

# ISOTROPIC DAMAGE IN VISCOPLASTIC FLOW CONDITIONS, FEM APPLICATIONS WITH PRACTICAL EXAMPLES

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## **Abstract**

*The damage mechanics is a very important branch of solid mechanics. Although it is still developing, it has already been applied to many engineering problems. Among many various types of damage models, which are commonly used, the main subject of this paper is the isotropic continuous damage in viscoplastic flow conditions.*

*This paper include information about the viscoplastic type of constitutive modelling, the presentation of mechanical representation of damage by the scalar variable  $D$ , effective stress concept, equations of the chosen isotropic damage model, the Chaboche viscoplastic constitutive model including damage effects and the identification procedure of damage material parameters with practical example.*

*The chosen constitutive model is easy to apply to the finite element analysis. Using the MSC.Marc system, which great advantage is possibility of a user subroutine application, the geometry nonlinear finite element analysis of plate and shell structures including damage is possible.*

*This paper includes description of the applied program and user subroutines applied: UVSCPL - viscoplastic modelling (the standard MSC.Marc system supports the Chaboche model, but not in the damage variant) and UACTIV - deactivation of finite elements.*

*The last part of the paper includes the practical numerical example of a clamped bar of 0.3 m long and 0.1x0.01 m cross-section size. In calculations the four-node thin-shell, divided into five layers, elements were used. Dynamic, geometrically nonlinear analysis using Newmark integration algorithm has been performed. As the results, screenshots of displacements with maps of damage parameter for the bar in four time moments are presented.*

**Keywords:** *FEM, viscoplasticity, isotropic damage, constitutive modelling, numerical modelling*

## **1. Introduction**

Damage in metals is mainly the process of initiation and growth of microcracks and cavities in material's structure. The first, who introduce basics of the continuous damage mechanics, is Kachanov in 1958 [6]. He proposed a damage variable related to the density of defects and the effective stress concept.

Since Kachanov many theories of the continuous damage mechanics have been developed. In this paper the isotropic damage concept, proposed by Lemaitre [8], is used. This approach is introduced into FE analysis with the viscoplastic Chaboche model [3] employed. Additionally the practical example of identification of damage material parameters is shown.

The main goal of the present paper is to present an application of this damage concept in viscoplastic flow conditions to the geometrically non-linear FE analysis of plate and shell structures in the commercial program MSC.Marc.

## 2. Mechanical representation of damage

### 2.1. Damage variable

Let us consider a volume element in a damaged body at macro-scale that is of a size large enough to contain many defects and small enough to be considered as a material point of mechanics of continua (Fig. 1).

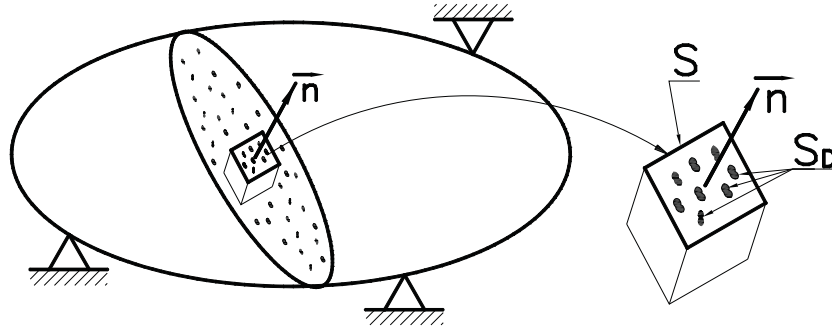


Fig. 1. Damaged element

Let us consider a plane cross-section, defined by its normal vector  $\mathbf{n}$ , cutting the volume element, where  $S$  is the overall cross-section area of undamaged material and  $S_D$  is the effective area of the intersections of all microcracks or cavities of damaged material. Thus, the damage variable  $D_{(n)}$  is defined by equation [8]:

$$D_{(n)} = \frac{S_D^{(n)}}{S^{(n)}}. \quad (1)$$

According to this definition, the damage variable depends on the choice of the normal  $\mathbf{n}$  and a tensor formulation should be used. Assuming isotropy of damage, which means uniform distribution of cracks and cavities in all directions,  $D_{(n)}$  does not depend upon  $\mathbf{n}$  and becomes to be a scalar value  $D$ .

### 2.2. Effective stress concept

The introduction of the damage variable, Equation (1), leads to the concept of the effective stress that is a stress calculated over the effectively resisting section [7]. In the presence of the isotropic damage the effective resistance area is given by equation:

$$\tilde{S} = S(1 - D) \quad (2)$$

and by definition the effective stress  $\tilde{\mathbf{s}}$  is given as:

$$\tilde{\mathbf{s}} = \frac{\mathbf{s}}{1 - D}, \quad (3)$$

where  $\mathbf{s}$  is the Cauchy stress tensor.

### 2.3. Hypothesis of strain equivalence

We assume as valid the strain equivalence hypothesis [9]: every strain behaviour of a damaged material is represented by constitutive equations of an undamaged material in the potential of which the stress is simply replaced by the effective stresses.

### 3. Constitutive equations

The constitutive equations of an isotropic material used in this paper are based on additive decomposition of the strain rate into its elastic  $\dot{\epsilon}^E$  and inelastic  $\dot{\epsilon}^I$  parts, which is valid for small inelastic strains:

$$\dot{\epsilon} = \dot{\epsilon}^E + \dot{\epsilon}^I \tag{4}$$

#### 3.1. Thermodynamics

The derivation of the constitutive equations for a given material can be done in the framework of the thermodynamics of irreversible phenomena through a certain number of variables called state variables [1]. The summary of the internal and associate variables is presented in (Tab. 1). To characterize material, a state potential is introduced. Usually, the Helmholtz free energy  $\psi$ , that is a scalar function of all the internal variables, is used.

The expression of the state potential can be determined taking into account the state coupling between variables [4]. It is possible to uncouple the state potential into the elastic behaviour with damage and inelastic (hardening), with the specific free energy being decomposed as:

$$\psi = \psi^E(\mathbf{e}^e, T, D) + \psi^I(\mathbf{a}, r, T) \tag{5}$$

Tab. 1. Summary of the internal and associate variables [10]

	State variables		
	Observable variables	Internal variables	Associated variables
Elasticity	Temperature, $T$ Strain tensor, $\epsilon_{ij}^E$		Specific entropy, $S$ Stress tensor, $\sigma_{ij}$
Viscoplasticity		Inelastic strain tensor, $\epsilon_{ij}^I$ Isotropic hardening variable, $r$ Kinematic hardening variable, $\dot{\mathbf{O}}$	Stress tensor, $\sigma_{ij}$ Isotropic hardening stress, $R$ Kinematic hardening stress, $\mathbf{X}$
Damage		Damage, $D$	Damage energy release rate, $Y$

The elastic part of the free energy function can be expressed as follows [8]:

$$\psi^E = \frac{(1-D)}{2\rho} (C_{ijkl} \epsilon_{ij}^E \epsilon_{kl}^E), \tag{6}$$

where  $\rho$  is density of a material,  $C_{ijkl}$  are components of the elasticity tensor.

According to the strain equivalence principle the stress component can be calculated as:

$$\sigma_{ij} = \rho \frac{\partial \psi^E}{\partial \epsilon_{ij}^E} = (1-D) C_{ijkl} \epsilon_{kl}^E. \tag{7}$$

The elastic strain components can be calculated by reversing of Equation (7):

$$\epsilon_{ij}^E = \frac{1+\nu}{E} \frac{\sigma_{ij}}{1-D} - \frac{\nu}{E} \frac{\sigma_{kk}}{1-D} \delta_{ij}. \tag{8}$$

The damage strain energy is defined as follows [8]:

$$Y = \rho \frac{\partial \psi^E}{\partial D} = -\frac{1}{2} C_{ijkl} \varepsilon_{ij}^E \varepsilon_{kl}^E. \quad (9)$$

If  $W_E$  is density of the elastic strain energy:

$$W_E = \int dW_E = \int \sigma_{ij} d\varepsilon_{ij}^E = \frac{1}{2} (1-D) C_{ijkl} \varepsilon_{ij}^E \varepsilon_{kl}^E, \quad (10)$$

we can establish the relation between the damage strain energy  $Y$  and  $W_E$ :

$$-Y = \frac{W_E}{1-D}. \quad (11)$$

Splitting the density of the elastic strain energy  $W_e$  into two parts: the shear energy part and the hydrostatic energy part we get the function  $Y$  [8]:

$$-Y = \frac{\sigma_{eq}^2}{2(1-D)^2 E} \left[ \frac{2}{3} (1+\nu) + 3(1-2\nu) \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2 \right], \quad (12)$$

where  $\nu$  is the Poisson's ratio,  $E$  is the Young's modulus of undamaged material,  $\sigma_{eq}$  is the Huber-Misses equivalent stress and  $\sigma_H$  is the hydrostatic stress both expressed as:

$$\sigma_{eq} = \left[ \frac{3}{2} (\sigma_{ij} - \sigma_H)(\sigma_{ij} - \sigma_H) \right]^{1/2}, \quad \sigma_H = \frac{1}{3} \sigma_{kk} \text{ sum over } k. \quad (13)$$

### 3.2. Dissipation potential

The thermodynamic state potential allows us to write relations between observable state variables and associated variables. For the internal variables it allows only the definition of their associated variables. To describe the evolution of the internal variables (the dissipation process) the dissipation potential function  $\varphi$  is needed [3].

In case of the viscoplasticity and damage it is possible to note from experimental observation that damage does not depend explicitly upon  $\tilde{O}$ ,  $R$ ,  $\mathbf{X}$ . Therefore, it is possible to separate the dissipation effect in damage and the viscoplastic flow contribution [2]:

$$\varphi = \varphi_D(Y, \dot{p}, D) + \varphi_I(\mathbf{s}, R, \mathbf{X}, D), \quad (14)$$

$$\dot{D} = -\frac{\partial \varphi_D}{\partial Y}, \quad \dot{\varepsilon}_{ij}^I = \frac{\partial \varphi_I}{\partial \sigma_{ij}}. \quad (15)$$

To describe the evolution of the damage variable the isotropic damage concept proposed by Lemaitre is used [7]. The dissipation function  $\varphi_D$  is written as a power function of  $Y$  and as a linear function with respect to accumulated plastic strain rate  $\dot{p}$ :

$$\varphi_D = \frac{S}{s+1} \left( \frac{-Y}{S} \right)^{s+1} \dot{p}, \quad \dot{p} = \sqrt{\frac{2}{3} \varepsilon_{ij}^I \varepsilon_{ij}^I}, \quad (16)$$

where  $S$  and  $s$  are damage material parameters.

The damage function is derived from Equation (15):

$$\dot{D} = \left( -\frac{Y}{S} \right)^s \dot{p}. \quad (17)$$

To describe the evolution of the inelastic strain, the viscoplastic constitutive Chaboche model [3] and the effective stress concept have been chosen. The dissipation function  $\varphi_I$  is written as:

$$\varphi_I = \frac{K}{n+1} \left\langle \frac{(J(\mathbf{s}' - \mathbf{X}')/(1-D)) - R - k}{K} \right\rangle^{n+1}, \quad J(a_{ij}) = \sqrt{\frac{3}{2} a^{ij} a_{ij}}, \quad (18)$$

where  $\mathbf{s}'$  and  $\mathbf{X}'$  are the deviatoric parts of the stress and kinematic hardening tensors, respectively,  $k$  is the initial yield stress,  $K$  and  $n$  are the viscous material parameters. The angle brackets  $\langle x \rangle$  are referred to the McCauley brackets:  $\langle x \rangle = \frac{1}{2}(x + |x|)$ .

The inelastic strain rate is derived from Equation (15):

$$\dot{\varepsilon}^I = \frac{3}{2} \dot{p} \frac{\mathbf{s}' - \mathbf{X}'}{J(\mathbf{s}' - \mathbf{X}')}, \quad (19)$$

where accumulated plastic strain rate is given by:

$$\dot{p} = \left\langle \frac{(J(\mathbf{s}' - \mathbf{X}')/(1-D)) - R - k}{K} \right\rangle^n. \quad (20)$$

The kinematic hardening tensor  $\mathbf{X}$  and the isotropic hardening scalar  $R$  are expressed as:

$$\dot{\mathbf{X}} = \frac{2}{3} a \dot{\varepsilon}^I - c \mathbf{X} \dot{p}, \quad \dot{R} = b(R_1 - R) \dot{p}, \quad (21)$$

where  $a, b, R_1$  are hardening material parameters.

#### 4. Damage material parameters – identification

The easiest method of damage material parameters identification is to carry out suitable quantity of uniaxial tensile experiments. A single experiment consists of a set of constant strain rate cycles with the constant amplitude of strain to obtain weakening of the elastic modulus (Fig. 2). As the result of experiments: elastic modulus, plastic strain and maximum value of the stress, for each cycle, are recorded (Tab. 2). This method assumes homogeneous character of damage.

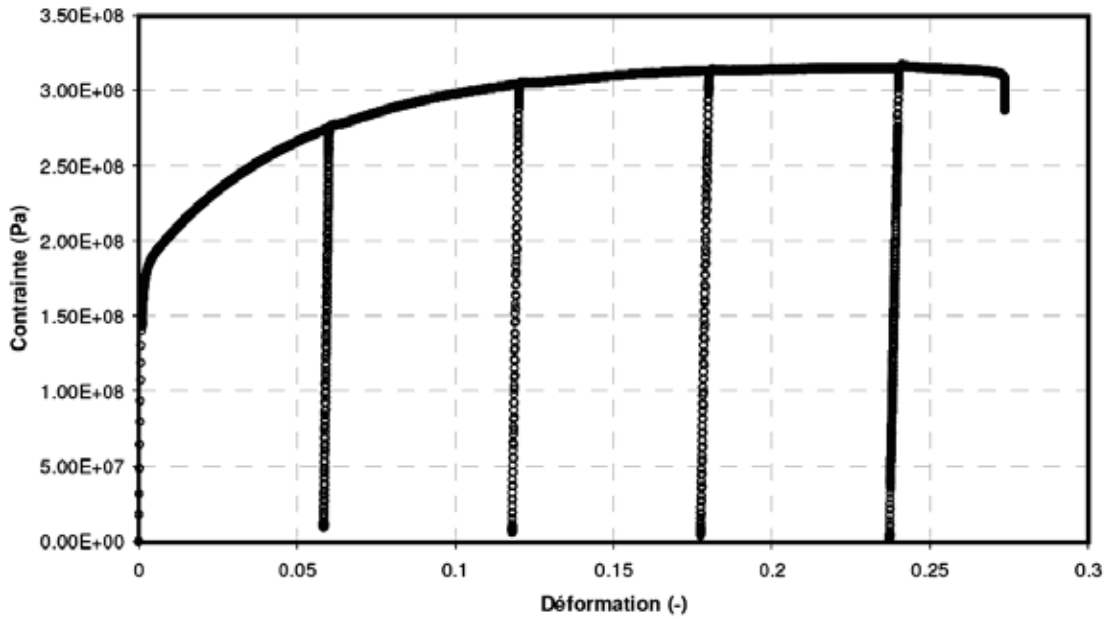


Fig. 2. Cyclic tension test – stress vs. strain – Steel DC01 [7]

According to hypothesis of effective stress the elastic strain can be expressed as follows [8]:

$$e^E = \frac{s}{E(1-D)} \quad (22)$$

Using Equation (22) it is possible to obtain the effective elastic modulus  $\tilde{E}$ :

$$\tilde{E} = E(1-D) \quad (23)$$

Reversing Equation (23) we can get the damage variable depending from the initial and effective elastic modulus emerges, which helps us to calculate  $D$  variable in every cycle (Tab. 2):

$$D = 1 - \frac{\tilde{E}}{E} \quad (24)$$

Tab. 2. Experimental results and calculated damage variable  $D$

DC01	Initial	Cycle 1	Cycle 2	Cycle 3	Cycle 4
Young modulus E [GPa]	163.4	150.2	131	118.5	109
Plastic strain [-]	–	0.0585	0.118	0.178	0.237
Stress [MPa]	–	274.8	304.1	313	314.6
D	0	0.0808	0.1983	0.2748	0.3329

Taking the damage function, Equation (17), and the damage strain energy function, Equation (12), in uniaxial loading conditions and substituting second to first, we get the function:

$$\dot{D} = \frac{dD}{dt} = \left( \frac{\sigma^2}{2E(1-D)^2 S} \right)^s \frac{d\varepsilon_{pl}}{dt}, \quad (25)$$

where  $\varepsilon_{pl}$  is inelastic strain in uniaxial loading conditions.

Then by reducing  $dt$  the following formula is obtained:

$$\frac{dD}{d\varepsilon_{pl}} = \left( \frac{\sigma^2}{2E(1-D)^2 S} \right)^s. \quad (26)$$

Selecting an appropriated function, which will approximate the function (26) will enable to identify the damage material parameters  $S$  and  $s$ :

$$D(\varepsilon_{pl}) = D_0 + a(1 - \exp(-b\varepsilon_{pl})). \quad (27)$$

On the basis of experiments data, it is possible, using the last square method, to calculate  $a$  and  $b$  parameters.

By comparison of the derivate function (27) with inelastic strain, which  $a$  and  $b$  parameters are known, with formula (26), using the last square method, we approximate the damage material parameters  $S$  and  $s$ :

$$\frac{dD(\varepsilon_{pl})}{d\varepsilon_{pl}} = ab \exp(-b\varepsilon_{pl}) = \left( \frac{\sigma^2}{2E(1-D)^2 S} \right)^s. \quad (28)$$

The approximation of the function  $D(\varepsilon_{pl})$  gives:  $a = 0.5392$ ;  $b = 6.1474$ , that leads to:  $\frac{dD(\varepsilon_{pl})}{d\varepsilon_{pl}} = 3,3147 \exp(-6,1474 \cdot \varepsilon_{pl})$ . Finally, using formula (28) the damage material parameters, which were sought, are:  $S = 0.6537$ ,  $s = -0.7986$ .

## 5. Application of the Chaboche model with damage to FEM

### 5.1. Description of the applied program and subroutines used

In the numerical analysis the MSC.Marc system has been used. It is a multipurpose FEM program for advanced engineering simulations, which great advantage is possibility user-defined subroutines application.

The viscoplastic Chaboche model with damage has been applied to the program using the UVSCPL subroutine (viscoplastic modelling). The algorithm used in the UVSCPL is presented in the form of flow chart (Fig. 3).

In the program there are also PLOTV (postprocessing of element variables) and UACTIV (deactivation of finite elements) subroutines applied. The first one to plot the user variable  $D$  defined in the UVSCPL, the second to deactivate finite elements, for which the value of the damage parameter  $D$  at all integration points of the element is greater then 0.17.

Additionally, the re-meshing feature with the equivalent plastic strain criteria (the value of 0.05) to subdivide elements were used. This approach creates additional nodes, but unlike the nodes of the virgin mesh, they are not independent – they translations and rotations are calculated on the basis of nodes of original mesh.

In calculations the four-node thin-shell elements (Element 139) divided into five layers were used. Dynamic, geometrically non-linear analysis using the Newmark integration algorithm has been performed.

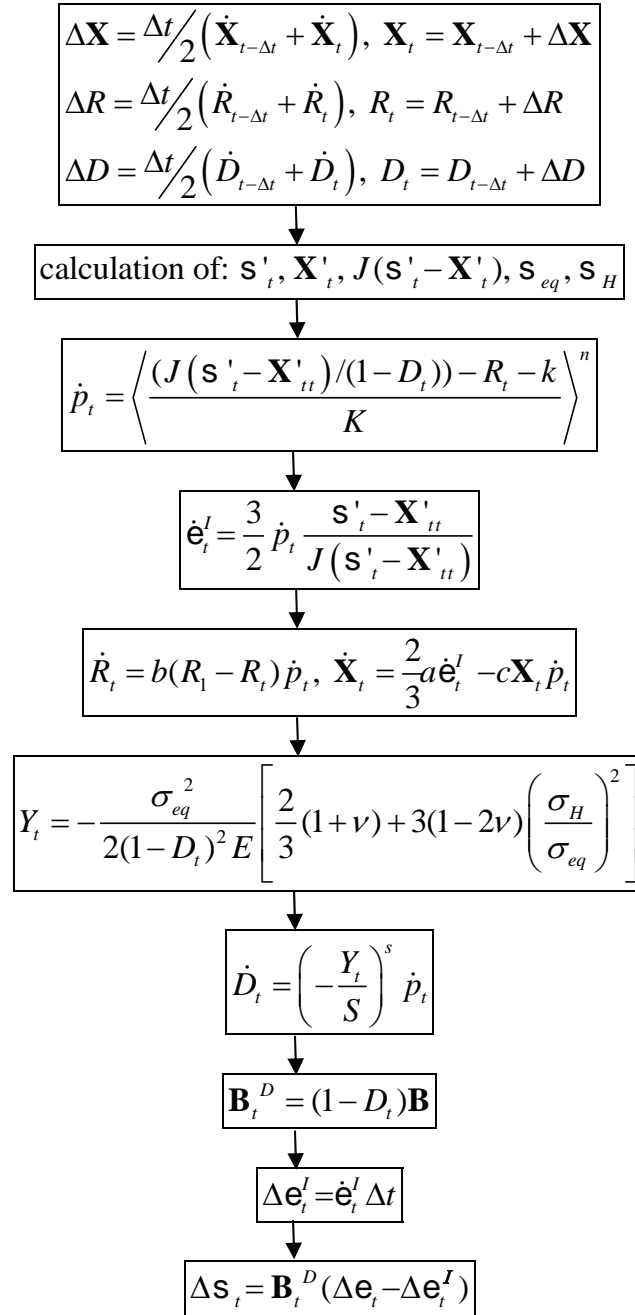


Fig. 3. UVSCPL subroutine – calculation algorithm

## 5.2. Example – a clamped bar

In the present example a clamped bar of 0.3 m long and 0.1x0.01 m cross-section size is considered (Fig. 4). The bar is loaded by edge force increasing linearly from 0 to  $F_{\max} = 8 \cdot 10^6$  N/m during 0.2 s.

The following material parameters were taken:  $E = 219$  GPa,  $\nu = 0.3$ ;  $\rho = 7900$  kg/m<sup>3</sup> the Chaboche model parameters [5]:  $k = 210$  MPa,  $b = 9.18$ ,  $R_1 = 138.48$  MPa,  $a = 535.5$  MPa,  $c = 64$ ,  $n = 1$ ,  $K = 267$  (MPa·s)<sup>1/n</sup> and the damage parameters  $S = 0.6537$  MPa,  $s = -0.7986$ .



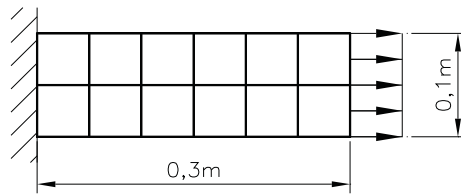


Fig. 4. Geometry and load imposing of a bar

As the results of the numerical calculation, the graphs of damage variable and strain in the time domain are presented (Fig. 5). For the presentation node of the element which is first deactivated is chosen.

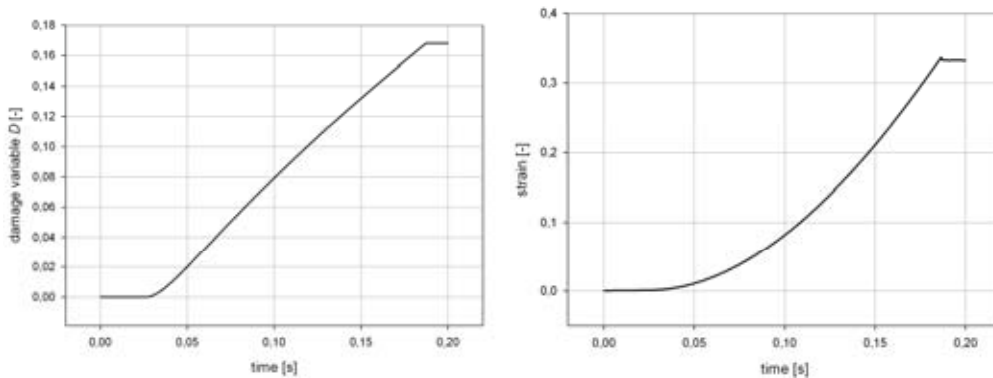


Fig. 5. Time function of the damage variable and strain in first deactivated element's node

Additionally, screenshots of displacement with maps of damage parameter for the bar in four time moments are presented in Fig. 6.

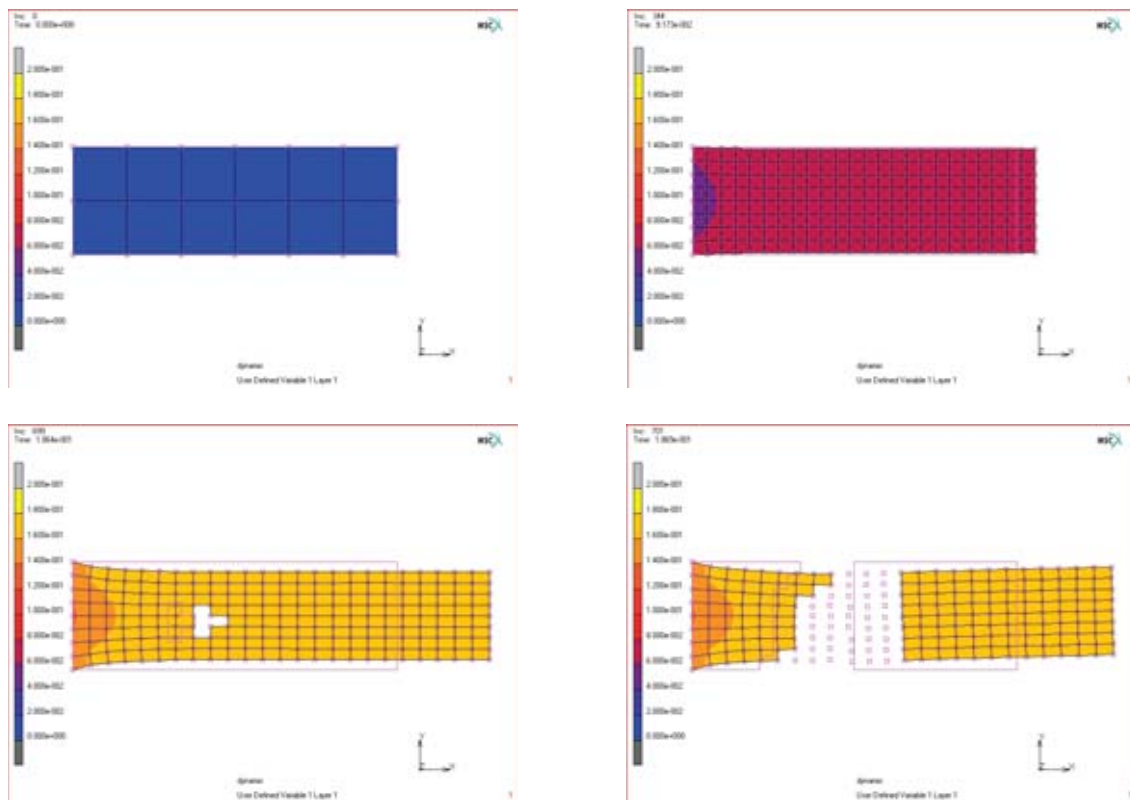


Fig. 6. Bar damage analysis ( $t_1 = 0$  s,  $t_2 = 0.0917$  s,  $t_3 = 0.1864$  s,  $t_4 = 0.1869$  s)

## 6. Conclusions

In the study the authors presented the model of isotropic damage based on a continuum damage variable and the concept of effective stress, which can be directly applied in numerical calculations, what has been done. Additionally the damage material parameters identification procedure is proposed. The obtained results encourage the authors to continue the research. More numerical examples will be shown during the conference presentation.

## Acknowledgements

Calculations presented in the paper were performed at the Academic Computer Centre in Gdansk (TASK).

Research was supported by Polish-French cooperation program Polonium.

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