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# **IEEE 802.11 LAN CAPACITY: INCENTIVES AND INCENTIVE LEARNING**

For an ad hoc IEEE 802.11 WLAN we investigate how backoff attacks (configuring minimum and maximum CSMA/CA contention windows in pursuit of a larger-than-fair bandwidth share) affect a proposed capacity-fairness index *CFI*. If the backoff mechanism is mandatory, the CSMA/CA game that arises has a unique Nash equilibrium. In the more realistic opposite case there is no single compelling outcome; we envisage that a station then calculates backoff attack incentives to predict imminent play. We link *CFI* to the network size, "power awareness," a station's perception of the other stations' susceptibility to incentives, and the way of learning how the other stations perceive the other stations' susceptibility to incentives. We demonstrate that if the stations are few and "power aware," cooperative behavior emerges quite frequently.

### 1. INTRODUCTION

Estimated limits of network performance become the more realistic, the richer is the assumed network model; most existing estimates account for PHY-layer bandwidth, MAC and transport protocol overhead, DATA frame size, number of network stations, station mobility, and channel characteristics. In the context of an ad hoc IEEE 802.11 WLAN [8] we bring into the picture the stations' noncooperative behavior in the form of a *backoff attack*. Each station *n* is free to select an arbitrary CSMA/CA configuration  $w_n = \langle w_{n,\min}, w_{n,\max} \rangle$  (the minimum and maximum contention windows). This may lead to a larger-than-fair long-term bandwidth share [2, 5]. Perpetrators of backoff attacks cannot be detected, since an ad hoc WLAN station remains virtually anonymous to non-recipients. We propose a capacity-fairness index (CFI), a synthetic performance measure equal to the product of the total goodput (bandwidth utilization) and the Jain index of the stations' bandwidth shares. CFI favors cooperative station behavior and compensates for the contention overhead such behavior entails. To calculate CFI we reflect the dynamics of CSMA/CA at saturation using a Monte Carlo-oriented Markovian model; this alleviates the "decoupling approximation" of analytical models [4, 11, 14] and allows arbitrary CSMA/CA configuration profiles  $(w_1, \ldots, w_N)$ . Our approach therefore consists in taking the functional dependence of IEEE 802.11 performance metrics on  $w_n$  as an empirical fact rather than deriving it from some analytical model.

We next define *CFI* for the case where each station pursues a maximum payoff (bandwidth share) in a noncooperative *N*-player *CSMA/CA game* [1, 5, 11, 12, 15]. It can be shown that if the *greedy* <1, 1> configuration is excluded (the backoff mechanism is mandatory) then the game has a unique Nash equilibrium (NE) [7]. Otherwise, one disincentive to configure <1, 1> is a certain "power awareness" i.e., fear of another station also configuring <1, 1>, for all the transmission

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power is then spent on frame collisions. This we reflect as a nonpositive "penalty bandwidth share." With payoffs so modified, the CSMA/CA game has multiple Nash equilibria. In the absence of a compelling unique NE one needs to calculate the chances of particular CSMA/CA configuration profiles. We propose a simple calculus of backoff attack incentives, a form of seeking a best reply to the beliefs as to the other stations' imminent play. We link *CFI* to the network size, the stations' "power awareness," a station's perception of the other stations' susceptibility to incentives, and a station's way of learning how the other stations perceive the other stations' susceptibility to incentives. In particular, we demonstrate via Monte Carlo simulation that for small enough networks and "power aware" enough stations, cooperative behavior may ultimately emerge. Although the focus in this paper is on IEEE 802.11, the presented approach can easily be extended to other existing or future contention MAC protocols.

In Sec. 2 we outline the network model and Monte Carlo-oriented Markovian approximation of saturated CSMA/CA. Sec. 3 discusses how the Prisoners' Dilemma payoff structure is altered by bringing the greedy configuration into the picture and describes the proposed method of calculating incentives. In Sec. 4 we introduce the noncooperative *CFI* and noncooperative learning *CFI* to predict the effect of conscious (incentive-driven) backoff attacks upon the network performance. Sec. 5 concludes the paper.

### 2. NETWORK MODEL AND CFI

For an ad hoc IEEE 802.11 WLAN with *N* stations using basic access CSMA/CA, assume: (A1) a single-hop network topology and error-free channel, (A2) saturation load (each station is always ready to transmit a long DATA frame), and (A3) fixed-length DATA frames. Assumption (A1) serves to factor out inefficiency due to transmission errors and hidden stations; error freedom also invalidates selfish selection of the data rate [15]. Assumption (A2) is motivated by our concern about noncooperative behavior (under light or moderate load it is pointless, as each station mostly gets the required bandwidth); moreover, the total goodput (bandwidth utilization) is then close to the network capacity. Assumption (A3) simplifies the calculation of the stations' bandwidth shares, owing to a fixed duration of a DATA frame collision. It can easily be relaxed [4].

The dynamics of CSMA/CA at saturation load lead to a simple Monte Carlo simulationoriented Markovian model. The contention window and backoff counter constitute station *n* state at the *k*th instant immediately following a backoff slot or termination of a DATA frame transmission:  $S_n^k = [CW_n^k, BC_n^k]$ , with  $w_{n,\min} \le CW_n^k \le w_{n,\max}$  and  $0 \le BC_n^k < CW_n^k$ ). Then  $((S_1^k, ..., S_N^k))_{k=1,2,...}$  is a 2*N*-dimensional Markov chain whose one-step state transitions are governed by the principles of CSMA/CA:

$$S_{n}^{k+1} = \begin{cases} [CW_{n}^{k}, BC_{n}^{k} - 1], \text{ if } BC_{n}^{k} > 0 \text{ and } BC_{-n}^{k} > 0 \\ S_{n}^{k}, \text{ if } BC_{n}^{k} > 0 \text{ and } BC_{-n}^{k} = 0 \\ [W_{n,\min}, U(w_{n,\min})], \text{ if } BC_{n}^{k} = 0 \text{ and } BC_{-n}^{k} > 0 \\ [\min(2CW_{n}^{k}, w_{n,\max}), U(\min\{2CW_{n}^{k}, w_{n,\max}\})], \text{ if } BC_{n}^{k} = 0 \text{ and } BC_{-n}^{k} = 0, \end{cases}$$
(1)

where  $BC_{-n}^{k} = \prod_{m \neq n} BC_{m}^{k}$  and U(j) is a random integer from  $\{0, \dots, j-1\}$ . The Markov chain version of the strong law of large numbers states that the ergodic probability of a given state is almost surely the Cesaro limit of the current visit count at that state [6]. That limit can be

approximated by pursuing the evolution of  $S_n^k$ , leading to statistically credible estimates of the form

$$e = \lim_{k \to \infty} \frac{\mathbf{1}_{E^1 = 0} + \dots + \mathbf{1}_{E^k = 0}}{k},$$
(2)

where **1** is the indicator function and  $E^k$  depends on the  $BC_n^k$  relative to a specific estimate. If *e* is station *n* frame transmission rate  $t_n$  [incurred collision rate  $c_n$ ] then  $E^k = BC_n^k$ [ $E^k = BC_n^k + BC_{-n}^k$ ], whereas if *e* is the total frame transmission rate *T* then  $E^k = BC_n^k \cdot BC_{-n}^k$ . Station *n* success rate (per busy backoff slot) is  $s_n = t_n \cdot (1 - c_n)/T$ . Following the standard renewal process argument, station *n*'s bandwidth share (goodput of successful DATA frames normalized to the PHY-layer bandwidth) can be expressed as [4]:

$$b_n = \frac{\tau_{\text{DATA}\_\text{payload}} \cdot s_n}{\tau_{\text{DATA}+\text{DIFS}} - \tau_{\text{slot}} + \tau_{\text{slot}} / T + \tau_{\text{SIFS}+\text{ACK}} \cdot \sum_{m=1}^N s_m},$$
(3)

where  $\tau_{(.)}$  is the duration of a specified frame transmission or time interval. Note that in existing IEEE 802.11 parameter settings,  $\tau_{DATA+DIFS} > \tau_{slot}$  [4].

A proposed measure that reflects both the total goodput (bandwidth utilization) and fairness is the capacity-fairness index (*CFI*), equal to the product of  $\Sigma_n b_n$  and the Jain index of bandwidth shares [9]:

$$CFI = \left(\sum_{n=1}^{N} b_n\right) \cdot \frac{\left(\sum_{n=1}^{N} b_n\right)^2}{N \cdot \sum_{n=1}^{N} b_n^2},\tag{4}$$

*CFI* favors cooperative behavior in that a single station monopolizing the bandwidth with no contention overhead typically produces a smaller *CFI* than *N* stations sharing the bandwidth equally and so incurring a significant contention overhead.

#### 3. BACKOFF ATTACK INCENTIVES

Let us complement the network model with two more assumptions: (A4)  $w_{min}$  and  $w_{max}$  are user configurable, and (A5) a station remains anonymous to non-recipients in that, based on a set of on the medium, a non-recipient cannot reliably detect that any two sensed frames have a common sender. Assumption (A4) sets the scene for backoff attacks and assumption (A5) ensures impunity for their perpetrators, since it renders backoff attacks undetectable by means of statistical traffic analysis.

With each station *n* free to select its  $w_n$  configuration, a noncooperative *N*-player CSMA/CA game arises in which the bandwidth shares (3) are payoffs. A likely outcome with rational players is a *Nash equilibrium* (NE) [7, 10], a configuration profile where each station has selected a best reply to the other stations' configuration profile, hence no station wants to deviate unilaterally.

If the *greedy* configuration <1, 1> is ruled out (i.e., the backoff mechanism is mandatory) then one can prove that there is no use playing less selfishly than  $w_s = <2$ , 2>. Consequently, the

configuration profile all- $w_s = (w_s, ..., w_s)$  is a unique NE [11]. One thus envisages a scenario where x stations out of N are *selfish* and configure  $w_s$  (have the capability and desire to tamper with  $w_n$ ), and N - x are *honest* and stick to a standard-prescribed  $w_h$ . Let the respective bandwidth shares be denoted by  $b_s(N, x)$  and  $b_h(N, x)$ .

Table 1 presents  $b_h(N, x)$  and  $b_s(N, x)$  for 54 Mb/s IEEE 802.11a with  $w_h = <16$ , 1024> and 1500-byte DATA frames. We see that  $b_h(N, 0) > 0$  for not too large N (e.g.,  $b_h(N, 50) \approx 0.9\%$ ) and  $b_h(N, x) \approx 0$  for x > 0 and any N. Note that  $b_s(N, x + 1) > b_h(N, x)$  i.e., a station always benefits by switching from honest to selfish. Yet  $b_s(N, x)$  decrease in x with  $b_s(N, x) < b_h(N, 0)$  for large enough x; in particular,  $b_s(N, N) < b_h(N, 0)$ . Similar conclusions hold for any existing IEEE 802.11 setting. Such a payoff structure fits the description of an N-player Prisoners' Dilemma [16].

	$b_{\rm h}(N, x)$ and $b_{\rm s}(N, x)$ , % of PHY bandwidth					
x	N = 10		N = 20		N = 50	
0	5.3		2.5		0.9	
1	0	68.0	0	67.4	0	65.7
2	0	18.3	0	18.3	0	18.1
3	0	11.2	0	11.2	0	11.1
4	0	7.6	0	7.6	0	7.6
5	0	5.7	0	5.7	0	5.7
10		2.3	0	2.3	0	2.3
20				1.0		1.0
50						0.3

Table 1. Bandwidth shares from Monte Carlo model (95% confidence intervals are within 1% of sample averages)

Suppose now that  $w_g = <1$ , 1 > i.e., disengaging the backoff scheme, is allowed. A station that is the only one to configure  $w_g$  cuts off the other stations regardless of their configurations and so incurs no contention overhead; hence, it enjoys the highest possible bandwidth share  $b_G = \tau_{DATA_payload}/\tau_{DIFS+DATA+SIFS+ACK}$ . (For a 54 Mb/s IEEE 802.11a,  $b_G = 69.5\%$ .) However, selecting *any* configuration in the presence of another station configuring  $w_g$  yields a zero bandwidth share. It is therefore reasonable to confine feasible configurations to  $w_g$ ,  $w_s$ , and  $w_h$  (some stations play greedy, some play as selfish as possible without cutting off all the other stations, and the rest are honest). Let  $b'_s(N, x, y)$ ,  $b'_g(N, x, y)$ , and  $b'_h(N, x, y)$  be a selfish, greedy, and honest payoff if x, y, and N - x - y stations configure  $w_s$ ,  $w_g$ , and  $w_h$ , respectively. Besides the received bandwidth these payoffs should reflect the fact that greedy stations cut each other off from the medium, while spending considerable transmission power on DATA frame collisions. Let a greedy (though "power aware") station in this case perceive a "penalty bandwidth share"  $b_C \le 0$ . Assume that "power awareness" does not reflect upon selfish or honest stations (which either receive nonzero bandwidth shares or are cut off and so spend no power on frame transmission). The stations' payoffs now are:

$$b'_{g}(N, x, y) = \begin{cases} b_{G}, \text{ if } y = 1\\ b_{C}, \text{ if } y > 1 \end{cases} \quad b'_{s[h]}(N, x, y) = \begin{cases} b_{s[h]}(N, x), \text{ if } y = 0\\ 0, \text{ if } y > 0. \end{cases}$$
(5)

Is a station compelled to configure  $w_g$ , as it is to configure  $w_s$  if  $w_g$  is ruled out? There are reasons to believe the configuration profile all- $w_g = (w_g, ..., w_g)$  is likely but not compelling. First, stations engaged in multiparty communication may not wish to cut off the other parties. Second, one may argue against all- $w_g$  on game-theoretic grounds. If  $b_c < 0$  then any configuration profile with y = 1 is a NE; indeed,  $w_g$  is the best reply to any configuration profile that does not contain  $w_g$ , and the worst reply to any configuration profile that does. If  $b_c = 0$  then not only all- $w_g$ , but also any configuration profile with y > 0 is a NE. Thus the game is no longer a Prisoners' Dilemma. We predict particular CSMA/CA configuration profiles by calculating *backoff attack incentives*.

A backoff attack incentive can be evaluated on the assumption that a station considering selfish or greedy play in near future knows all the payoffs  $b'_{s}(N, x, y)$ ,  $b'_{g}(N, x, y)$ , and  $b'_{h}(N, x, y)$  for the present N. These it can obtain offline from the Monte Carlo model. Note that although assumption (A5) precludes exact knowledge of N, it can be estimated based on observation of  $b_{h}(N, 0, 0)$  whenever all the stations play honest.

**Definition 1**: A selfish or greedy backoff attack incentive is measured as the ratio of the payoff that a station considers likely upon switching from  $w_h$  to  $w_s$  or  $w_g$ , respectively, and the honest payoff  $b'_h(N, 0, 0)$ .

Depending on what a station "considers likely," Definition 1 may translate into various numerical values. The simplest selfish and greedy incentive measures are:

$$I_{g,0} = \hat{b}'_g(N,0,1) = \hat{b}_G \quad I_{s,0} = \hat{b}'_s(N,1,0), \tag{6}$$

where the hats symbolize normalization with respect to  $b'_{h}(N, 0, 0)$ . One could term the above approach *naïve*, or *0-order sophisticated*, as it neglects similar calculations being done by the other stations (hence, assumes that they will stay at  $w_{h}$ ).

To depart from the naïve approach, a station must form a model of how the other stations' play is susceptible to the calculated incentives, and how the predicted play of the other stations is reflected in the calculated incentives. The former leads to the concept of a *susceptibility map*, whereas the latter relies on Definition 1, with "likely" defined in terms of statistical expectation.

A susceptibility map  $\Phi: \mathbf{R}^2_+ \to [0,1]^3$  returns for any pair  $(I_s, I_g)$  of selfish and greedy incentive measures the probabilities  $p_g$ ,  $p_s$ , and  $p_h$  of, respectively, switching to  $w_g$  and  $w_s$ , and staying at  $w_h$  by any other station, with the constraint  $p_g + p_s + p_h = 1$ .  $\Phi$  should be continuous and agree with the following intuitions:

$$p_{\rm g}$$
 increases in  $I_{\rm g}$  and is insensitive to  $I_{\rm s}$ ,  
 $p_{\rm s}$  increases in  $I_{\rm s}$ , (7)  
 $p_{\rm h}$  decreases in both  $I_{\rm s}$  and  $I_{\rm g}$ ,

The insensitivity of  $p_g$  to  $I_s$  is motivated by the greedy payoff being insensitive to the number of selfish (as well as honest) stations.

Although the susceptibility map may vary from station to station, it is natural for a station to form a rough estimate of incentive measures assuming that the other stations' susceptibility maps are the same as its own. Both backoff attack incentive measures can then be calculated as the expected payoffs b' normalized with respect to  $b'_{\rm h}(N,0,0)$  (therefore appearing with hats):

$$I_{s,1} = \sum_{\substack{x,y,z \ge 0 \\ x+y+z \le N-1}} \frac{(N-1)!}{x! \, y! \, z!} \, p_s^x \, p_g^y \, p_h^z \cdot \hat{b}'_s(N, x+1, y) = \sum_{\substack{x=0 \\ x=0}}^{N-1} \binom{N-1}{x} p_s^x \, p_h^{N-1-x} \cdot \hat{b}_s(N, x+1)$$

$$I_{g,1} = \sum_{\substack{x,y,z \ge 0 \\ x+y+z \le N-1}} \frac{(N-1)!}{x! \, y! \, z!} \, p_s^x \, p_g^y \, p_h^z \cdot \hat{b}'_g(N, x, y+1) = \hat{b}_G \cdot (1-p_g)^{N-1} + \hat{b}_C \cdot [1-(1-p_g)^{N-1}],$$
(8)

where  $(p_g, p_s, p_h) = \Phi(I_{s,0}, I_{g,0})$ . This approach can be termed *1-order sophisticated*, as it does account for the other stations also calculating incentive measures, though neglects their use of  $\Phi$ .

One can imagine sophistication of higher orders, considering that (8) has the form  $(I_{s,1}, I_{g,1}) = F(N, (I_{s,0}, I_{g,0}))$ . Upon applying (8), a station substitutes the calculated incentive measures  $I_{s,1}$  and  $I_{g,1}$  for  $I_{s,0}$  and  $I_{g,0}$ , and re-applies (8) to account for the other stations using  $\Phi$ , then repeats these steps to account for the other stations accounting for the other stations using  $\Phi$  etc. Hence, it recursively applies (8) i.e., takes,  $(I_{s,k}, I_{g,k}) = F(N, (I_{s,k-1}, I_{g,k-1}))$ . In game-theoretic terms, continuation of the above steps ad infinitum implies that  $\Phi$  is taken to be *common knowledge* [7]. Depending on  $\Phi$  and the payoff values, the sequence  $((I_{s,0}, I_{g,0}), (I_{s,1}, I_{g,1}), \ldots)$  may or may not converge. However, if it does converge, the limiting  $(I_{s,\infty}, I_{g,\infty})$  solves:

$$(I_{\mathrm{s},\infty}, I_{\mathrm{g},\infty}) = F(N, (I_{\mathrm{s},\infty}, I_{\mathrm{g},\infty})).$$
(9)

We propose to take the solution of the fixpoint-type equation (9) as the  $\infty$ -order sophisticated incentive measures regardless of whether the convergence occurs. The following proposition (where, as noted above, condition (ii) is satisfied in existing IEEE 802.11 settings) states that (9) does admit a solution, which is unique.

**Proposition**: Let (i)  $\Phi$  agree with the intuitions (7), and (ii)  $b_s(N, x)$  decrease in x for all N. Then (9) has a unique solution  $(I_{s,\infty}^o, I_{g,\infty}^o)$  regardless of N and  $b_c$ .

*Proof*: We offer semi-formal arguments to avoid uninformative rigor. The right-hand side of the first equation in (9) is continuous and decreases in  $I_{s,\infty}$ . To see the latter, it is convenient to regard the binomial coefficients at the  $b_s(N, x+1)$  as weights summing up to  $(p_s + p_h)^{N-1} = (1 - p_g)^{N-1}$ . This sum being insensitive to  $I_{s,\infty}$ , and the weights themselves growing relatively faster with  $I_{s,\infty}$  for larger x, one can represent the right-hand side for a smaller and a larger  $I_{s,\infty}$  in the form of convex linear combinations, namely  $\sum_x v_x \cdot b_s(N, x+1)$  and  $\sum_x v_x \cdot \xi_x \cdot b_s(N, x+1)$  respectively, where  $v_x$ ,  $\xi_x \ge 0$ ,  $\sum_x v_x = \sum_x v_x \cdot \xi_x = 1$ , and  $\xi_x$  increases in x. That is, the contribution of the  $\hat{b}_s(N, x+1)$  with larger x increases in  $I_{s,\infty}$ , thus decreasing the whole expression. Consequently, given an  $I_{g,\infty}$  we get a unique solution for  $I_{s,\infty}$  i.e., a well-defined implicit function  $I_{s,\infty} = f_1(I_{g,\infty})$ . The right-hand side of the second equation in (9) decreases in  $p_g$ , therefore in  $I_{g,\infty}$ , while being insensitive to  $I_{s,\infty}$ . The implicit function it defines,  $I_{g,\infty} = f_2(I_{s,\infty})$ , is thus constant. One concludes that the (continuous) curves along which the two sides of each equation balance will intersect exactly once when projected onto the  $(I_{s,\infty}, I_{g,\infty})$  plane.

An example of a feasible susceptibility map  $\Phi$  is:

$$p_{g} = \varphi(I_{g,\infty}), \ p_{s} = \varphi(I_{g,\infty} + I_{s,\infty}) - \varphi(I_{g,\infty}), \ p_{h} = 1 - \varphi(I_{g,\infty} + I_{s,\infty}).$$
(10)

Here, the susceptibility function  $\varphi: \mathbf{R}_+ \to [0, 1]$  measures a station's willingness to switch from  $w_h$  to  $w_s$  or  $w_g$ , given  $I_s$  and  $I_g$ . It is continuous and nondecreasing, and by convention,  $\varphi(0) = 0$  and  $\varphi(\infty) = 1$ . Note that (10) therefore agrees with (7).

Other common-sense properties of (10) are:

- $p_s = 0$  if  $I_{s,\infty}$  is small enough and  $p_s = \varphi(I_{s,\infty})$  if  $I_{g,\infty}$  is small enough (analogous property holds for  $p_g$ ),
- $p_s + p_g \rightarrow 1$  if  $I_{s,\infty} \rightarrow \infty$  and  $I_{g,\infty} \rightarrow \infty$  (i.e.,  $p_h$  then tends to zero), and

•  $p_{\rm s} = p_{\rm g}$  if  $I_{{\rm s},\infty} = I_{{\rm g},\infty}$ .

#### 3. NONCOOPERATIVE CAPACITY-FAIRNESS INDICES

By (4),  $CFI = N \cdot b_h(N, 0)$  for cooperative behavior; denote this value by c-*CFI*. At saturation load and in the absence of selfish or greedy stations, c-*CFI* approximates the WLAN capacity. On the other hand, if the CSMA/CA game is played, we use the probabilities of configuring  $w_g$ ,  $w_s$ , and  $w_h$ , determined by  $\infty$ -order incentives.

**Definition 2**: The noncooperative CFI is the expected value of (4) with respect to the probabilities (10) calculated from (9) for  $\infty$ -order incentives:

$$\mathbf{n} - CFI = \mathbf{c} - CFI \cdot p_{\mathrm{h}}^{N} + N \cdot p_{\mathrm{g}} \cdot (1 - p_{\mathrm{g}})^{N-1} \cdot \frac{b_{\mathrm{G}}}{N} + \sum_{x=1}^{N} {N \choose x} p_{\mathrm{s}}^{x} \cdot p_{\mathrm{h}}^{N-x} \cdot \frac{x}{N} \cdot x \cdot b_{\mathrm{s}}(N, x), \quad (11)$$

(recall that if exactly x out of N bandwidth shares are nonzero and equal then the Jain index is x/N [9]).

Fig. 1 compares c-*CFI* and n-*CFI* for various  $b_C$  ("power awareness" levels), including  $b_C = -b_G$  ("highly power-aware") and  $b_C = 0$  ("power-unaware"). As the susceptibility function  $\varphi$  we take

$$\varphi(I) = 1 - e^{-aI}, \tag{12}$$

with the steepness parameter a = 1. Both capacity-fairness indices deteriorate as N grows, but the discrepancy between them remains distinct all along and becomes huge for  $N \ge 10$ . Taking a  $b_C$  closer to zero enhances the picture, n-*CFI* visibly becoming the smallest for "power-unaware" stations (which feel the most incentives to configure  $w_g$ ). It seems that the observed performance deterioration further aggravates that resulting from hidden stations, higher-layer protocol overhead, and/or imperfect channel. Obviously, network goodput as determined from the network and protocol setting alone is by far inadequate when estimating network capacity; factoring in noncooperative (selfish or greedy) behavior driven by  $\infty$ -order incentives is more realistic.



Fig. 1. c-CFI and n-CFI for various "power awareness" levels.

Fig. 1 may be criticized on the following grounds. First, the choice of a common steepness parameter a = 1 at all the stations looks somewhat arbitrary; a network capacity estimate should abstract from a particular a. Second, when calculating  $p_g$ ,  $p_s$ , and  $p_h$ , a station assumes the function

 $\varphi$  to be common knowledge – in the lack of a unique (compelling) NE, this is a best reply to its beliefs as to the other stations' imminent play. In reality the other stations' susceptibility to incentives may differ from what is believed, but may be learned by repeatedly playing the CSMA/CA game and observing successive configuration profiles. Note that the latter task is possible even under assumption (A5): each station *n* can observe  $b_n$  and the total goodput  $\Sigma_m b_m$  (successful DATA frames followed by short ACK frames), as well as sense the presence or absence of empty backoff slots. It is therefore easy to recognize the cases y > 1 (with  $\Sigma_m b_m = 0$  and no empty backoff slots sensed), y = 1 (with  $\Sigma_m b_m = b_G$  and no empty backoff slots sensed), and y = 0 (with empty backoff slots sensed); in the latter case, x > 0 (with  $b_n \approx 0$ ) and x = 0 (with  $b_n > 0$ ) can also be distinguished.

In a simplified scenario, the longer y = 0 is observed, the more firm becomes station *n*'s belief that the other stations believe the other stations to be less susceptible to  $I_{g,\infty}$  (i.e., believe that the graph of  $\varphi$  rises, hence are more likely to configure  $w_g$ ). Such a "balance of threats" keeps station *n* from configuring  $w_g$  in the future, as it is the worst reply to a configuration profile with y > 0. (Formally, the solution of (9) then becomes smaller, as do  $p_g$  and  $p_s$ .) Similarly, observed x = 0raises the belief in the other stations' increased likelihood to configure  $w_s$ . Hence, in spite of the incentives to configure  $w_g$  and  $w_s$  it is possible that cooperative behavior (x + y = 0) ultimately emerges provided that the stations are sophisticated enough. In view of the above, Fig. 1 may be regarded as too pessimistic

We model these intuitions by manipulating the exponential susceptibility function (12) through the steepness parameter *a* (the graph of  $\varphi$  rising as *a* increases). In the *i*th instance of the repeated CSMA/CA game, station *n*'s perception of the other stations' susceptibility to incentives is thus reflected by  $a_n^i$ , with the following dynamics:

$$a_n^{i+1} = \max\{0, a_n^i + \delta_n(X^i, Y^i)\},\tag{13}$$

where the function  $\delta_n$  describes the learning process at station *n*, initial steepness parameter  $a_n^0$  is arbitrary (e.g., can be selected at random), and  $X^i$  and  $Y^i$  are the numbers of selfish and greedy stations. We have:

$$(X^{i}, Y^{i}) = \sum_{n=1}^{N} (X^{i}_{n}, Y^{i}_{n}),$$
(14)

where  $(X_n^i, Y_n^i)$ , n = 1, ..., N, are independent random vectors with the joint probability distribution

$$Pr((X_n^i, Y_n^i) = (x, y)) = \begin{cases} p_s, \text{ if } x = 1 \text{ and } y = 0\\ p_g, \text{ if } x = 0 \text{ and } y = 1\\ p_h, \text{ if } x = y = 0, \end{cases}$$

and  $p_s$  and  $p_g$  are calculated from (10) using the steepness parameter  $a_n^i$ .

Whether and how often cooperative behavior will emerge in the course of the repeated CSMA/CA game depends on the learning model (i.e., the  $\delta_n$ ,  $a_n^0$ , and  $\Phi$ ). One way of analyzing the influence of noncooperative behavior upon *CFI* is to fix a reference learning model and compare the outcome of the game for various *N* and  $b_c$ . For example, let  $\delta_n(x, 0) = -\Delta_n$  if x > 0,  $\delta(0, 0) = -r\Delta_n$ , and  $\delta(x, y) = \Delta_n$  if y > 0, where *r* is a constant and  $\Delta_n$  is proportional to  $a_n^0$  (thus relative

changes of the steepness parameter are the same at each station). Then  $((a_1^i, ..., a_N^i), i = 1, 2, ...)$  is an *N*-dimensional random walk with two absorbing states. Clearly, one of them is (0, ..., 0), since the solution of (9) then yields  $p_s = p_g = 0$  and  $p_h = 1$ . This corresponds to indefinitely long cooperative behavior of all the stations, producing c-*CFI*. At state  $(\infty, ..., \infty)$ , the solution of (9) yields  $p_g = 1$  and  $p_s = p_h = 0$ , hence this too is an absorbing state corresponding to indefinitely long greedy behavior and producing *CFI* = 0. Define  $a_{max}$  so that the corresponding solution of (9) yields  $p_g$  arbitrarily close to 1.

**Definition 3**: The noncooperative learning CFI is given by:

$$nl-CFI = \pi_N \cdot c-CFI, \tag{15}$$

where  $\pi_N$  is the probability of eventually reaching the cooperative absorbing state (0,...,0), given that at the game start each station *n* selects  $a_n^0 \in [0...a_{max}]$  at random.

Fig. 2 depicts nl-*CFI* assuming  $\Delta_n = 0.2 a_n^0$  and r = 2. The values of  $\pi_N$  were obtained via Monte Carlo simulation of (13) and (14), which explains the "ragged" look of the curves for large *N*. Compared to Fig. 1 we get distinctly higher network performance in the case of "power aware" stations. Our approach quantitatively illustrates a few intuitions:

- the network's ability to provide high and fair bandwidth shares to all stations, as measured by *CFI*, diminishes as *N* increases, partly on account of growing contention overhead, but mostly because of the stations' limited willingness to behave cooperatively; these two factors are graphically illustrated at N = 50 by the gray and colorless arrow, respectively,
- incentive calculus dictates that the willingness to behave cooperatively grow with "power awareness" for fear of spending all the transmission power on frame collisions without getting any bandwidth share; accordingly, *CFI* improves as  $b_{\rm C}$  goes negative,
- the prediction of network performance depends on a station's perception of the other stations' susceptibility to incentives, reflected by the susceptibility map Φ, and the learning process, reflected by the functions δ<sub>n</sub>, and
- each of the nl-*CFI* curves lies between the n-*CFI*<sub> $\infty$ </sub> and c-*CFI* curves; its bias towards the latter measures the chance  $\pi_N$  of emergence of cooperative behavior; this is almost certain for small enough *N* assuming enough "power awareness," and is hardly possible for large *N* and/or little "power awareness."



Fig. 2. c-CFI and nl-CFI for various "power awareness" levels.

### 3. CONCLUSION

The introduction of  $w_g$  and "power awareness" changes the CSMA/CA game into one with multiple Nash equilibria i.e., without a compelling outcome. We envisage that each station then calculates common-knowledge incentives to configure  $w_s$  and  $w_g$ , and the corresponding probability distribution of imminent configuration profiles. This permits to assess the effect of a conscious backoff attack scenario, where a station's configuration depends on the other stations' predicted susceptibility to incentives. It can also model a more realistic scenario in which the other stations' predicted susceptibility is learned by playing the game repeatedly. Using several versions of the proposed capacity-fairness index we have quantified the effect of noncooperative behavior and indicated the possibility of emergence of cooperative CSMA/CA configuration profile. Our approach can be extended to nonhomogeneous "power awareness" in order to study the coexistence of devices with diverse battery lifetimes, and can serve as a framework for analyzing the impact of noncooperative behavior under *any* contention MAC protocol that yields a similar payoff structure.

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