

# Linear array of microstrip antennas with sector radiation pattern operating in the X-band

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**Abstract**—In this paper a linear antenna array utilizing sector radiation pattern is presented. The Fourier transform method is used for synthesis, from which the obtained excitation coefficients determine the resultant radiation pattern for an arbitrary sector. Moreover, the influence of a single radiating element radiation pattern is considered which is used to act as a counterweight to compensate the degradation which occur at the sector boundaries, by modifying the array factor. Theoretical examples for this phenomenon are presented, whereas for the purpose of this paper a nine-element linear array with 90° sector was designed and manufactured in order to verify this concept.

**Keywords**—sector radiation pattern, linear antenna array, Fourier synthesis method, microwave engineering

## I. INTRODUCTION

Modern wireless communication system tends to provide high throughput, stable connection, low latency and be broadband making the device less sophisticated from the RF viewpoint. All of these requirements are met when the wireless part of the system is properly designed, so there is a need to pay special attention on the antenna design. Nowadays it is done with the use of printed microstrip antennas [1, 2], although such single element structure is not capable of exhibiting specific type of radiation pattern and the bandwidth is limited which is crucial for fast transmission systems, hence antenna array concept was proposed [3, 4]. This solution consists of many microstrip radiating elements that are specifically distributed geometrically to enlarge the effective aperture and therefore gain as compared to a single radiator. The aim of this idea is to concentrate/intercept the EM energy in an arbitrary narrow sector. Such radiation pattern is formulated during the array synthesis [4, 5]. Depending on the application there is a tremendous freedom on the overall shape while designing an antenna array and that is why nowadays there is a demand on this type of antennas especially in beamforming, beam scanning, Direction of Arrival (DoA), Angle of Arrival (AoA) areas.

In this paper we present an analytical method for synthesis of an arbitrary sector radiation pattern, which compensates the drawbacks of a single element in the pattern multiplication rule.

## II. THEORETICAL DESCRIPTION

### A. Linear Antenna Array

Let us consider a linear array distributed evenly by distance  $d$ , along z-axis consisting of  $N$  identical radiating elements with the exact same radiation pattern, amplitudes and a constant progressive phase  $\beta$  between two consecutive antennas. For the simplicity of the assumptions the mutual coupling phenomena will be neglected [5]. For such array as presented in Fig. 1 the total far-field  $E_t$  is a sum of all

individual sources with constant phase difference  $\psi$  in respect to neighboring antennas:

$$E_t(\theta) = E(\theta) + E(\theta)e^{j\psi} + \dots + E(\theta)e^{jN\psi} \quad (1)$$

where

$$\psi = kd\cos(\theta) + \beta \quad (2)$$

With simple factorization of (1) one obtains:

$$E_t(\theta) = E(\theta)[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{jN\psi}] \quad (3)$$

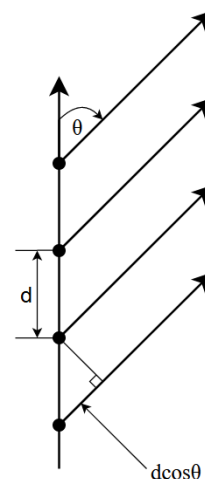


Fig. 1. Linear antenna array

Formula (3) is an analytical way to express the antenna array total field but it is also known as the pattern multiplication rule for continuous sources, in which  $E(\theta)$  is the radiation pattern of a single element whereas the factor in the square brackets is referred to as an array factor  $AF$ . This rule may be also applied to any defined array with exact same elements having arbitrary amplitudes, phases, distance and arrangement.

Predominantly array factor is a function of the number of radiating sources, its geometrical distribution, relative displacements and excitation which shapes the desired radiation pattern. One may notice that  $AF$  is independent of the radiating properties of each element, hence leads to the expression for normalized array factor:

$$AF(\psi) = \frac{1}{N} \left[ \frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{\psi}{2})} \right] \quad (4)$$

### B. Fourier Transform Method

This method makes use of the theorem proposed by Fourier which assumes that any sophisticated mathematical function can be described in terms of Fourier expansion coefficients. Since the transformed signal consists of

harmonic components, it perfectly matches with the beam shaping concept. Given a general description of the radiation pattern the weighted coefficients are determined, thus one obtains the necessary excitation signals that should be realized in the array. If the desired radiation pattern may be trivially expressed in the analytical way, then this method is the right choice. The approximated radiation pattern is more accurate if more expansion coefficients are determined which in practical viewpoint means that more elements in the array should be included. Whenever the demanded pattern has at some point a drastic change in the shape or contains a discontinuity, the resultant radiation pattern derived from the Fourier method will include at these sensitive points an oscillatory overshoot behavior which is referred to as Gibbs' phenomena. The only way to minimize that effect is to use a sufficient number of expansion coefficients while synthesizing the antenna array [6,7].

While dealing with this method few clarifications should be made, firstly the point of interest is the AF around the symmetry axis of the  $N$  discrete sources placed in the array, so two cases must be considered. One for odd number of elements ( $N = 2M + 1$ ) and second one for even number of elements ( $N = 2M$ ). In addition, when the average value over all angles of the synthesized radiation pattern is different than zero, then the case with odd number of elements must be considered, since the  $a_0$  expansion series coefficient acts as a DC term in classical Fourier series [5,6,8]. Secondly, since the  $AF$  is a periodic function  $\psi$  with period  $2\pi$ , the distance between consecutive discrete sources must be equal to  $d = \lambda_0/2$  for the sake of the condition  $2kd = 2\pi$  to satisfy the periodicity requirement for the visible region. Shortening the distance  $d$  will lead the  $AF$  to be non-periodic, hence in order to proceed with the Fourier series calculations it must be made pseudo-periodic with the use of fill-in functions. Such functions will lead to nonunique solutions since it may be arbitrary chosen, thus concluding in different results while synthesization process. Moreover, such displacement implies in superdirective behavior which means that the array will have a narrow beamwidth and bandwidth, not to mention about the mutual coupling effect. On the other hand, by making the distance  $d$  longer the level of coupling should be lower but in matter of fact more sidelobes will occur in the visible region, often grating lobes may also appear [3].

Assuming that the phase center of the linear array is its physical center at  $z = 0$ , then the array factor may be determined as:

- Odd number of elements ( $N = 2M + 1$ )

$$a_m = \frac{1}{2\pi} \int_{-\pi}^{\pi} AF(\psi) e^{-jm\psi} d\psi \quad -M \leq m \leq M \quad (5)$$

- Even number of elements ( $N = 2M$ )

$$a_m = \frac{1}{2\pi} \int_{-\pi}^{\pi} AF(\psi) e^{-j\left[\frac{(2m+1)}{2}\right]\psi} d\psi \quad -M \leq m \leq -1 \quad (6)$$

$$a_m = \frac{1}{2\pi} \int_{-\pi}^{\pi} AF(\psi) e^{-j\left[\frac{(2m-1)}{2}\right]\psi} d\psi \quad 1 \leq m \leq M \quad (7)$$

As proposed in [8], the excitation coefficients were determined in analytical way leading to difficult numerical problems at that time and in consequence the outcome results were sophisticated. In this particular case, if the desired sector was chosen, then for each radiating interval a unique solution of excitation coefficients had to be obtained. Therefore, we introduced an automated way of performing this synthesis

process, implying in more opportunities while designing systems where calculus is obligatory. The most common numerical methods that support solving definite integrals are: forward Euler, backward Euler and the trapezoidal methods. For the purpose of this paper only the last-mentioned technique was used for implementation of the Fourier transform method.

### C. Sector Radiation Pattern

The mathematical description of the sector radiation pattern is a square function and can be compared to a digital signal. It takes values  $0$  – for no radiation and  $1$  – for radiation, for arbitrary given interval  $\langle a; b \rangle$  as presented in Fig. 2. In addition, the radiation pattern is an even function around  $90^\circ$  due to broadside type of antenna. Therefore, the array factor may be expressed as [7]:

$$AF(\theta) = \begin{cases} 1 & , \psi \in \langle a; b \rangle \\ 0 & , \psi \notin \langle a; b \rangle \end{cases} \quad (8)$$

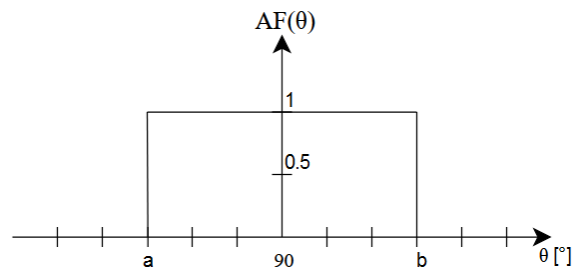
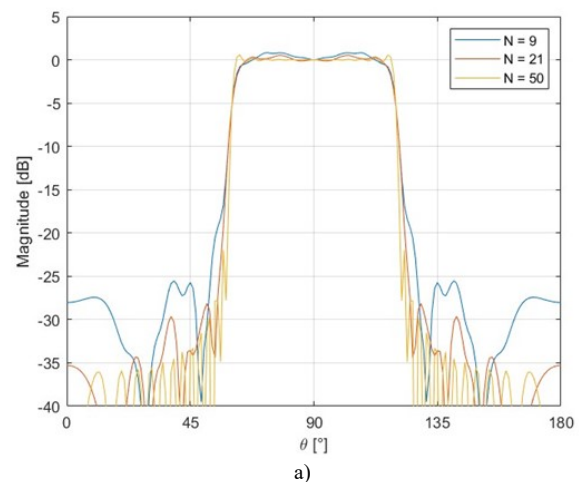


Fig. 2. Ideal array factor for sector radiation pattern

Fig. 3 represents the array factor for few cases of different aperture while varying the number of radiating elements used in the synthesis. If more radiators are used to approximate the sectoral radiation pattern, then the AF is converging to a flat response, recalling a square wave with less oscillations, hence damping the Gibbs' phenomena. This effect is due to the property of the Fourier Transform which states, that if the number of expansion coefficients tends to infinity, then the function is approximated more accurately. On the other hand, the number of sidelobes is also increasing because more discrete sources are present in the visible region.



a)

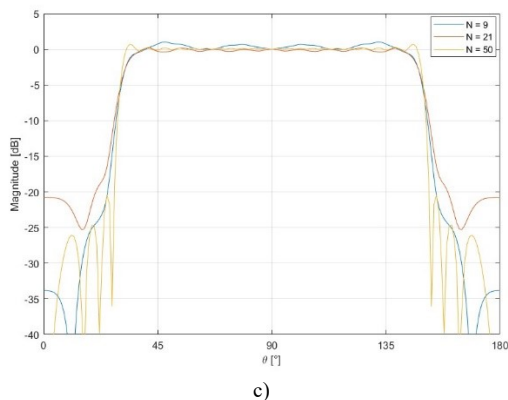
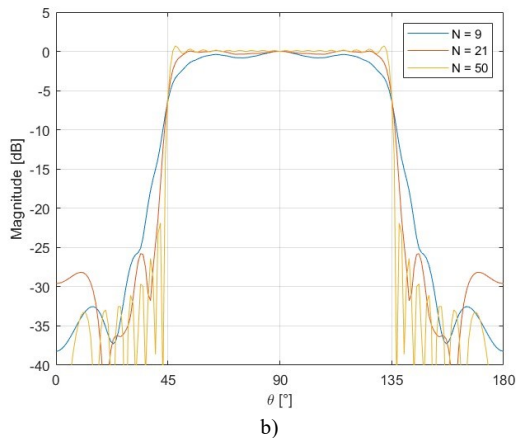


Fig. 3. Comparison of normalized array factors for 60° (a), 90° (b) and 120° (c) aperture

According to the pattern multiplication rule, the resultant pattern is a product of a single element radiation pattern and an array factor, the graphical representation is depicted in Fig. 4. While looking at the overall radiation pattern, it is noticeable that it does not remind the desired one. The main reason standing behind this unwanted phenomenon is the impact of the single radiating element resulting from the pattern multiplication rule. It is the consequence of exhibiting a pseudo parabolic type of radiation pattern which leads to a drastic drop at the boundaries of the sector.

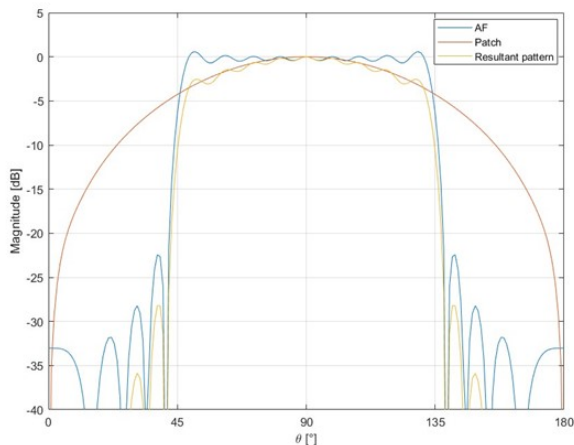


Fig. 4. Resultant radiation pattern with the influence of a single radiating element

In order to eliminate this side effect there are two options. The first one is to modify the radiation pattern of a single

element by making it omnidirectional or isotropic, nevertheless it is rather impossible to achieve such behavior. The second approach and it seems to be the key approach to this problem is to modify the desired radiation pattern with the use of the single radiator. The outcome of this maneuver will be a modified array factor which will compensate the influence of the single element on the resultant pattern. Therefore, the modified array factor can be precisely obtained by taking the exact radiation pattern of any designed patch antenna in the sector of interest and simply flip it to counterweight the original pattern of the radiator. In general case, the modified  $AF$  may be expressed as [8]:

$$AF(\theta) = \begin{cases} -p\psi^2 + q\psi + 1 & , \psi \in \langle a; b \rangle \\ 0 & , \psi \notin \langle a; b \rangle \end{cases} \quad (9)$$

For demonstration of the compensation effect, the same case as presented in Fig. 4 is taken into consideration. Fig. 5 concludes the entire investigation on the influence of the single radiating element, leading to satisfactory results.

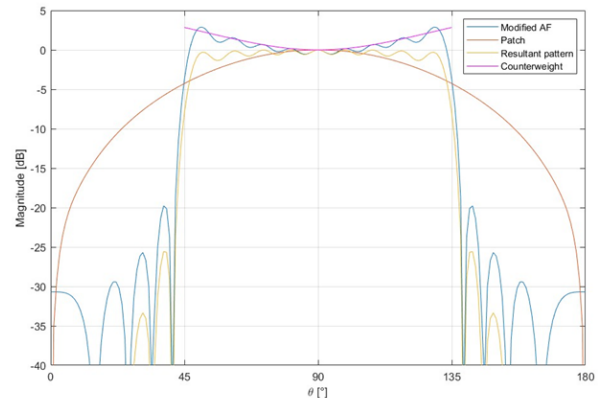


Fig. 5. Compensated radiation pattern

### III. DESIGN VERIFICATION

In order to proof the described concept a linear array with nine microstrip antennas was designed. The operating frequency  $f_o = 10$  GHz and  $\Theta = 90^\circ$  aperture. Isola I-Tera MT3.45 30 mils substrate was used with  $\epsilon_r = 3.45$  and  $\tan\delta = 0.0035$ .

#### A. Single Radiating Element

Firstly, a rectangular patch antenna with inset feed to maintain the planar structure of the antenna array. The overall performance of the patch is shaped by few factors like: width and length of the patch, inset placement with corresponding feeding line width responsible for impedance matching. After obtaining descent preliminary results the dimensions of the patch itself and the line impedance of  $50\Omega$  was kept fixed.

Table I. Microstrip patch dimensions

$W_{\text{patch}}$ [mm]	10.05
$L_{\text{patch}}$ [mm]	7.83
$W_{\text{inset}}$ [mm]	3.14
$L_{\text{inset}}$ [mm]	2.4
$W_{\text{line}}$ [mm]	1.71

By only optimizing the inset location, the final version of the radiator is obtained. The proposed dimensions and layout can be found in Table I and Fig. 6 respectively.



Fig. 6. Microstrip patch antenna

According to Fig. 7 the resultant curve for reflection coefficient is on descent level of almost -20 dB. Moreover, it is clearly visible that the patch resonates at the desired frequency of interest, specifically 10 GHz. The BW is approximately 2%. The radiation pattern exhibited by the designed radiator is a classical pattern closely related to the parabolic function, which radiates in the broadside direction as illustrated in Fig. 8.

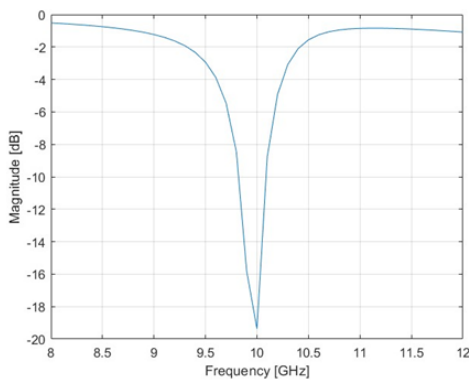


Fig. 7. Reflection coefficient for designed patch antenna

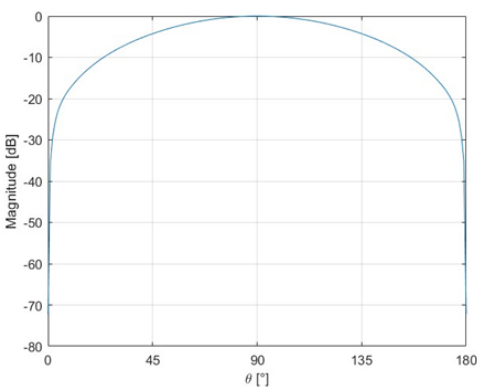


Fig. 8. Normalized radiation pattern for designed patch antenna

### B. Antenna Array

The Fourier transform method was used to determine the excitation coefficients for the sector radiation pattern. Values in Table II presents the initial values which does not take the influence of the single element radiation pattern and the compensated ones which were used in the design. Series feeding network was chosen due to several advantages.

Namely, the compactness, ease of controlling excitations and low degradation of the signal. Moreover, as one may noticed the excitation distribution for the assumed number of elements is consecutively descending, hence this configuration yields the best manipulation for optimization purposes. The fundamental concept standing behind the amplitude division in this type of network is based on quarter-wave impedance transformers. If four transformers, each having their specific characteristic impedance value of  $Z_o$  are alternately interconnected, then as a result a signal division is made from one end to another, such arrangement is referred to a single section as depicted in Fig. 9. In addition, since each transformer is quarter-wave long, thus has an electrical length of  $EL = 90^\circ$ . In consequence a constant net phase is obtained, ideally  $0^\circ$  shift. Mathematically it may be expressed as:

$$\frac{I_2}{I_1} = \frac{Z_2 Z_4}{Z_1 Z_3} \quad (10)$$

$$\sum_{i=1}^4 (EL)_i = 0^\circ \quad (11)$$

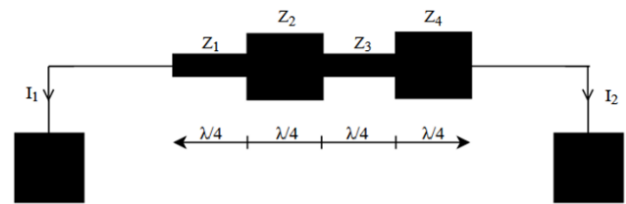


Fig. 9. Single feeding section

Table II. Derived excitation coefficients

Excitation coefficients	Initial AF	Modified AF
$a_0$	1	1
$a_{\pm 1}$	0.3581	0.306
$a_{\pm 2}$	-0.2168	-0.268
$a_{\pm 3}$	0.0558	0.082
$a_{\pm 4}$	0.0576	0.0627

Fig. 10 presents the simulated return loss. One may notice a minor shift of the resonance in comparison to the assumed one which occurs at 9.95 GHz. Such small error is acceptable since simulations only approximates the EM conditions. Moreover, the BW is near 3% which is the consequence of the narrowband behavior of the feeding network and patch itself. Fig. 11 depicts the designed antenna array.

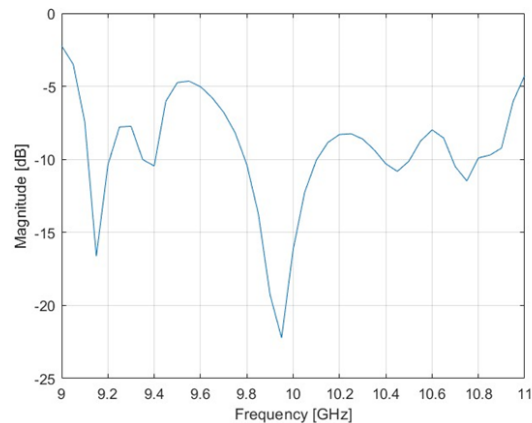


Fig. 10. Reflection coefficient of the designed antenna array



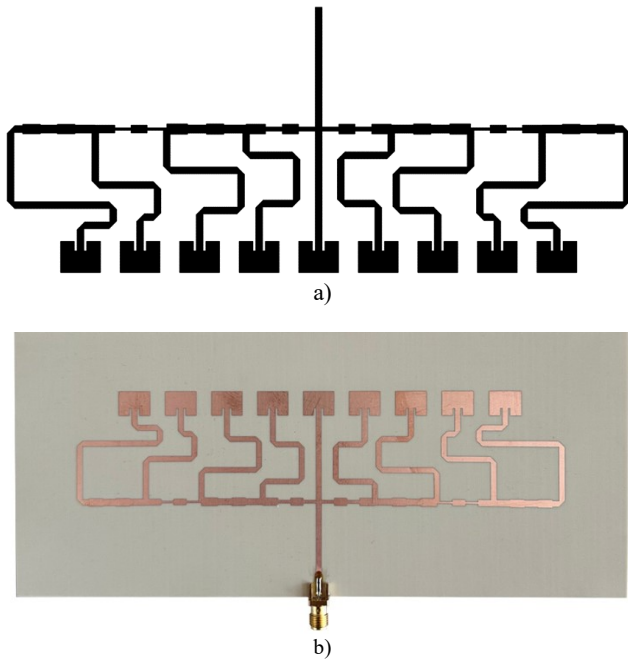


Fig. 11. Antenna array – layout (a), fabricated (b)

### C. Results

The most important stage was the measurement of the antenna array itself in real conditions, in order to verify if the desired and synthesized radiation pattern was achieved. For this purpose, the measurements were carried out in the anechoic chamber to minimize the reflections and propagation negative impacts as shown in Fig. 12.

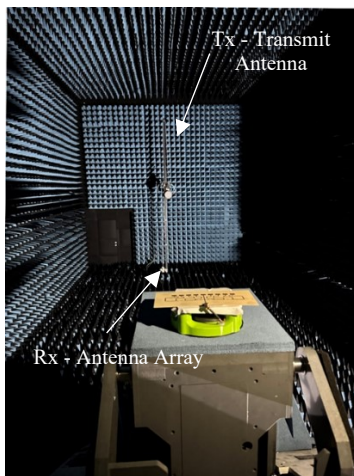


Fig. 12. Anechoic chamber setup

The first measurement carried out was the one to check the reflection coefficient. Fig. 13 presents the results and in comparison, to the theoretical ones as shown previously the main resonance got shifted to around 9.7 GHz, although the useful BW for  $VSWR < 2$  got doubled to value of 6%. Such bandwidth allows to investigate more radiation patterns around the design frequency with descent array performance. Fig. 14 shows the measured radiation pattern at nominal design frequency of 10 GHz in which the sectoral shape in the radiating range is retained, the sector is close to the required one, whereas the sidelobes are not clearly present. Fig. 15 - 16 presents the radiation patterns for different

neighboring frequencies, in which the sectoral behavior is similar to the nominal one, in addition for many frequencies the sidelobes are noticeable.

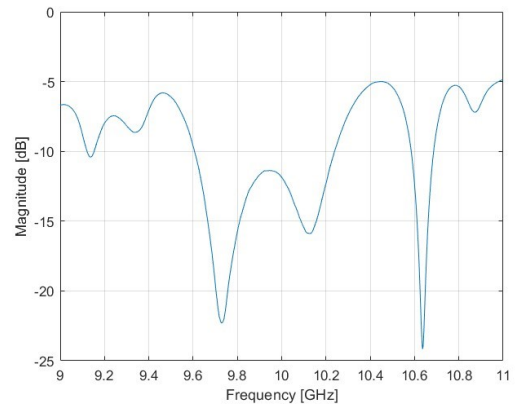


Fig. 13. Measured S11 of the antenna array

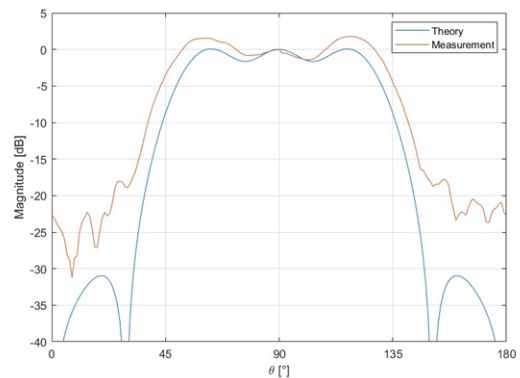


Fig. 14. Comparison of theoretical and measured radiation pattern at 10 GHz

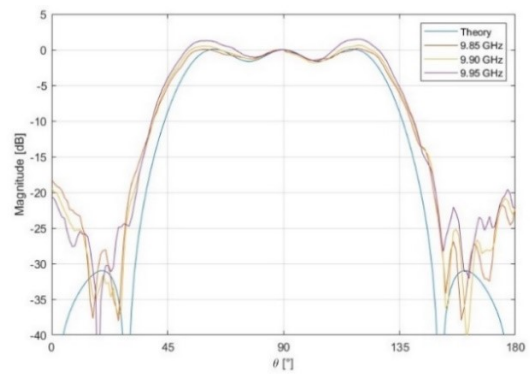


Fig. 15. Comparison of theoretical and measured radiation patterns for neighboring at lower frequencies

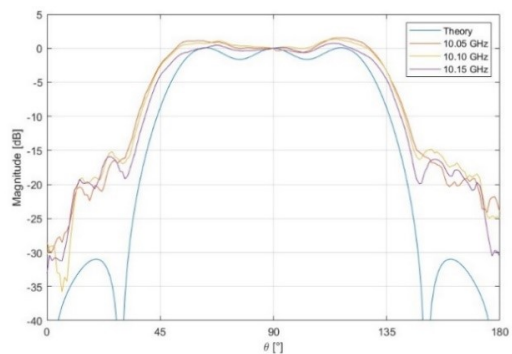


Fig. 16. Comparison of theoretical and measured radiation patterns for neighboring at higher frequencies

#### IV. CONCLUSION

The Fourier transform method is capable of synthesizing such analytical radiation pattern as the one described in this paper. With a simple enhancement of the array factor and with the use of modern numerical methods, one is able to eliminate potential drawbacks while using this method. It is important to guarantee a constant signal reception/transmission in the main beam without any fluctuations especially in this type of radiation pattern.

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