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INFRARED TECHNIQUES FOR NATURAL CONVECTION INVESTIGATIONS IN CHANNELS BETWEEN TWO VERTICAL, PARALLEL, ISOTHERMAL AND SYMMETRICALLY HEATED PLATES.

Witold M. Lewandowski* Michał Ryms Hubert Denda

Faculty of Chemistry, Department of Chemical Apparatus and Theory of Machines, Gdansk University of Technology, ul. G. Narutowicza 11/12, 80-233 Gdansk, Poland *wlew@pg.gda.pl

Keywords

natural convection, infrared imaging, vertical channels, vertical symmetrically heated plates

Abstract

The effect of the gap width between two symmetrically heated vertical, parallel, isothermal plates on intensity of natural convective heat transfer in a gas (Pr = 0.71) was experimentally studied using the balance and gradient methods. In the former method heat fluxes were determined based on measurements of the voltage and electric current supplying the heaters placed inside the walls. In the latter, heat fluxes were calculated from the temperature distribution in the air in the plane perpendicular to the surface of the heating plates. Temperature fields were visualised using a thermal imaging camera. The analysis was conducted on two parallel vertical plates of height H = 0.5 m and width B = 0.25 m with the heated surfaces facing each other. Vertical planes with peripherally open channels and three different distances s = 0.045, 0.08 and 0.18 m were created this way. The surface temperature of the heating plates t_w was changed every 5 K and set at $t_w = 30$, 35, 40, 45, 50, 55, 60, 65, 70, 75 and 80 °C, while the ambient temperature range was from 18 to 25 °C.

1. Introduction

Heat transfer by natural convection is not only a theoretically difficult research problem; it is also a tricky one to tackle experimentally. Measurements of temperature, velocity and heat fluxes, are very hard to make owing to their very small and continually changing values in moving convective streams. However, this method of heat transfer is attractive because it is reliable, simple and cost-effective, especially in construction, electronics and power engineering. Natural convection, on the other hand, despite the low intensity of heat exchange due to the large amount of heat transferred from the surfaces of buildings, industrial facilities, power transmission lines, etc., causes gigantic energy losses. An important, but rarely studied research topic relating to naturally occurring convection is heat transfer within a channel formed by two vertical walls. We come across this in heating technology (radiators), buildings and electronics, energetics, household devices, and many other situations.

Natural convection in a vertical plane channel is not an unequivocally defined problem [1], [2], as the following configurations of heat transfer within the channel can occur:

- vertical cavity (by Rayleigh-Bénard convection in a closed space) [3], [4], [5], [6], [7], [49],

- vertical flat gap partially heated [48] or opened (from the bottom, top or from all sides) [8], [9],

- vertical natural convection arrays for the following conditions:

- asymmetric (hot-cold, hot-adiabatic, warm-hot) isoflux [18], [21], [40], [41], [46], [48] or isothermal heating plates [10], [11], [12], [13], [14], [16], [21], [40], [41], [47], [50], [52], [53], [60]

- symmetric: isoflux [40], [41], [51] or isothermal heating plates [12], [21], [40], [41], [45],

- vertical plane channel with different wall temperature (hot-cold, hot-adiabatic, warm-hot) [16], [17], [26]

- vertical plane with an open-ended channel and isothermal, symmetrically heated walls [19], [20], [21],

Any of these configurations can be investigated theoretically (analytically: [21], [40], [38], numerically: [7], [11], [14], [19], [20], [24], [26], [27], [28], [32], [35], [36], [37], [48], [50], [52], [53], [58], [59]), experimentally: [7], [8], [12], [16], [18], [20], [29], [30], [31], [35], [39], [52], [53], [60] or tested by visual methods [42], [43], [49].

Within the context of the above division, the current work concerns the experimental and visual study of natural convection heat transfer within a vertical plane channel formed by two vertical, parallel, isothermal, symmetrically heated walls.

The first results of an experimental and theoretical study of natural convective heat transfer in a vertical channel were published in 1942 by Elenbass [21], who carried out investigations on square vertical plates. By analysing a simplified set of equations, and by adjusting constants to fit experimental data, he proposed the following equation for the Nusselt number as a function of the modified Rayleigh number Ra^* , called the Elenbass Rayleigh number:

$$Nu_0 = \frac{Ra^*}{24} \cdot \left(1 - e^{-35/Ra^*}\right)^{3/4} \tag{1}$$

This equation, confirmed by experimental studies, is valid for the range $10^{-1} < Ra^* < 10^5$ and for fairly short vertical plates in air. In addition, for $10^4 < Ra < 10^9$ it has two asymptotes: one for small values of the s/H ratio (fully developed flow regime) and the other for large values of s/H (boundary layer flow regime).

Further studies of natural convection in vertical open channels were published by Raithby and Hollands [22] and [23], and Aung, Fletcher and Sernas [17], who derived a different relation that also captures both limiting cases $s \to 0$ and $s \to \infty$. After modifying the relation (1), they obtained a new version of the Elenbass equation:

$$Nu_0 = \left(Nu_{fd}^m + Nu_{bl}^m\right)^{1/m}; \quad m = -1.9 \quad , \tag{2}$$

where Nu_{bl} is the Nusselt number for the boundary layer regime near the entrance and Nu_{fd} is the Nusselt number for fully developed flow throughout the flow passage along the greater part of the channel.

$$Nu_{\rm bl} = 0.62 \cdot (Ra^*)^{1/4}$$
, for $b \to \infty$ (3)

$$Nu_{\rm fd} = \operatorname{Ra}^*/24$$
 , for $b \to 0$, (4)

A comparison of equations (1) and (2) with Sparrow and Bahrami's experimental data [8] and Ormiston's numerical solutions [24] is given in [25].

Further research by Churchill and Usagi [45], performed for natural convective heat transfer within a vertical channel with isothermal, symmetrically heated rectangular plates, led to an equation similar to but simpler than (2):

$$Nu_0 = \left[\left(\frac{Ra^*}{24} \right)^{-m} + \left(0.59 \sqrt[4]{Ra^*} \right)^{-n} \right]^{-1/m} , \qquad (5)$$

which is valid for fairly short vertical plates in air and when $10^4 < Ra < 10^9$.

From the correlating procedure described by Churchill and Usagi [45], the exponent m in Eq. (5) is equal to approximately 2, so the relationship for the vertical channel with two isothermal, symmetrically heated surfaces takes the form:

$$Nu_0 = \left[\left(\frac{576}{Ra^*}\right)^2 + \frac{2.873}{\sqrt{Ra^*}} \right]^{-1/2},\tag{6}$$

In turn, Martin, Raithby and Yvanowich [25] focused on short, wide channels $H/b \le 10$, in which the proportion of fully developed natural convection is larger and interaction with the region below the bottom inlet into the channel cannot be neglected. As a result they proposed a modified version of the relationship between the Nusselt and Rayleigh numbers for the fully developed regime. It can be written thus:

$$\widetilde{Nu}_{fd} = \frac{\widetilde{Ra}}{6} \cdot \left(1 + \sqrt{1 + \frac{12}{\widetilde{Ra}}} \right) , \tag{7}$$

with two asymptotes:

$$\widetilde{Nu}_{fd} = \sqrt{\frac{\widetilde{Ra}}{3}} \qquad \text{for } \widetilde{Ra} \to 0 \quad , \tag{8}$$

$$\widetilde{Nu}_{fd} = \frac{Ra}{3} \qquad \text{for } \widetilde{Ra} \to \infty , \qquad (9)$$

Eq. (8) represents a new asymptote that accounts for the effect of upstream conditions. Because of the transition to the boundary layer regime, the range of Ra over which the Elenbass asymptote is valid is limited. Eq. (9) is an already known asymptote of the Elenbass equation (1), (4).

In order to span convective heat transfer from the fully developed to the boundary layer regime Martin, Raithby and Yvanowich [25] transformed Eq.3 into:

$$\widetilde{Nu}_{bl} = 0.62 \cdot \widetilde{Ra}^{1/4} \cdot \left(\frac{H}{b}\right)^{3/2} \tag{10}$$

Bar-Cohen and Rohsenow [40] and Bar-Cohen and Schweitzer [41] compared convective heat transfer in a vertical channel between two isothermal, symmetrically heated plates with other configurations, *i.a.* with the specific conditions of isothermal but asymmetrically heated plates and isoflux, and symmetrically and asymmetrically heated ones. No less important, however, is the optimisation of the distance between the heated plates s_{opt} from the intensification point of view for convective heat transfer $Nu_{0,opt}$. Hence, these researchers also provided optimum values of s_{pot} and $Nu_{0,opt}$ for each configuration of two vertical plates (symmetric, asymmetric, isothermal, isoflux) they examined.

For the case considered in this paper (two isothermal, symmetric, vertical plates), expecting heat transfer maximisation, they give the following optimal relations:

$$s_{\text{opt}} = 2.714 \ P^{-0.25} \quad \text{and} \ Nu_{0,\text{opt}} = 1.31$$
, (11)

The optimum values of the Rayleigh and Nusselt numbers, calculated for the channel from (11), are 54.3 and 1.31 respectively, [40], which exceeds the Elenbass optimum spacing by only 4 % [21].

Another research direction of convective heat transfer within a vertical channel between two parallel vertical plates leads to the analysis of the mechanisms of this phenomenon. These studies can be performed either by determining the values of the temperature and velocity fields in the channel numerically, or through their visualisation. With the values of these fields to hand it is then possible to determine the local heat transfer coefficients and their distribution to heat transfer surfaces.

The numerical modelling of convective structures was carried out by Baïri et al. [7], Herwig using DNS (direct numerical simulation) [10], Zoubir et al. [20], Salih [26] and many others [1], [2].

Visualisation research of this problem was carried out by Wright et al. [42]: they used smoke and interferometry technique images to visualise convective flow patterns within channels.

In turn, Guo, Song, Zhao attempted to investigate these structures using laser speckle photography [43]. Ambrosini, Paoletti and Tanda studied local heat transfer coefficients and isotherm patterns using Schlieren and holographic interferometry. The flow patterns in an asymmetrically heated vertical channel were shown by both flow visualisation techniques based on laser tomography and velocity field measurements in the plane of symmetry of the channel using the 2D - PIV (Particle Image Velocimetry) technique of Polidori et al. [55]. A laser Doppler velocimeter LDV and thermocouples were used by Kato et al. to determine velocity and temperature fields within the channels of electronic devices [44]. Those methods allowes to obtain mainly qualitative results, like convective flow paterns determinations,

separation of convective streem or its transformation into free bouiant chimney [42]. The other groups of methods are focused on studies which gives aspecially quantitative information about particular convection heat transfer problems [44], [55].

Above mentioned methods of temperature and velocity fields indication in natural convective heat transfer phenomenons however usually needs expensive devices and time consuming procedures. Alternatively to both method groups the infrared imaging method can be proposed. It was proviously introduced and tested with promising results [57] and can be to used for quantitative and qualitative studies as well.

The aim of the present work is to investigate convective heat transfer within a channel consisting of two parallel, vertical plates at three given spacings base of thermal infrared imaging method. These isothermal plates were heated symmetrically with ten different power heating settings. Local heat transfer coefficients and isotherm patterns were recovered by the procedure from [57]. Once the results have been found consistent with those of other researchers, they will offer a better understanding of the scientifically and technically very important issue of heat transfer in channels.

This study should alows to obtain the distributions of local heat transfer coefficients on isothermal vertical walls creating opan channels with different distances between heated surfaces s as well as the convective flow patterns between them.

The experiments were carried out at the Department of Chemical Apparatus and Theory of Machines, Faculty of Chemistry, Gdańsk University of Technology, and the results compared with those stated above.

2. Methodology

2.1. Testing device

The testing device used in this investigation is shown in Fig.1. It consists of two identical vertical plates, symmetrically heated to a constant isothermal temperature of both surfaces. Each plate is constructed from three aluminium sheets of height 0.495 m and width 0.247 m but with different thicknesses (0.012 m external, 0.010 m central and 0.010 m internal). Two flat heaters are placed inside each plate acording to scheme in Fig.2. The main heater (300 W) is sandwiched between the external aluminium sheet, a 0.006 m thick rezotex layer and the central sheets. The second, auxiliary heater (150 W) is placed between the internal aluminium sheet and the bottom rezotex layer (0.006 m).



Fig.1 The experimental device consisting of two perpendicular panels and a grid placed across the gap between them.

A cross-section of one of the two identical vertical heating plates used in this study is shown in Fig.2.

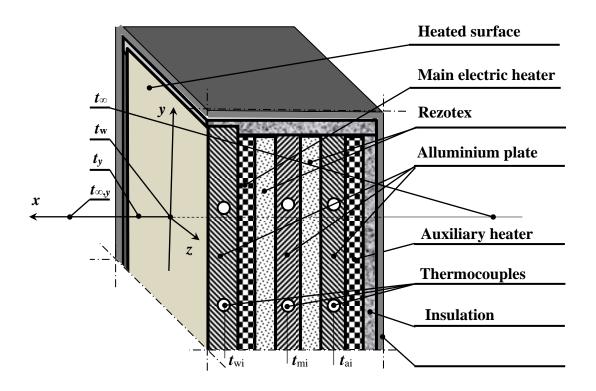


Fig.2 Cross-section of the upper left corner of the left-hand heated vertical plate from Fig.1.

On each experimental plate the values indicated by the thermocouples were taken to be the following temperatures: of the surface of the vertical heating plates t_w (average of 4 thermocouples placed within the external aluminium panels), of the surface of the rezotex plate on the side of the main heater t_m (4 thermocouples), and of the side of auxiliary one t_a (4 thermocouples). 32 thermocouples were used in the experimental device: 12 in one and another 12 in the second vertical heating plate, with the remaining 8 being distributed throughout the surrounding area undisturbed by heat transfer.

The regulation system automatically maintains the pre-set temperature of the heating surface t_w by adjusting the power of the main heater $N = U \cdot I$ and simultaneously that of the auxiliary heater, so that the temperature difference on both sides of the rezotex partition remains equal to zero $\Delta t = t_m - t_a = 0$. With $\Delta t = 0$ and narrow plates it can be assumed that the whole heater power from both heated surfaces will be transformed into a heat flux $Q = N_1 + N_2 = U_1 \cdot I_1 + U_2 \cdot I_2$, and then transferred by convection to the air inside the channel between the two vertical plates. The aluminium outer surfaces of both plates were polished in order to minimise heat transfer by radiation.

One of these plates, previously used to test the new method of measuring heat fluxes in air using a thermal imager, is described in detail in [57]. In the present study we use the same method of measuring the heat flux using a thermal imaging camera; hence, the theoretical basis and test procedures of this method are also given in [57]. Consequently, the method of measuring convective heat flows and heat transfer based on the visualisation of temperature fields of convective flow patterns in air using a thermal imaging camera in this work is described only briefly. Instead, we focus on the results that were obtained using it.

2.2. Description of the measurement method

Since air within the range of natural convection temperatures (0 - 100 °C) does not emit radiation in the mid-infrared (wavelengths from approx. 1 to 15 μ m) range of the camera's sensitivity, a plastic mesh was used in order to detect this radiation. The grid, placed parallel to the flow of the convective heat flux and perpendicular to the heating surface, has the temperature of the surrounding air. The heated mesh filaments are already visible to the IR camera. The low coefficient of thermal conductivity and the small diameter of the fibres prevents the equalisation of the temperature on the mesh surface. The frequency of convective streem fluctuations in the air is much greater than thermal inertia of the mesh, threfore detected temperature is averaging in the time. It is close to stationary conditions, assumed in our consideration.

The results of preliminary tests of different grids cooperating with the IR-FlexCam® Fluke Ti35 enabled the most suitable one to be selected for visualising the temperature fields in the vertical gaps during convective heat transfer. The chosen grid had the following parameters: cotton impregnated with polyester material, with a thermal conductivity $\lambda = 0.02$ W/(m·K), mesh size a = 1.6 mm, and fibre diameter d = 0.4 mm.

Stretched over the frame (Fig.1) and stiffened with polyester lacquer spray, the mesh was placed perpendicular to the heating surfaces. A grid was cut out for each slot width s = 0.045, 0.08 and 0.18 m and placed perpendicular to the plates at their half-width z = B/2.

The analysis was performed using two vertical plates of height H = 0.5 m and a width B = 0.25 m with the heating surfaces arranged in parallel and close to each other. Vertical planes with peripherally open channels and three different spacings s = 0.045, 0.08 and 0.18 m were created this way.

The surface temperature of the heating plates t_w was changed at 5 K intervals: $t_w = 30$, 35, 40, 45, 50, 55, 60, 65, 70, 75 and 80 °C, while the ambient temperature ranged from 18 to 25 °C. The temperature field in the channel, in the plane perpendicular to the heated surfaces, located exactly between the plates z = B/2, was identified with the IR camera and the grid as air radiation detector. This decision was taken as a result of previous studies which had shown that the *z* coordinate had no significant impact on the results. The temperature fields in the three cross-sections of the boundary layer z = 0.0, z = 0.25 B and z = 0.5 B are qualitatively convergent [57].

All the tests described in this paper were conducted using a FlexCam® Fluke Ti35 IR camera with a resolution specified by the manufacturer as ≤ 0.1 K in the temperature range - 20 - 100°C. It was set up at a constant distance of 2 m from and perpendicular to the grid and parallel to the heating surfaces.

3. Results and discussion

3. 1. Results of temperature field and temperature gradient investigations

The results of the temperature field visualisation within the cross-sectional plane inside the gaps between the two vertical isothermal left- (L) and right-hand (R) plates obtained using an IR camera are shown in Fig. 3 for s = 0.045 m, Fig. 4 for s = 0.085 m and Fig. 5 for s = 0.18 m.

For technical reasons, it is impossible to illustrate graphically on one photograph the temperature distribution of the heating surface and the temperature field on the mesh: when the IR camera's sensitivity is focused on higher plate temperatures, the temperature field on the mesh lies beyond the temperature scale and vice versa. Therefore, when the temperature fields of the air are visualised, the surface temperature of the plate, which is beyond the temperature scale, is shown in grey. However, this has no effect on the results of the

calculations, since these are carried out with the temperature field data digitally stored by the IR camera.

The digital values of temperature distributions on the (x, y, z=B/2) plane, perpendicular to the two vertical heating walls at both ends of the channel, are presented graphically in Fig. 3 - 5 and also listed in Table 1.

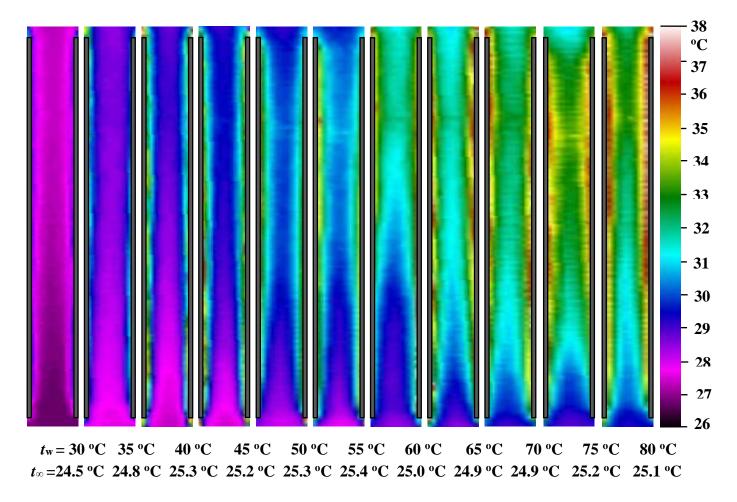


Fig.3 Temperature fields in the gap s = 0.045 m between the vertical isothermal heated plates as a function of their surface temperature t_w and the ambient temperature t_∞ in the x, y, z=B/2 planes.

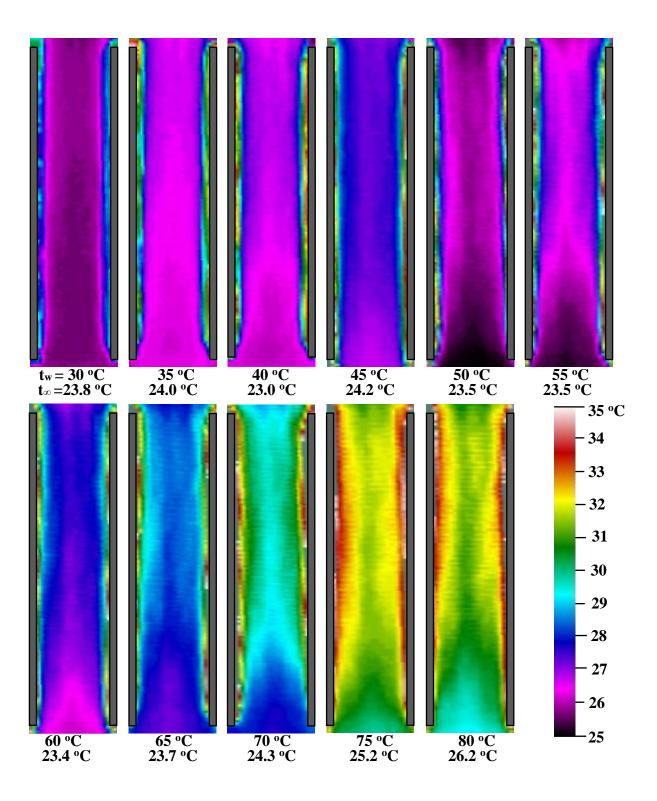


Fig.4 Temperature fields in the gap s = 0.085 m between the vertical isothermal heated plates as a function of their surface temperature t_w and the ambient temperature t_∞ in the x, y, z=B/2 planes..

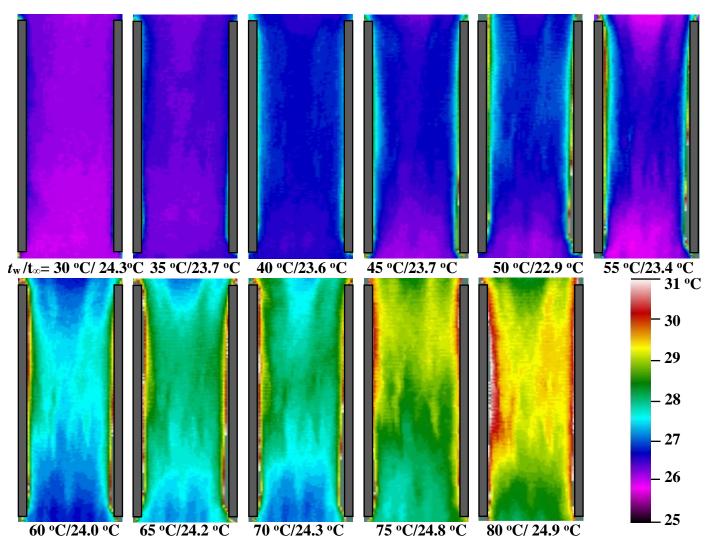


Fig.5 Temperature fields in the gap s = 0.18 m between the vertical isothermal heated plates as a function of their surface temperature t_w and the ambient temperature t_{∞} in the x, y, z=B/2 planes.

Some examples of these distributions inside the three channels (s = 0.045, 0.85 and 0.18 m) in the (x, y, z = B/2) plane at the leading edges y = 0, at mid-height y = H/2 and at the trailing edges y = H of the channels are shown in Fig.6.

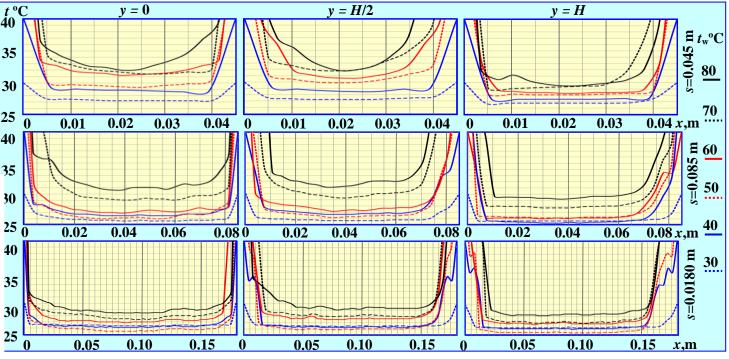


Fig.6 Temperature distribution across three gaps (s = 0.045, 0.85, and 0.18 m) in the *x*-axis perpendicular to the heated plates, as a function of their surface temperature ($t_w = 30$, 40, 50, 60, 70 and 80 °C) measured on three specific levels y = 0 (leading edge), H/2 and H (trailing edge).

In this figure the temperature range has been limited to 40 °C for better clarity. The temperature variations in the channels along the lines perpendicular to the heating surfaces x, over a wider range of temperature variation from 25 to 80 °C, but only at level y = H/2 for set temperatures of the heated vertical plates $t_w = 30$, 40, 50, 60, 70 and 80 °C are shown in Figure 7.

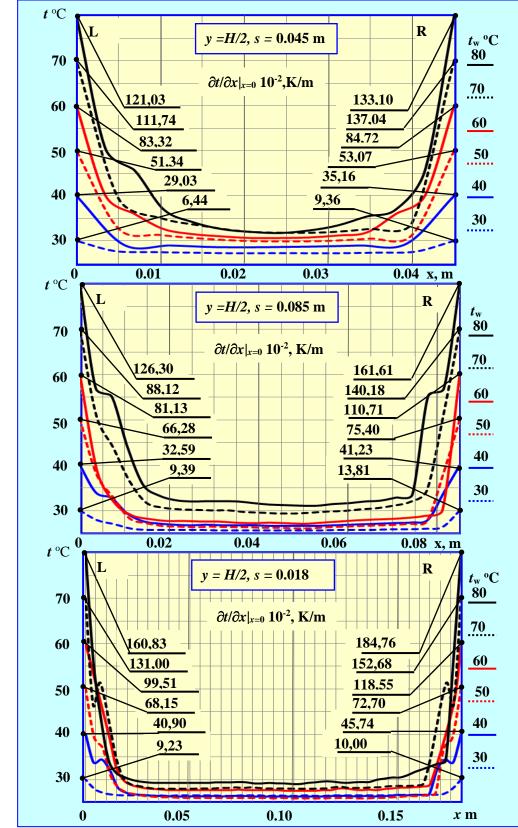


Fig.7 Temperature and temperature gradient distributions perpendicular to two vertical parallel heated surfaces that form three channels with spacings s = 0.045, 0.85, and 0.18 m, measured at y =

H/2 and z = B/2, as a function of the temperature $t_w = 30, 40, 50, 60, 70$ and 80 °C of the heated surfaces.

In addition to the temperature distributions Fig. 7 also shows the calculated values of the temperature gradients $dt/dx|_{x=0}$ on both heating surfaces (L) and (R) at points y = H/2 and z = B/2.

From the digital data generated, apart from the photos, by the IR camera and partially listed in Table 1, the temperature gradients $dt/dx|_{x=0}$ and their distributions on the left- and right-hand heated walls of the vertical channels were calculated on the basis of equation (12):

$$\frac{\partial t}{\partial x}\Big]_{x=0,L} = \frac{(t_0 = t_w) - t_1}{x_1 - (x_0 = 0)} \quad (L) \text{ and } \quad \frac{\partial t}{\partial x}\Big]_{x=0,R} = \frac{(t_n = t_w) - t_{n-1}}{(x_n = s) - x_{n-1}} \quad (R).$$
(12)

The results of the numerical calculations, independently for the left- (L) and righthand (R) heated walls, for the four channels (s = 0.045; 0,085 and 0,18 m) in the form of temperature gradients $dt/dx|_{x=0,y}$ for wall temperatures from $t_w = 30$ to 80 °C at 5 K intervals at different heights of the channel ranging from y=0 (leading edges of the channels) to y = H(trailing edges of the channels) for z = B/2 = constant are listed in Tables 2, 3 and 4.

Table 2. Temperature gradient distributions along the left- and right-hand (separated by a slash) vertical plates of constant surface temperature $t_w = 30, 35, 40, 45, 50, 55, 60, 65, 70, 75$ and 80 °C within a channel of width between the heated plates s = 0.045 m.

		$\frac{\partial t}{\partial x} \begin{vmatrix} x=0, y(\mathbf{L}) / \partial t / \partial x \end{vmatrix} _{x=0, y} (\mathbf{R}), \qquad \mathbf{K/m}$										
t _w , °C	30	35	40	45	50	55	60	65	70	75	80	
t∞, °C	24.5	24.8	25.3	25.2	25.3	25.4	25	24.9	24.9	25.2	25.1	
y [.] , m	644/	1630/	2868/	4222/	5765/	6722/	8332/	10252/	11816/	10728/	12630/	
0.0	982	2506	3574	4940	5608	7253	8530	12262	14119	14000	13929	
	543/	1407/	2648/	3796/	5524/	6586/	8000/	9851/	11273/	9946/	12171/	
0.035	936	2485	3536	4902	5395	7143	8357	12173	13959	13980	1331	
	543/	1443/	2716/	3921/	5393/	6551/	8036/	9890/	11174/	9981/	12138/	
0.070	936	2485	3436	4821	5341	7051	8339	12151	13889	13790	13214	
	527/	1481/	2648/	3796/	5339/	6519/	7908/	9931/	11174/	9846/	11934/	
0.105	936	2485	3516	4860	5216	7086	8298	12105	13796	13707	13310	
	576/	1517/	2749/	3887/	5301/	6551/	8036/	10069/	11174/	9998/	11899/	
0.140	936	2438	3475	4741	5270	6920	8298	12062	13748	13647	13291	
	611/	1556/	2817/	3850/	5301/	6654/	8148/	10031/	11235/	9998/	11983/	
0.175	936	2438	3475	4741	5307	6845	8357	12062	13704	13603	13291	
	611/	1443/	2852/	3887/	5134/	6551/	8110/	9989/	11215/	10185/	11899/	
0.210	936	2485	3516	4699	5253	6827	8394	12062	13796	13603	13291	
	644/	1591/	2903/	4009/	5134/	6586/	8332/	10130/	11174/	10354/	12103/	
0.245	936	2438	3516	4699	5307	6827	8472	12062	13704	13603	13310	
	712/	1630/	2988/	4117/	5095/	6654/	8462/	10211/	11315/	10625/	12255/	
0.280	982	2485	3536	4619	5341	6881	8512	12062	13748	13647	13424	
	712/	1630/	3056/	4188/	5042/	6722/	8628/	10252/	11414/	10813/	12086/	
31.5	982	2485	3536	4661	5429	6955	8648	12062	13889	13647	13483	
	780/	1630/	3123/	4330/	5134/	6790/	8815/	10310/	11636/	11016/	12222/	
0.350	1025	2485	3574	4699	5537	7086	8840	11994	13796	13750	14006	
	780/	1647/	3175/	4418/	5170/	6858/	8924/	10352/	11674/	11272/	12222/	
0.385	1025	2485	3616	4661	5679	7143	8840	12015	13796	13853	13870	
	815/	1647/	3243/	4577/	4929/	6942/	8963/	10310/	11636/	11560/	12222/	
0.420	1071	2485	3655	4481	5838	7143	8744	12015	13704	13937	13833	
	847/	1686/	3327/	4702/	5060/	7045/	9037/	10390/	11716/	11899/	12526/	
0.455	1071	2485	3696	4500	5963	7143	8821	12015	13841	14040	14102	
0.400	883/	1778/	3395/	4932/	6027/	7198/	9295/	10451/	11716/	12374/	13257/	
0.490	1114	2485	3857	5143	6159	7979	9536	11947	13748	14563	15148	
Average	682/	1581/	2967/	4176/	529/	6729/	8468/	10161/	11423/	10706/	12237/	
L//R	987 825	2477	3568	4745	5510	7086	8599 8524	12070	13816	13825	13654	
Channel	835	2029	3268	4461	5400	6908	8534	11116	12620	12266	12946	

	plates $s = 0.085$ m.										
	$\partial t/\partial x \mid_{x=0,y} (L) / \partial t/\partial x \mid_{x=0,y} (R), K/m$										
$t_{\rm w}$. °C	30	35	40	45	50	55	60	65	70	75	80
t∞, °C	23.8	24	23	24.2	23.5	23.5	23.4	23.7	24.3	25.2	26.2
<i>y</i> [.] , m	744/	1934/	3175/	4261/	6462/	6978/	7741/	9512/	7314/	10580/	12086/
0.0	1422	2848	4373	6293	7707	9207	11471	12429	14329	13661	16551
	648/	1510/	2613/	3599/	6222/	6111/	5872/	7863/	5926/	10142/	1134/
0.035	1381	2803	4250	6161	7500	8980	11271	12363	14150	13500	16420
	713/	1698/	2852/	3802/	6258/	6315/	6434/	8377/	6652/	10096/	11543/
0.070	1381	2803	4207	6161	7417	8897	11250	12322	14150	13619	16354
	809/	1817/	2988/	3971/	6332/	6687/	7062/	8908/	7407/	10207/	11815/
0.105	1381	2762	4123	6071	7373	8897	11114		13971	13481	16205
	843/	1901/	3088/	4039/	6332/	6823/	7317/	9032/	7716/	10321/	12103/
0.140	1381	2762	4083	6050	7373	8790	11114	12125	13971	13481	15967
	874/	1969/	3140/	4074/	6427/	6978/	7553/	9316/	8025/	10466/	12323/
0.175	1381	2762	4123	6114	7500	8750	11204	12256	14018	13481	16009
	907/	2070/	3208/	4175/	6519/	7130/	7792/	9688/	8519/	10614/	12494/
0.210	1422	2803	4123	6114	7540	8833	11250	12256	14107	13539	16074
	939/	2138/	3259/	4226/	6628/	7282/	8113/	9867/	8812/	10630/	12630/
0.245	1381	2717	4123	6050	7540	8603	11071	12256	14018	13439	16161
	1003/	2205/	3327/	4329/	6702/	7401/	8368/	10043/	9136/	10809/	12749/
0.280	1336	2717	4083	5961	7583	8373	10961	11911	13861	13400	15967
	1037/	2241/	3395/	4414/	6776/	7502/	8556/	10134/	9368/	10889/	12901/
31.5	1295	2676	4123	5961	7623	8123	10804	11780	13907	13400	15967
	1037/	2241/	3463/	4378/	6815/	7605/	8520/	10222/	9444/	10920/	12953/
0.350	1250	2589	4000	5961	7623	7750	10536	11565	13771	13359	16009
	1068/	2273/	3531/	4481/	6924/	7741/	8675/	10364/	9659/	11019/	13105/
0.385	1208	2458	3897	5961	7707	7500	10289	11220	13771	13259	16009
	1037/	2309/	3563/	4566/	6999/	7841/	8724/	10452/	9906/	11179/	13309/
0.420	1163	2310	3707	6004	7790	7083	10000	10789	13907	13118	15881
	1068/	2341/	3599/	4650/	7037/	7877/	8827/	10506/	10109/	11358/	13545/
0.455	1208	2310	3563	6004	7833	7270	9643	10595	13929	13079	15922
	1102/	2377/	3735/	4786/	7147/	8012/	9063/	10611/	10323/	11537/	13681/
0.490	1422	2803	4000	6204	8040	8937	10246	11479	14621	13940	17089
	922/	2068/	3262/	425/	6639/	7219/	7908/	966/	8554/	10718/	12572/
L/R	1334	2675	4052	6071	761	8400	10815	11834	14032	13450	16172
Channel	1128	2372	3657	5161	7125	7810	9362	10747	11293	12084	14372

Table 3. Temperature gradient distributions along the left- and right-hand vertical plates of constant surface temperature $t_w = 30, 35, 40, 45, 50, 55, 60, 65, 70, 75$ and 80 °C within a channel of width between the plates s = 0.085 m.

Table 4. Temperature gradient distributions along the left- and right-hand vertical plates of constant surface temperature $t_w = 30, 35, 40, 45, 50, 55, 60, 65, 70, 75$ and 80 °C within a channel of width between the plates s = 0.180 m.

		$\partial t/\partial x \mid_{x=0,y} (L) / \partial t/\partial x \mid_{x=0,y} (R), K/m$											
t _w , °C	30	35	40	45	50	55	60	65	70	75	80		
t_{∞} , °C	24.3	23.7	23.6	23.7	22.9	23.4	24	24.2	24.3	24.8	24.9		
y , m	957/	3049/	4127/	5864/	6815/	9026/	10211/	11716/	13199/	13253/	16667/		
0.0	1167	3414	4661	6107	7957	10998	12381	14141	15386	18170	19093		
	886/	2969/	4049/	5710/	6702/	8705/	10069/	11495/	13000/	12596/	16187/		
0.035	1083	3393	4574	6027	7833	10808	12236	14023	15338	18071	18998		
	886/	2969/	4049/	5670/	6667/	8647/	9931/	11235/	13019/	12559/	16000/		
0.070	1083	3393	4574	5985	7707	10571	12023	13855	15315	17997	18857		
	886/	2969/	4049/	5633/	6667/	8606/	9851/	11273/	13019/	12596/	15937/		
0.105	1040	3393	4530	5905	7540	10427	11950	13760	15268	17875	18857		
	886/	3008/	4049/	5633/	6741/	8705/	9890/	11235/	13019/	12691/	15937/		
0.140	1040	3393	4574	5905	7457	10381	11905	13691	15268	17824	18712		
	886/	3008/	4049/	5593/	6741/	8747/	9890/	11273/	12920/	12827/	15937/		
0.175	1000	3393	4574	5905	7417	10213	11950	13642	15268	17824	18712		
	•	•	•	•			•	•	•	•	•		

Channel	946	3130	4286	5762	6946	9151	10963	12539	14176	15335	16949
L/R	970	3248	4488	5852	7059	9414	11844	13618	15232	17547	17638
Average	922/	3012/	4083/	5671/	6832/	8888/	10081/	1146/	1312/	13122/	16259/
0.490	1123	3036	4315	6027	7667	9570	11737	13379	15268	16940	16690
	1028/	3049/	4167/	5941/	7073/	9389/	10390/	11816/	13462/	14003/	16937/
0.455	647	2811	4143	5464	5270	7048	11238	13048	14898	15593	13882
	957/	2927/	4127/	5633/	6924/	9106/	10172/	11495/	13061/	12769/	16373/
0.420	750	2946	4271	5545	5520	7284	11429	13284	15061	16598	14739
	957/	2969/	4127/	5670/	6963/	9106/	10252/	11536/	13199/	13426/	16583/
0.385	833	3036	4357	5625	6187	7810	11619	13379	15109	17164	15928
	886/	3049/	4090/	5710/	6963/	9068/	10252/	11594/	13279/	13580/	16500/
0.350	873	3168	4488	5744	6623	8309	11737	13547	15223	17580	17120
	923/	3049/	4090/	5633/	6924/	8946/	10211/	11536/	13199/	13562/	16333/
31.5	873	3257	4530	5866	6937	8739	11855	13642	15223	17899	17787
	923/	3049/	4090/	5633/	6889/	8866/	1013/	11536/	13160/	13426/	16270/
0.280	1000	3304	4574	5905	7167	9333	11787	13596	15268	17875	18118
	923/	3049/	4049/	5593/	6815/	8827/	10069/	11456/	13141/	13272/	16167/
0.245	1000	3393	4574	5905	7270	9714	11855	13596	15268	17899	18476
	923/	3049/	4090/	5556/	6815/	8785/	9951/	11353/	13100/	13272/	16083/
0.210	1040	3393	4574	5866	7333	10000	11950	13691	15315	17899	18594
	923/	3008/	4049/	5593/	6776/	8785/	9951/	11353/	13019/	13000/	15980/

3.2. Results of the balance method for measuring convective heat transfer within the channels

The results of studies carried out using the balance method in accordance with the procedure described in [57] for the left- and right-hand heated plates of the channels for different distances s can be written as:

$$Nu_{\rm b} = 0.489 \cdot Ra_{\rm b}^{1/4}$$
 (L), =0.492 $\cdot Ra_{\rm b}^{1/4}$ (R), =0.491 $\cdot Ra_{\rm b}^{1/4}$ (total), $s = 0.045$ m, (17)

$$Nu_{\rm b} = 0.477 \cdot Ra_{\rm b}^{1/4}$$
 (L), =0.482 $\cdot Ra_{\rm b}^{1/4}$ (R), =0.480 $\cdot Ra_{\rm b}^{1/4}$ (total), $s = 0.085$ m, (18)

$$Nu_{\rm b}=0.482 \cdot Ra_{\rm b}^{1/4}$$
 (L), =0.511 $\cdot Ra_{\rm b}^{1/4}$ (R), =0.497 $\cdot Ra_{\rm b}^{1/4}$ (total), $s = 0.180$ m, (19)

$$Nu_{b}=0.461$$
 $\cdot Ra_{b}^{1/4}$ (L), =0.510 $\cdot Ra_{b}^{1/4}$ (R), =0.486 $\cdot Ra_{b}^{1/4}$ (total), $s = \infty$. (20)

The left-hand vertical plate (L) was already used in our previous study of convective heat transfer, and the result was published in [57] as:

$$Nu_{\rm b} = 0.494 \cdot Ra_{\rm b}^{-1/4} \,. \tag{21}$$

The phisical properties in Nusset and Rayleig numbers λ , β , a and ν are calculated according to avearage temperature of air $t_{av} = (t_w + t_\infty)/2$.

The divergence error between equation (20), currently obtained for the two plates, and the previously obtained equation (21) varies from 3.1% (R) to 7.2% (L). This small divergence demonstrates the reliability of the results obtained using the balance method and in the subsequent experiments.

Because of its good compatibility (1.8%) and reproducibility of its results, the total average Nusselt-Rayleigh relation obtained for the two vertical plates (20) has been taken as a reference for the balance method results obtained for the channels. This comparison shows that the convective convectional heat transfer within the channels s = 0.045 m and s = 0.18 m is comparable to that obtained with the separate vertical plates and that the differences – +1.02% for s=0.045 m, -1.23% for s = 0.085 and +2.2% for s = 0.18 m – are within the range of error.

Above results clearly show that the balance method is not precisely enough to perform a more detailed study of free convective heat transfer mechanisms in the channels, neither does it allow the local nature of the phenomenon to be examined. Therefore, it was decided to continue further studies with the use of gradient method.

3.3. Results of gradient method investigations

Assuming that heat transfer from the isothermal vertical flat surfaces into the air inside the channel is two-dimensional, the local heat fluxes in a steady state at level y, described by Fourier's and Newton's relationships, should be are the same, and can be written as:

$$-\lambda \cdot A \cdot \frac{\partial t}{\partial x}\Big]_{x=0,y} = \alpha_y \cdot A \cdot \left(t_w - t_{\infty,y}\right) \text{ and then } \alpha_y = \lambda \cdot \frac{-\frac{\partial t}{\partial x}\Big]_{x=0,y}}{t_w - t_{\infty,y}},$$
(22)

where t_w and $t_{\infty,y}$ °C are, respectively, the wall temperature and the lowest air temperature inside the channel at level y, α , W/(m²K) is the heat transfer coefficient and λ , W/(m·K) is the thermal conductivity of air.

The distributions of the local heat transfer coefficients $\alpha_{y,L}$ and α_{yR} , calculated using equation (22), from both vertical heated surfaces (L) and (R), as a function of temperature t_w and level y, for the three different channels (s = 0.045; 0.085 and 0.180) and two vertical heating plates (s = ∞), are given in Fig.8 The average values of the heat transfer coefficients for the left- and right-hand heated surfaces, determined from (23) and for the whole channels (24), are shown in the frames in Fig.8.

$$\overline{\alpha_{L/R}} = \frac{1}{H} \int_0^H \alpha_{y,L/R} dy = \frac{\lambda}{H} \int_0^H \frac{-\frac{\partial t}{\partial x}\Big|_{x=0,y}}{t_w - t_{\infty,y}} dy,$$
(23)

$$\alpha = \frac{\overline{\alpha_L} + \overline{\alpha_R}}{2}.$$
(24)

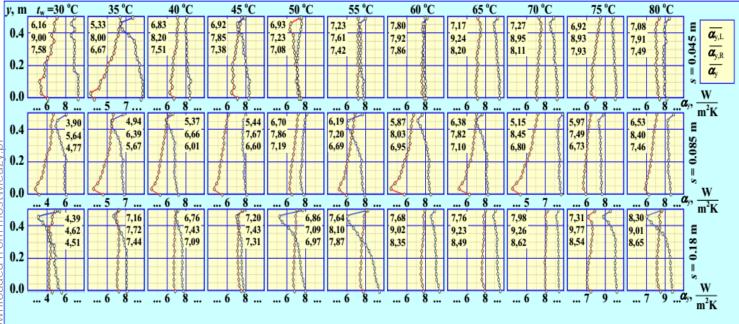


Fig.8 Distributions of local $\alpha_{y,L/R}$, average $\alpha_{L/R}$ and total α heat transfer coefficients on two (left – red diamonds, and right – blue circles) heated surfaces, which form gaps s = 0.045, 0.085 and 0.18 m at the channel half-width z = B/2, as a function of the temperature t_w of both heated surfaces.

Considering the influence of the variable properties of the surrounding air on the heat flow, the Nusselt Nu and Rayleigh Ra numbers for natural convection in the channels were defined as:

$$Nu = \frac{\alpha \cdot H}{\lambda}$$
 and $Ra = \frac{g \cdot \beta \cdot (t_w - t_\infty) H^3}{a \cdot v}$, (25)

where H, m – height of the heated plates (the characteristic linear dimension), g, m/s² – acceleration due to gravity, β , 1/K – expansivity coefficient of cubic expansion and a, ν , m²/s – thermal diffusivity and kinematic viscosity (see eq.17-20).

All the physical properties of the air a, β, v and λ were specified for the average air temperature t_{av} . The investigation was carried out in an insulated room in order to prevent accidental air movements due to ventilation and draughts. For the surface temperature of both plates t_{wL} and t_{wR} , measured by thermocouples and the lowest air temperature in the channel at level $y t_{\infty,y}$ the average air temperature was determined as $t_{av} = (t_{wL} + t_{wR} + 2t_{\infty,y})/4$. In the case of the two vertical heated plates, which, because of the large distance $s = \infty$ between them did not influence each other, the surface temperature was additionally measured with the IR camera. If $t_{wL}=t_{wR}$ and $t_{\infty L}=t_{\infty L}$ the average air temperature t_{av} can be traditionally determined as $t_{av} = (t_{wL} + t_{wR})/2$.

The results of convective heat transfer for the channels, calculated using equation (25) in the form of the average Nusselt and Rayleigh numbers can be written as:

$$Nu = 1.094 \cdot Ra^{1/4}, \ s = 0.045 \text{ m}, \tag{26}$$

$$Nu = 0.945 \cdot Ra^{1/4}, s = 0.085 \text{ m},$$
(27)

$$Nu = 1.073 \cdot Ra^{-1/4}$$
, $s = 0.180$ m, (28)

Divergences between (17)-(19) and (26)-(28) are due to differences in the definition of Nusselt numbers Nub and Nu. In the case of balance method averaging of local values does not occurs, while in the gradient method it was necessary to perform.

In the gradient method the convective heat transfer from two vertical plates was investigated for seven wall temperatures $-t_w = 40$; 45; 50; 55; 60; 65 and 70 °C – with the local heat transfer coefficients being averaged independently for each plate α_{yL} and α_{yR} , in accordance with the procedures in (23) and (24). For sixteen temperatures t_w and moreover, instead of α_y , local Nusselt numbers Nu_y were averaged according to the formula:

$$Nu = \frac{1}{H} \int_0^H Nu_y dy = \frac{1}{H} \int_0^H \frac{\alpha_y}{\lambda} \cdot y \cdot dy = \frac{1}{H} \int_0^H \frac{-\frac{\partial t}{\partial x}}{t_w - t_\infty} \cdot y \cdot dy.$$
(29)

Despite of the quantitative discrepancies the gradient method using the IR camera is practically more useful for qualitative investigations of heat transfer in channels than the balance one.

The results (26)-(28) shows that the convective heat transfer intensity intensity of natural convective heat transfer is the function of distance between plates *s*. The divergences are more visible in comparison with (17)-(19). The results with respect to the *s* =0.085 with minimal effect, are the following: 15.8% for s = 0.045 m and 13.5% for s = 0.180 m.

The results obtained for the channels based on a characteristic linear dimension other than the height H, i.e. the distance between the plates s, were also calculated. The result, which is the relation $Nu^* = f(Ra^*)$ presented in the graph of Fig.9 can be written as:

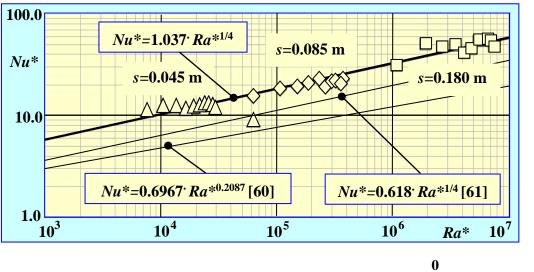
$$Nu^* = 1.037 \cdot Ra^{*1/4} \quad . \tag{30}$$

where the Nusselt and Rayleigh numbers, modified for the channels, are defined as:

$$Nu^* = \frac{\alpha \cdot s}{\lambda}$$
 and $Ra^* = \frac{g \cdot \beta \cdot (t_w - t_\infty) s^3}{a \cdot v} \frac{s}{H}$, (31)

It should be borne in mind that these results are one-dimensional, because they were obtained in the vertical planes in the centre of the space between the plates z = B/2 and perpendicular to their heated walls. A generalisation of these results is all the more true if $B/H \ge 1$.

Fig.9 shows the comparison of obtained results and the literature data presended by Fossa [60] and Rohsenow [61]. Visible descrypances are caused by differences, in the case of paper [60] investigated channels were asimetrically heated, while the corelation proposed by Rohsenow is the compilation of results of Churchill [45], Aung [17] and Elenbaas [21].



9 Convective heat transfer in open channels as a function of distance between isothermal vertical heated surfaces *s* shown in the form of modified Nusselt-Rayleigh relations.

Fig.

3.4. Mechanism of convective heat transfer inside the channels.

Analysis of the results, especially the visual ones, allows the structure of the convective flows of air inside the channels to be specified. Among these structures, one can specify four characteristic situations of overall natural convection heat transfer from the two vertical parallel isothermal plates forming the channel with an open circuit. Two of them are borderline and the other two intermediate:

- a the first borderline case, when the spacing between the plates *s* is very close and much smaller than the height *H* or width *B* of the plates ($s_a << H$ and $s_a << B$): a fully developed flow pattern inside the channel is observed, which should, and as shown by further research, does indeed intensify heat transfer from the heated walls into the air.
- b the second extreme situation occurs when the space between the plates is large $s_b >> H$. Then, each plate transfers heat to air independently of the other – there is no interaction between the two plates either thermally or hydrodynamically. This case should be treated as the well-known problem of natural convective convectional heat transfer from a single vertical plate.

It was observed that in the other two models the mechanism of convective heat transfer inside the channels is intermediate between the two extreme cases a) and b). Thermally developing regions also exist in both but in different forms:

- c in the model with the plates also close together ($s_c \ll H$), but with the distance between the heating walls slightly larger than in the case of a) ($s_a \ll s_c$). This increase of *s* caused a partial inhibition of heat transfer.
- d a further increase in the distance between the plates sd>sc, the thermal and flow activities of the two plates continue to interact with each other and likewise, the temperature profile along the channel continues to develop. However, with limited wall friction and air flow retardation inside the channel, the chimney effect intensifies, which in turn, as in the first extreme case, intensifies convective heat transfer.

4. Conclusions

This paper analyses natural convective heat transfer in air gaps between two vertical, parallel, isothermal and symmetrically heated plates, investigated using infrared techniques and the gradient method. It was demonstrated that this method is also applicable to investigations of more complex cases of convective heat transfer than those described in [57].

The sequentially presented results (temperature fields t(x,y), temperature gradients distributions $\partial t/\partial x |_{x=0,y}$, distributions of heat transfer coefficients $\alpha(x,y)$ and averaged Nusselt numbers Nu(Ra)) show their logical correctness and consistency. Consequently, they can provide a basis for describing and interpreting the convective heat transfer phenomenon examined in this work.

This study confirmed that there is a simple correlation between the width of the gap between heated walls s on the intensity of convective heat transfer in open channels (30). The results demonstrated the inhibition or intensification of heat transfer in accordance with s. We hope that other researchers may find this method useful for practical application in the structural design and construction of air heat exchangers, radiators or coolers.

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Nomenclature

- a thermal diffusivity, m²/s
- *b* half-width of channel, $\equiv s/2$, m
- $A \text{area}, \equiv 2HB, \text{ m}^2; \text{ coefficient}$
- *B* plate width, m
- *C* coefficient
- g acceleration due to gravity, m/s²
- *H* channel or vertical plate length, m
- *I* amperage of electric current, A

- m exponent in Eqs. (2) and (5)
- $Nu_{\rm b}$ channel balance Nusselt number, $\equiv UI/(\lambda(t_{\rm w}-t_{\infty})B)$ dimensionless
- \widetilde{Nu} modified Nusselt number, $\equiv UIs/(\lambda(t_w-t_\infty)H^2)$ dimensionless
- Nu_y local Nusselt number, $\equiv \alpha y / \lambda$ dimensionless
- Nu total Nusselt number, Eq. (25), dimensionless
- Nu^* total channel Nusselt number, $\equiv \alpha s / \lambda$, dimensionless
- **P** plate/air parameter in Eq. (22), $\equiv \mathbf{Ra}/\mathbf{H}^4$, m⁻⁴
- q'' heat flux, $\equiv q/A$, W/m²
- Ra_b Rayleigh number, $\equiv g\beta \Delta t H^3/(\nu a)$, dimensionless
- Ra^* channel Rayleigh number, $\equiv g\beta \Delta ts^3/(\nu a)s/H$, dimensionless
- \widetilde{Ra} modified Rayleigh number, $\equiv g\beta \Delta t b^3 / (va) \cdot H/b$, dimensionless
- *s* plate spacing, m
- *t* temperature, ^oC
- U voltage of electric current, V
- *x,y,z* coordinates, m

Greek Letters

- α heat transfer coefficient, Eq. (13) W/(m² K)
- β volumetric coefficient of thermal expansion, 1/K
- λ thermal conductivity, W/(m K)
- $\boldsymbol{\nu}$ kinematic viscosity, m²/s
- ρ density, kg/m³
- ∞ entrance or ambient value

Subscripts

- a auxiliary
- av average,
- b balance method
- L left,
- m main,
- R right,
- w wall,
- y local