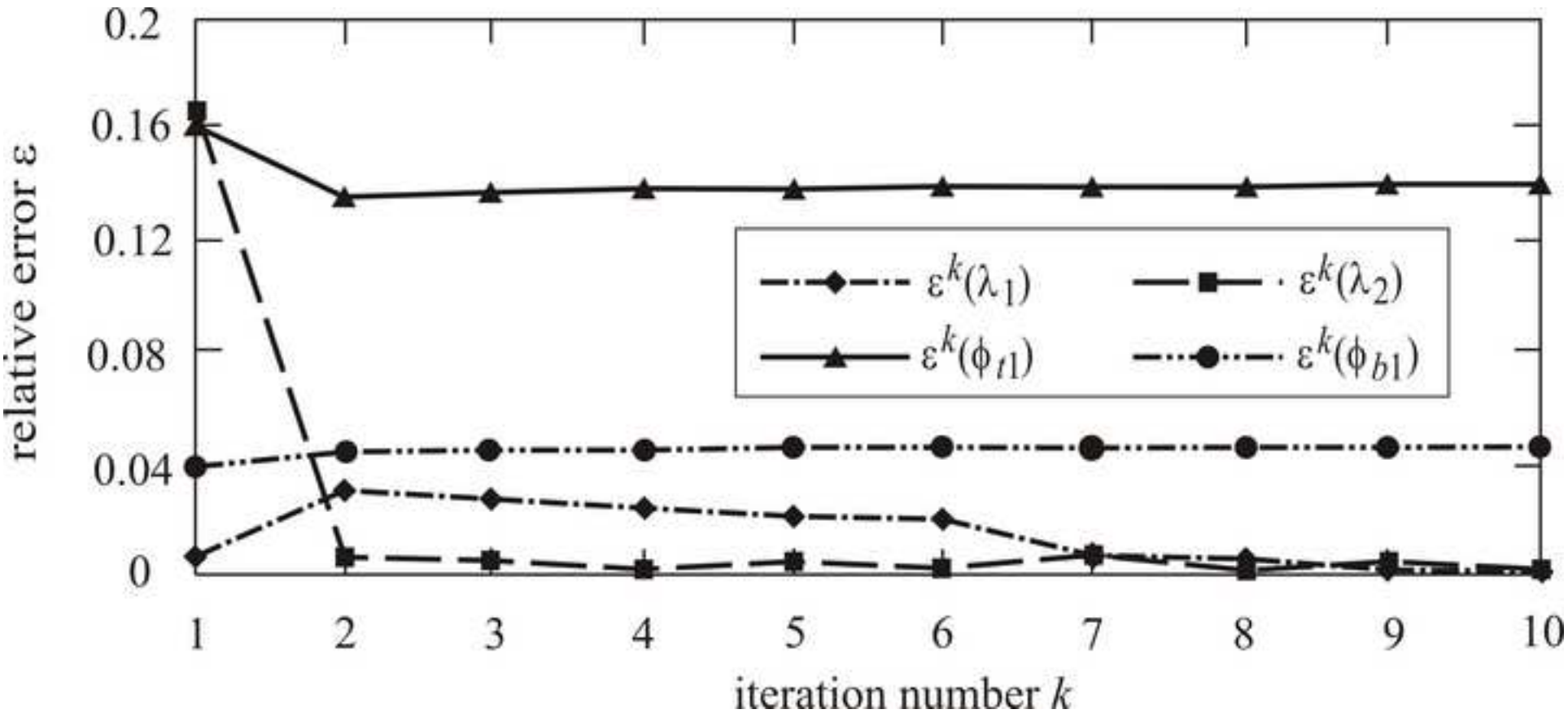
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