

## Installation of GNSS receivers on a mobile railway platform – methodology and measurement aspects

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### Abstract

Determining the course of a railway track axis using satellite methods relies on ensuring the precise assembly of GNSS receivers in dedicated measuring devices. Depending on the number of receivers, solutions that are based on placing the apparatus directly above the railway track axis (as well as in eccentric positions) are used to indirectly obtain data to form the basis of the desired results. This publication describes the installation of five GNSS receivers on a mobile measuring platform as part of the InnoSatTrack project. The methodology and the procedure of the geodetic measurements required to obtain the geometrical configuration of the measuring apparatus, specified in the technical project, are presented. The publication presents the principles of total station measurement methods as well as the steps taken to achieve precise results from staking out. The process of acquiring the same configuration of the GNSS receivers, based on the geometry of the squares on the two measuring platforms that were used in the research, has been shown. The final determination of the position of the receivers did not exceed an error of 0.010 m with an average error of 0.003 m, despite the occurrence of mounting difficulties. The results have demonstrated the high credibility and effectiveness of the presented solution.

### Introduction

Examining a railway track shape with a satellite device requires the right geometry of the measurement system; apart from satellite receivers, this can include accelerometers and inclinometers (Kreye, Eissfeller & Ameres, 2004; Akpınar & Gulal, 2011). The most frequent aim of such measurements is to precisely determine the position of a railway track

axis and to determine the extent of any track deformation (Chen et al., 2015). With the development of mobile laser scanning, interest is growing in automatic detection for rails and ancillary infrastructure objects in point clouds (Arastounia & Oude Elberink, 2016). The output of algorithms that create object models is not presented in a linear form, but as a percentage which indicates the accuracy of the fit-in procedure (Arastounia & Oude Elberink, 2016;

Lou et al., 2018). The first study in Poland concerning railway track shape through precision positioning with the global navigation satellite system (GNSS) was conducted in Kashubia in 2009 (Koc et al., 2009; Specht et al., 2011). The authors proved that it is justified to apply the method based on the accuracy parameters expressed by a mean error of 5 cm at a level of confidence of  $2\sigma$ . Moreover, studies have stressed the limitations in employing satellite measurements in railway engineering, which are caused by the presence of objects that obscure the sky in urban areas and forests (Koc & Specht, 2009). Subsequent upgrades to the space segment of existing GNSS systems with a full satellite constellation and the creation of new GNSS systems (Specht et al., 2015) favored the development of state-run and commercial active geodetic networks (Baran et al., 2008; Specht, Specht & Dąbrowski, 2017). Since the first study in 2009, the research team has conducted several studies focusing on GNSS positioning accuracy and availability (Specht et al., 2014; Specht & Koc, 2016; Specht et al., 2019) and supporting it with software solutions (Specht, Chrostowski & Koc, 2016). In another area of interest, the team focused on the application of determining coordinates in the design and use of railway tracks (Koc, 2012; Koc & Chrostowski, 2014; Koc, 2016; Koc et al., 2019). The research involved further work aimed at developing railway mobile measurement platforms (MMP).

The issue of satellite positioning for railways, as discussed in the literature, covers the application of additional measurement and numerical methods. Gikas and Daskalakis (Gikas & Daskalakis, 2008) proposed a solution combining conventional tachymetric geodetic measurements with GNSS observations. Multi-sensor solutions are important for the availability and reliability of satellite measurements (Li et al., 2017; Gao et al., 2018; Kurhan et al., 2018) and inert INS (Chen et al., 2015; Chen et al., 2018). Several by studies by Jiang et al. (Jiang et al., 2017a; 2017b) have presented a novel approach to the issue by applying external supporting measurement systems whose application requires the use of innovative methods of design and computation (Yoshimura & Naganuma, 2013; Sánchez, Bravo & González, 2016; Wang et al., 2018). One of the objectives of this research project is to analyze the application of multiple GNSS receivers for one MMP platform. This approach, which is in line with modern research methods, involves the use of a set of devices deployed with strict geometry. The main objective of the project was to precisely determine

the geometric layout of a railway track. This paper describes the preparatory phase of the measurement process, which involves a presentation of the method of the precision deployment of five GNSS receivers in a specific geometric configuration on a measurement platform. Locating the measurement instruments according to the assumed spatial relationships is a key condition of the study conducted as part of the BRIK project.

## Methodology

### Measurement platform

It was decided to verify the modified concept of conducting measurements on a railway track during a measurement campaign conducted at the end of 2018 on a railway track in Gdańsk. The track shape study employed a measurement set that consisted of three measurement platforms coupled with a tramway, a motor vehicle. The measurement platforms were made from trolleys from a pre-war DWF 300 tramway, with the dimensions: 2.73 m × 1.78 m × 0.75 m. The central GNSS receivers were mounted on the trolley's kingpin, which is situated above the track axis. Over the years, the original trolleys were covered with steel sheets, whose flat upper surfaces enabled additional construction elements to be installed for the measurement instruments. The study uses three platforms, two of which were used to mount two sets of five GNSS receivers and one to mount a satellite compass (Figure 1).

In order to make it possible to set the receivers in the spatial configuration that was predefined in the project, two transverse steel frames with dimensions of 1.80 m, 0.25 m, 0.22 m and a central support on the kingpin were installed on the measurement platforms. The frames consisted of two 6 cm wide parallel steel profiles, with a 13 cm space between them. Two transverse profiles, 8 cm wide, with a 3 cm space between them, were welded in place above the rail at the place where the tribrachs and receivers were installed. The steel profile gauge provided the necessary tolerances for the process of precisely positioning the receiver. A round hole with a diameter of 5 cm was cut out in the central support with the horizontal dimensions of 0.25 m and 0.25 m in order to make it possible to move the tripod's head. The height of the central support could be modified within the range of 0–30 cm owing to a system of screws. The frames and the supports were fixed to the measurement platform with screws in a manner that prevented

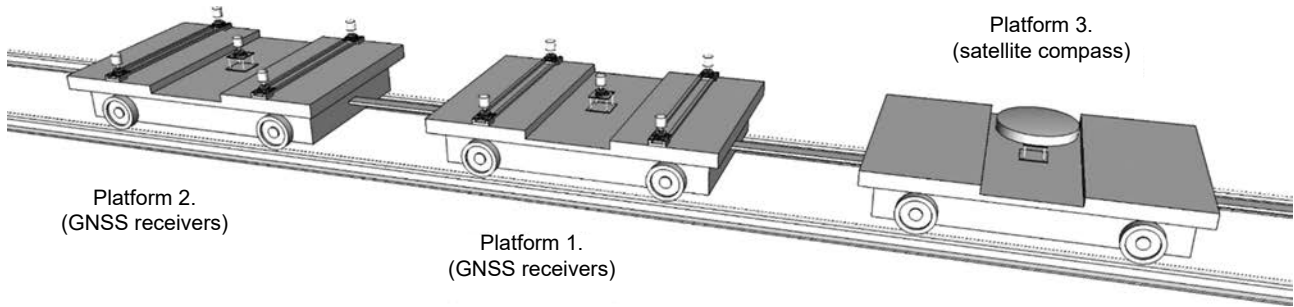


Figure 1. Railway measurement trolleys with the instruments mounted on them

their position from being changed. The GNSS receivers were later placed in a predefined configuration on the structure.

### Tachymetric measurement

The setup of the predefined geometric configuration of the measurement instruments was based on conventional tachymetric geodetic measurements. The essence of tachymetric measurements lies in simultaneous measurement of the angle distance: both horizontal and vertical in the local coordinate system of an electronic total station. As with laser scanners, the coordinate system at the measurement stand was the result of setting the device on a tripod. Thus, the horizontal wheel of the device receives a specific spatial configuration with the surroundings defined by the starting direction orientation (the zero value of the horizontal angle) (Figure 2). Currently, the practice of performing geodetic procedures in Poland usually involves conducting flat measurements in the state flat coordinate system PL-2000, which is based on the Gauss-Kruger projection (Hooijberg, 2008) and it is used in cartographic work and maps with scales of more than 1:10,000 (Regulation, 2012a). The horizontal control network is used to determine the georeference corresponding to

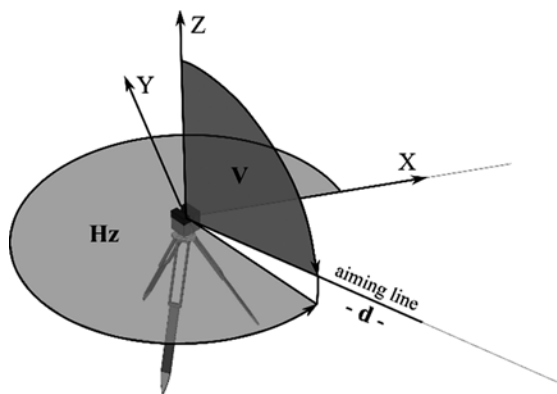


Figure 2. A local coordinate system for a total station and the observations registered during the measurement

the PL-2000 system (Regulation, 2012b). Based on known coordinates of the control network points, the total station computer then determines the rotation matrix coefficients and the translation vector coordinates (Korn & Korn, 2000), which enable coordinate transformation for points measured in the field to the PL-2000 system.

The distance is measured by the electromagnetic method which employs the relationship between the known electromagnetic waves (light) propagation velocity, time between the emission and reception of the beam and the distance covered (Heritage & Large, 2009). It is noteworthy that the beginning of the local coordinate system is situated at the emission and reception center of the measurement device. The horizontal and vertical angles are read from the code markers that are located on the horizontal and vertical wheels, which are integrated with the alidade and the instrument telescope (Wanic, 2007). With the rotation of the instrument around its vertical axis, the horizontal wheel rotates, the wheel's code marker is changed and the new horizontal angle value is read-out. Similarly, when the telescope rotates, a new vertical angle of the instrument's target axis is acquired. The integration of three measurement results (two angles and one distance) enables the determination of the three-dimensional ortho-Cartesian coordinates of the point being measured. Assuming a vertical angle measured downwards from the vertical direction, the coordinates in the right-hand local system of coordinates can be determined from the following formulas:

$$\begin{cases} x = d \sin(V) \cos(Hz) \\ y = d \sin(V) \sin(Hz) \\ z = d \cos(V) \end{cases} \quad (1)$$

where:

- $x, y, z$  – ortho-Cartesian coordinates of the point,
- $d$  – distance between the point and the measurement instrument,
- $V, Hz$  – vertical and horizontal angle of the instrumented target axis aimed at the point.

The transformation of the local system coordinates to another system of coordinates (e.g. state system PL-2000) in matrix notation has the following form:

$$\mathbf{P}' = \mathbf{R} \cdot \mathbf{P} + \mathbf{T} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \quad (2)$$

where:

- P, P'** – vectors of the point coordinates in the primary  $(x, y, z)$  and secondary system  $(x', y', z')$ ,
- R** – rotation matrix around the vertical axis  $OZ$  by the angle  $\theta$ ,
- T** – translation vector with coordinates  $T_x, T_y, T_z$ .

The observation and the point coordinates are then saved in the device's memory. Moreover, in the same way as with a total station, the recorded point coordinates can be used to carry out further computations or to perform separate measurement procedures, such as staking out.

As part of the tachymetric measurements, the orthogonal measurement method was applied to position the GNSS receiver positioning in the local total station coordinate system. The horizontal coordinates were used to mark out the measurement line which was used to calculate the distance along the baseline ( $l$ ) and the offset ( $h$ ). The procedure of staking out the position of the receiver is presented in Figure 3.

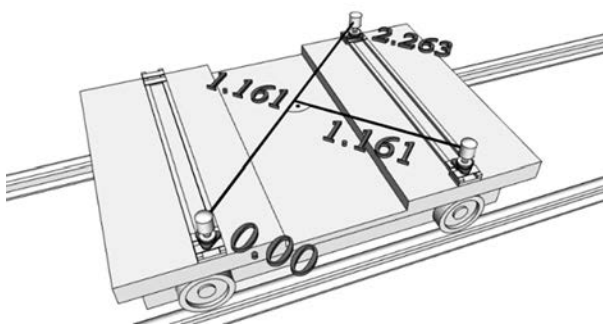


Figure 3. Staking out the position of the GNSS receiver on the platform using the orthogonal method

In a conventional geodetic approach, projecting the staked-out point onto the measurement line is followed by the calculation of the distance along the baseline and that of the offset. It is noteworthy that rectangular left-hand offsets for the measurement line have a negative sign. The flat coordinates of the point are calculated based on the known coordinates

of the points which mark out the measurement line from the following formula:

$$\mathbf{P} = \mathbf{P}_0 + \mathbf{R} \cdot \mathbf{O} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \cos(A) & -\sin(A) \\ \sin(A) & \cos(A) \end{bmatrix} \begin{bmatrix} l \\ h \end{bmatrix} \quad (3)$$

where:

- P, P<sub>0</sub>** – vectors of the coordinates for the point being measured  $(x, y)$  and the starting point of the measurement line  $(x_0, y_0)$ ,
- R** – rotation matrix for the azimuth  $A$ ,
- O** – linear observation matrix for the distance along the baseline  $l$  and the offset  $h$ .

In the context of the measurement assignment that is presented here, the opposite assignment was carried out. The measurement line was then created based on the position of two GNSS receivers that were facing each other. The point being staked out (the next GNSS receiver) was projected onto the measurement line, which made it possible to calculate the distance along the baseline as well as the rectangular offset. This task requires the canonical equations to be determined: a straight line co-linear with the measurement line and a line which is perpendicular to it and runs through the projected point  $P$  (Figure 4).

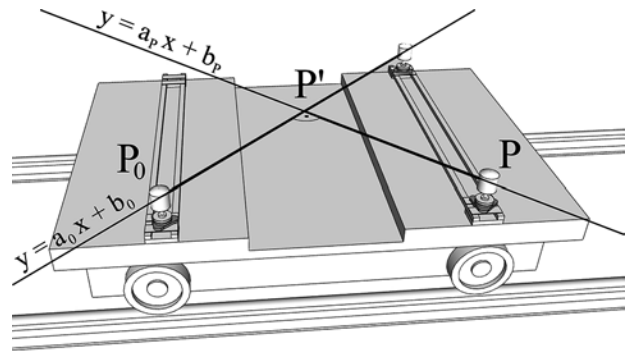


Figure 4. Staking out the position of the GNSS receiver on the platform using the orthogonal method

The slope  $a$  and the intercept  $b$  for the lines can be determined from the following formulas:

$$a_0 = \tan(90^\circ - A) \quad (4)$$

$$b_0 = y_0 - x_0 \tan(90^\circ - A) \quad (5)$$

$$a_p = -\cot(90^\circ - A) \quad (6)$$

$$b_p = y_p + x_p \cot(90^\circ - A) \quad (7)$$

The solution of a system of two equations of straight lines is the vector of the flat coordinates of

point  $P$  projected onto the measurement line ( $P'$ ). As a matrix, the vector is calculated from the following relationship:

$$\mathbf{X} = -(\mathbf{A}^T \mathbf{A})^{-1} (\mathbf{A}^T \mathbf{L}) \quad (8)$$

where:

$\mathbf{X}$  – vector of the coordinates of point  $P$ 's projection ( $P'$ ) onto the measurement line  $[x_{P'}, y_{P'}]^T$ ,

$\mathbf{A}, \mathbf{L}$  – matrix of the slopes as  $\begin{bmatrix} a_0 & -1 \\ a_P & -1 \end{bmatrix}$ , and the matrix of the intercepts as  $[b_0, b_P]^T$ .

The distance along the baseline and the offset can be calculated with the formulas:

$$l = \sqrt{(x_{P'} - x_0)^2 + (y_{P'} - y_0)^2} \quad (9)$$

$$h = \text{sign} \sqrt{(x_{P'} - x_P)^2 + (y_{P'} - y_P)^2} \quad (10)$$

where:

$$\text{sign} = \begin{cases} 1 & \text{for } (A_P - A) \geq 0 \\ -1 & \text{for } (A_P - A) < 0 \end{cases},$$

$A, A_P$  – azimuths of the measurement line and the section: starting point of the measurement line – projected point  $P$ .

The linear values determine the point's position relative to the assumed measurement line. This procedure is often used during geodetic work which involves the staking out of points to determine the point's positions in accordance with the design assumptions.

## Measurement and results

Preparations for the satellite measurements of the railway track as part of the research project involved

setting the correct geometric configuration of the ten receivers used on the two measurement platforms (Figure 5). The instrument settings presented here enabled the position of the track axis to be independently determined several times. The first determination was conducted by the central receiver of the measurement platform. Subsequent determinations came from the extreme receivers as the middle point of both diagonals. The last determination was obtained by calculating the coordinates of the point at which the lines (diagonals of the square) intersected. The solution applied here has an additional advantage: mutual control of successive determinations of the track position; this significantly improves their reliability.

All of the five GNSS receivers were placed on tripod heads; these were fixed to steel frames that were attached to the tramway measurement trolley. Ensuring the central position of the receiver, both with respect to the track axis and on the diagonals of the square formed by the other four GNSS receivers, was the key condition for the planned experiment. Therefore, it was then justified to start the geodetic work with the central receiver and, based on its position, to determine the position of the other GNSS receivers at the square's vertices. The measurements were conducted with a Leica TPS 1103 electronic total station. As the measurement trolley was rigidly positioned on the track, not every point situated on the longitudinal trolley axis was positioned above the track axis. Considering the receiver configuration, a compromise was adopted where the main receiver was situated on the kingpin. Only one point indicates the position of the track axis in curvilinear track sections. In this experiment, it was marked with a notch on the trolley's surface. In order to verify the reliability of the point, an independent staking out of the track axis was conducted based on the

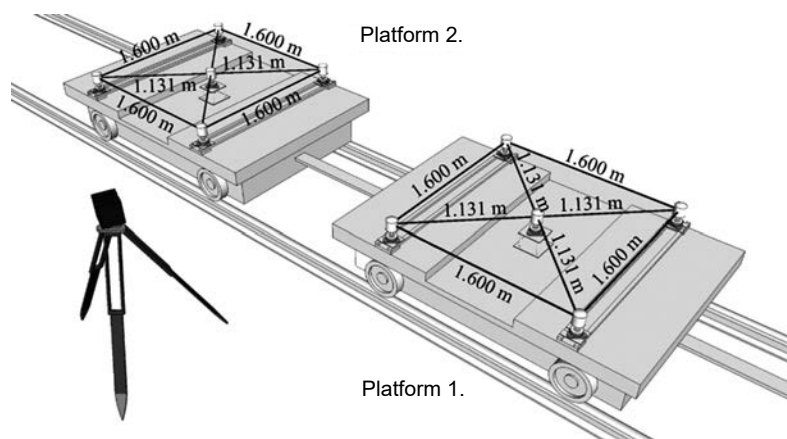


Figure 5. Mutual spatial relationships of the GNSS receivers on the measurement platforms



railway track's cross-sections in front of and behind the measurement trolley. The verification involved measuring points on the rails, calculating the central points of two sections of the cross-sections (points on the track axis) and subsequently, based on the calculated points, staking out a rectilinear section of the track axis. The measurements confirmed that the point on the trolley platform was marked correctly. A steel base with a 5 cm hole was then screwed to the welded bolts above the indicated central point. Owing to the construction of the tripod head, it was then possible to install various devices and measurement devices in its tang, such as the GNSS receivers or conventional measurement prisms. The others are applied in tachymetric measurements and allow millimeter accuracy of the distance between the instrument and the prism to be maintained. Taking advantage of the high accuracy of the method, the position of the central tripod head was staked out; after the measurement was completed, the prism was removed and replaced with a GNSS receiver. Therefore, one of the two conditions of the satellite measurement conditions was fulfilled.

The second stage of the geodetic work involved constructing a square, with the other four GNSS receivers at its vertices. An additional difficulty that was faced during staking out was the need to make sure that the diagonals intersected at the point at which the first receiver was situated. The external receivers were fixed to two metal frames which were situated transversely with respect to the track's direction. There were systems of two steel elements on the frame's edges, with a gap that enabled the installation of the tripod heads. The 3 cm distance between the longitudinal steel profiles ensured a limited range of movement of the tripod head. Staking out of the GNSS receiver's position was initiated by placing the tripod head with the prism onto one of the transverse frames on one of the track rails; therefore, one of the square diagonals was materialized. The position of both tripod heads was measured tachymetrically, which yielded three-dimensional coordinates  $x, y, z$  of the points in the local coordinate system of the electronic total station. In the next step, the position of the opposite square's vertex was then determined. The orthogonal staking out mode in the total station was applied for this purpose; the coordinates (1) obtained in the tachymetric measurement were relative to the measurement line determined by the two tripod heads installed on the mobile platform. Each time this procedure was carried out it resulted in presenting the distance along the baseline (9) and the offset (10) for the point being measured; three of

the points considered are co-linear if the offset value is zero. If the offset is not zero, it is necessary to move the tripod head within the tolerance range, which is provided by a gap between the steel elements crowning the frame. When the expected result was achieved, a tachymetric measurement was performed and the coordinates of the third point on the first square's diagonal were recorded.

The next part of the measurements involved positioning two vertical GNSS receivers on the other diagonal of the square. This was done using the geometric properties of the two diagonals of a square, which intersect at a right angle. Thus, in the case of the second diagonal of the square, a condition that has to be fulfilled is that the distance along the baseline (10) must be equal to the distance between the start of the measurement line (the external GNSS receiver) and its end (the middle GNSS receiver). Therefore, the position of the last two vertical receivers on the second diagonal will have an offset with the same absolute value, but with opposite signs. Staking out was based on the measurement line used earlier. The accuracy of the GNSS receiver's position was verified in a similar way as done previously, i.e., by placing the prism on consecutive tripod heads and adjusting any incorrect settings. When the assumed position was achieved, the last two tripod head's positions were measured tachymetrically and their three-dimensional positions were then recorded. There is one more noteworthy measurement aspect: when the prisms were removed from the tripod heads and replaced by the GNSS receivers, the phase centers of the antennas were not positioned at the same height as the prism's center; the vertical offsets of the receivers and the prisms were determined relative to the horizontal plane of the tripod's head in order to precisely determine the phase center position. They were then used as the basis for calculating the height coordinate, which was to be added to the height coordinate of the prism. The prism center coordinates for both measurement platforms are presented in Table 1.

**Table 1. Three-dimensional coordinates of the prism centers on the mobile measurement platform**

GNSS receiver	Platform 1			Platform 2		
	$x$ [m]	$y$ [m]	$z$ [m]	$x$ [m]	$y$ [m]	$z$ [m]
Front right	95.647	93.057	0.828	98.117	96.077	0.885
Back right	96.654	94.304	0.839	99.128	97.315	0.882
Back left	97.899	93.292	0.835	100.364	96.309	0.880
Front left	96.883	92.043	0.840	99.360	95.065	0.880
Middle	96.776	93.177	0.738	99.239	96.191	0.881

**Table 2. Deviations of the linear measures between the project data and the data measured on the platform**

Linear element	Platform 1			Platform 2		
	Design	Measurement	Difference	Design	Measurement	Difference
Top	1.600	1.599	-0.001	1.600	1.603	0.003
Right	1.600	1.603	0.003	1.600	1.598	-0.002
Bottom	1.600	1.604	0.004	1.600	1.594	-0.006
Left	1.600	1.610	0.010	1.600	1.599	-0.001
Semi-diagonal 1	1.131	1.135	0.004	1.131	1.128	-0.004
Semi-diagonal 2	1.131	1.134	0.002	1.131	1.129	-0.002
Semi-diagonal 3	1.131	1.129	-0.002	1.131	1.131	0.000
Semi-diagonal 4	1.131	1.139	0.008	1.131	1.132	0.001

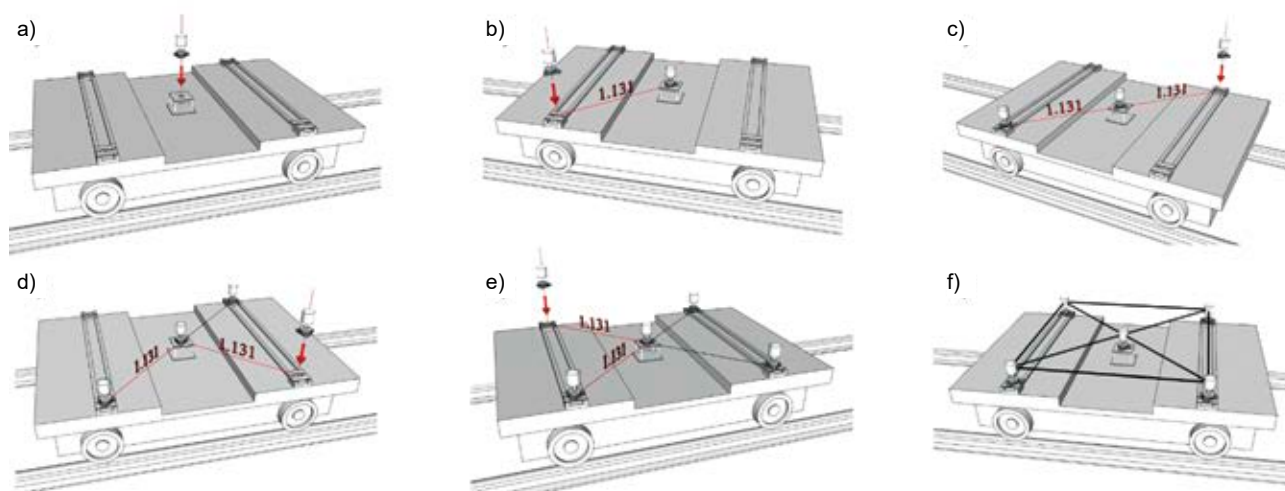
This spatial configuration was subsequently compared to the designed configuration. The differences in the linear measures between the assumed design data and the data obtained in the field for both measurement platforms are presented in Table 2.

The columns representing the differences between the design data and those staked out on the platform indicate that the receivers were accurately positioned with an error that did not exceed 0.010 m. It is noteworthy that only three sections out of the sixteen had an error that exceeded 0.004 m. Additionally, the prism coordinates on the central tripod heads differed from the coordinates of the point that was determined as the intersection of the two pairs of diagonals with the value of  $\Delta x = -0.008$  m and  $\Delta y = 0.000$  m on the first platform and  $\Delta x = 0.005$  m and  $\Delta y = 0.002$  m on the second. Thus, the second of the planned goals was achieved, i.e., another independent determination of the track axis, which coincides with the position of the central receiver with an error that did not exceed 1 cm. It is noteworthy that

some difficulty occurred as a result of it being impossible to place the transverse frames on the platform in the required position relative to the middle receiver. This was a result of the presence of fixed construction elements in the measurement trolleys. The stages of the staking out procedure for the receivers on the measurement platform are shown in Figure 6.

## Conclusions

The track axis at a certain point can be determined twice independently on the same run by applying several GNSS receivers deployed in a specific manner on a mobile measurement platform. The geometry of the five-receiver measurement equipment presented in this paper is based on the shape of a square. Four GNSS receivers are situated at the square's vertices and the fifth at its center of gravity, i.e. at the intersection of the diagonals. Satellite measurements (conducted both in real-time and by recording raw observation data) yielded synchronous positions of



**Figure 6. The procedure of staking out the five receivers in the predefined geometric configuration: the central receiver above the trolley kingpin (a), staking out the two receivers on the diagonal of the square (b, c), staking out the other two receivers at the vertices (d, e), the final spatial configuration of the receivers (f)**

the GNSS receivers. In the data processing stage, the coordinates were used to analyze the position that was measured by the central receiver and compare it with the position calculated from the diagonal intersection point in the square defined by the other four external receivers. Acquiring reliable results requires the use of precision geodetic techniques to properly deploy the equipment on the measurement platform. The compactness of the measurement infrastructure (the frame and the support) is a different aspect, as it is not made for one type of means of transport in particular. The compactness of the solution is a significant advantage as it allows the apparatus to be installed on both tramway trolleys and railway cars. On the other hand, despite their geometric stability, rigid systems are less versatile and are usually intended for use on one specific vehicle.

The methodology adopted in this study required that the satellite measurement preparation stage should include conducting staking out for the spatial configuration of the GNSS receivers. The tachymetric measurement method allowed for the determination of the position of the satellite receiver on the two platforms with an error not exceeding 0.010 m. This applied to both the linear measures in the square that was made from the frames and the position of the point that indicates the track axis. An important role in the staking out procedure was played by the orthogonal method, which determines the position of the points being analyzed with respect to the assumed measurement line. Despite the obstacles that resulted from the trolley's construction and occasional problems with mounting the frames for the receivers as planned, sub-centimeter accuracy was achieved. This is proof of the usability of the method at the principal satellite measurement stage, the main aim of which is to precisely determine the track axis. The measurement task can be performed within a relatively short time by a geodesist with an electronic total station. Staking out ten receivers and performing the control measurements took approximately 60 minutes in this measurement for the BRIK project. It is necessary to perform the staking out procedure for the GNSS receiver position with the utmost diligence and precision, given its huge impact on the accuracy of lengthy satellite measurements.

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