Paper

Low order autoregressive models for FDTD analysis of microwave filters

Piotr Kozakowski and Michał Mrozowski

Abstract — The forward-backward autoregressive (AR) model is applied to extract time signatures generated by the FDTD algorithm. It is shown that using simple techniques of model parameters selection one is able to reduce the model complexity for low and medium Q circuits.

Keywords* — *FDTD, signal processing, AR, filters.

1. Introduction

The finite-deference time-domain method is a versatile numerical technique that has been extensively used for solving electromagnetic problems. Due to its flexibility it has been applied to both simple problems and more complicated ones. However the main drawback of the FDTD method is a long computation time needed to analyze high Q circuits. In order to obtain the frequency domain characteristics of a circuit by means of the Fourier transform, a very long time-domain record of samples is needed. Premature termination of the simulation usually results in inaccurate extraction of narrow-band components in frequency-domain. In order to circumvent the aforementioned problem signal processing the system identification methods have recently been proposed to be used with the FDTD algorithm. Most of these techniques allow one to extract time signature features form a short segment of the original FDTD sequence. The number of methods such as autoregression, Prony's, general-pencil-of-function, to name but a few, have been employed to meet the challenge.

In this contribution we use a forward-backward autoregressive model for signal extrapolation. We show that using some simple techniques for model parameters determination it is possible to considerably reduce the numerical costs of signal extrapolation without the lost of accuracy of the method.

Applications of autoregressive method have been described in [3, 4, 6]. Nevertheless the model orders reported by the authors seem to be very large [3, 6]. This causes unnecessary numerical burden or even may lead to numerical instability of the algorithm used for extracting filter coefficients. Besides, most of the work has been confined to investigation of simple structures without considering the influence of Q-values on efficiency of the method.

Herein the application of AR method is illustrated by investigating the time signatures obtained from the FDTD simulation of three structures:

- a low Q pass-band microstrip filter [3],
- a three-section pass-band waveguide filter with medium Q [6],
- a high Q dual-mode waveguide filter.

All time signatures have been obtained using a commercial FDTD software QuickWave 3D [7].

2. Methodology

The autoregressive method is extensively described in signal processing literature [5]. For this reason we shall not describe the algorithm but rather concentrate on practical problems associated with its application to electromagnetics. Our purpose is to demonstrate that using the forward-backward autoregressive process [4, 5] with some simple techniques of model parameters selection it is possible to reduce model orders to the number of poles laying in the bound of interest.

If the sequence $x[1], x[2], \ldots, x[N]$ is to be modeled as an AR process of the p order the following model is assumed

$$x[n] = -a_1 x[n-1] - a_2 x[n-2] + \\ - \ldots - a_p x[n-p] + v[n], \qquad (1)$$

where $v[n]$ is a white-noise process. The constants $a_1, a_2, \ldots, a_p$ are determined using the least square method minimizing forward and backward errors.

The frequency response of the model is calculated from the filter coefficients using the following formula

$$H(j\omega) = \qquad (2)$$
$$= \frac{b_0 + b_1 \exp(-j\omega n\Delta t) + \ldots + b_p \exp(-j(p-1)\omega n\Delta t)}{1 + a_1 \exp(-j\omega n\Delta t) + \ldots + a_{p-1} \exp(-jp\omega n\Delta t)},$$

where the coefficients $b_1, b_2, \ldots, b_p$ are defined by the equation

$$\underline{\mathbf{b}}_p = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ a_1 & 1 & \ldots & 0 \\ a_2 & a_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{p-1} & a_{p-2} & \ldots & 1 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[p-1] \end{bmatrix}. \qquad (3)$$

The successful application of the AR method requires the knowledge of model parameters, namely the number of initial samples of the original FDTD record to be discarded, the number of samples required for model construction, a desampling factor and a model order.

Selection of training sequence. Selection of the segment to be used for training of the AR model is described in detail in [2]. In general the original FDTD record is divided into early and late time responses. The first part, dominated by transients, is discarded and only the other part is used for building model.

Desampling factor. Due to stability conditions of the FDTD algorithm the sampling period Δt_{FDTD} is much smaller that it is required by signal processing methods or the Nyquist formula. Leaving the value of Δt_{FDTD} unchanged entails problems with order selection and may lead to numerical instability of the method. This is because the model based on the oversampled sequence requires additional terms to extract the signal components situated outside the band. In our case the sampling period was increased by the factor:

$$k = \frac{1}{2\Delta t_{FDTD}\left(f_{max} + (t_0)^{-1}\right)}, \tag{4}$$

where t_0 corresponds to the time shift in the FDTD sequence. It can be shown that sampling the band-limited signals at exactly twice their maximum frequency is not sufficient, hence the additional factor $1/t_0$ in the formula (4).

Model order selection. Once the training sequence and desampling factor have been chosen one can pass on to the determination of the exact number of poles contained in the data. There are two commonly used statistics for selection of model orders the Akaike information criterion (AIC) and the minimum description length (MDL) [5]. Both of them are based on calculation of the predication error between the model and the data samples, using digital filtering as a result of modeling. The model order that ensures reaching the minimum of one of these measures is regarded as the best one. In our case the criteria are regarded as general guidelines for selecting model order and serve as a starting point for determination of the exact number of poles contained in the data. In order to investigate the possibility of model order reduction, the afore-mentioned criteria were modified in the following way. All terms outside the bands of interest were discarded. Thus the model order was lowered to the number of poles which lie in the band of interest. The approach, called a band limiting, considerably improved the efficiency of the model construction algorithm.

3. Numerical results

The method was verified by modeling of the three previously mentioned filters. In all cases model parameters were selected using the methods described above. The frequency-domain responses of the investigated structures were calculated directly from the filter coefficients of the AR model using an analytical formula. The error norms given in Table 1 were evaluated in the frequency-domain with reference to the Fourier transform of the original FDTD sequence.

In all cases, with model orders selected automatically (Figs. 1, 3, 5), the Fourier transforms obtained by combining the FDTD algorithm and the AR model were almost indistinguishable from those obtained directly from the FDTD record of samples.

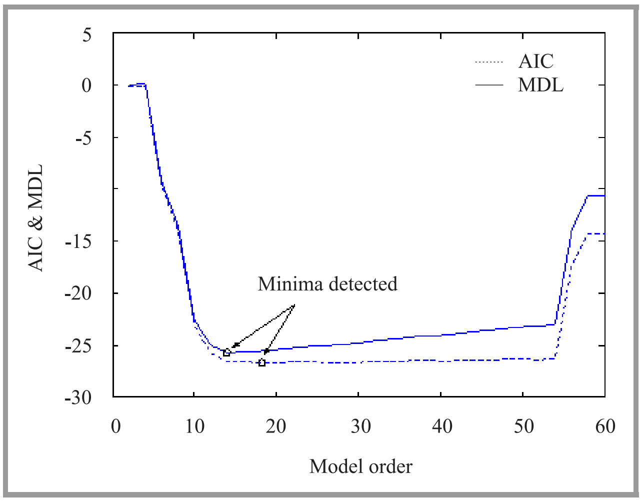

Fig. 1. Determination of the model order for the microstrip pass-band filter.

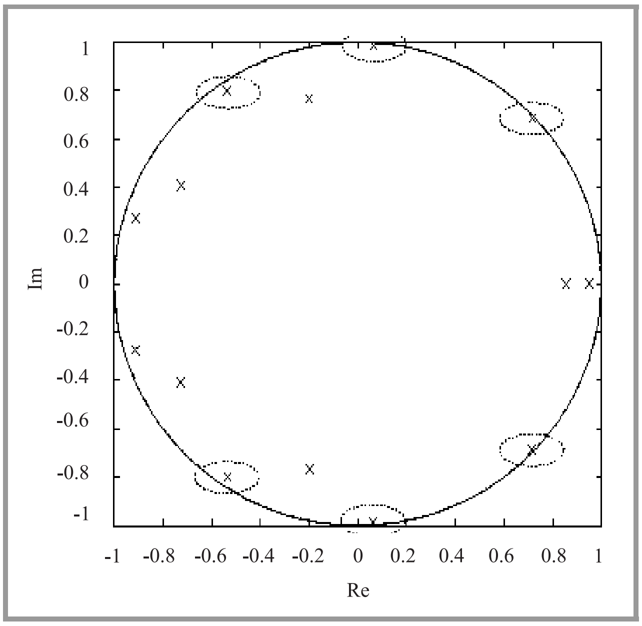

Fig. 2. Pole locations in complex plane for the microstrip pass-band filter (poles in band of interest are circled).

After reduction of filter orders (only poles from the band were taken into account (Figs. 2, 4, 6)) the original spec-

Table 1
Results of modeling

Parameters	Filter		
	low Q	medium Q	high Q
Frequency range	$0-12$ GHz	$33-38$ GHz	$10-12$ GHz
Desampling factor	56	25	27
First/last sample	3192/6832	3650/6150	5400/10044
Order selected automatically	14	28	56
Order selected after band limiting	6	6	8
Error norm with model (order selected automatically)	$2.0 \cdot 10^{-5}$	$2.35 \cdot 10^{-5}$	$2.74 \cdot 10^{-4}$
Error norm with model (order selected after band limiting)	$3.7 \cdot 10^{-5}$	$2.36 \cdot 10^{-5}$	$5.4 \cdot 10^{-2}$

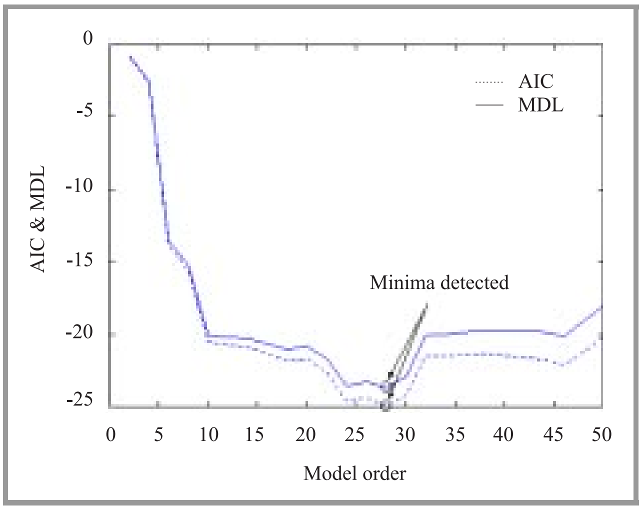

Fig. 3. Determination of the model order for the three-section waveguide filter.

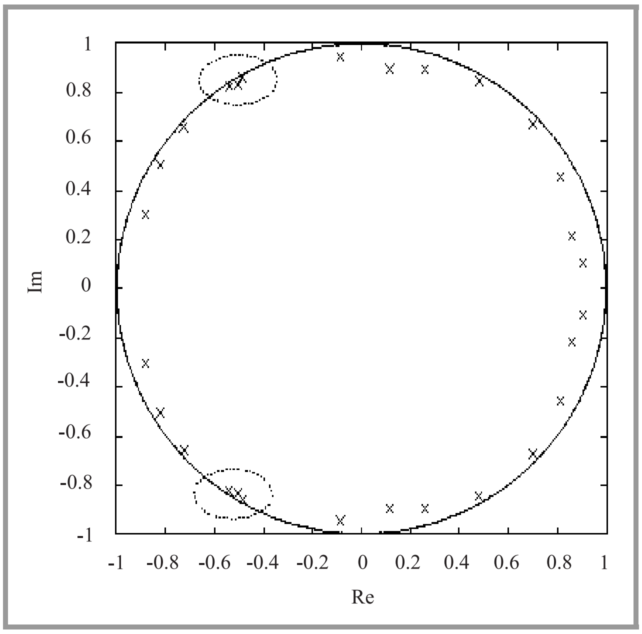

Fig. 4. Pole locations in complex plane for the three-section waveguide filter (poles in band of interest are circled).

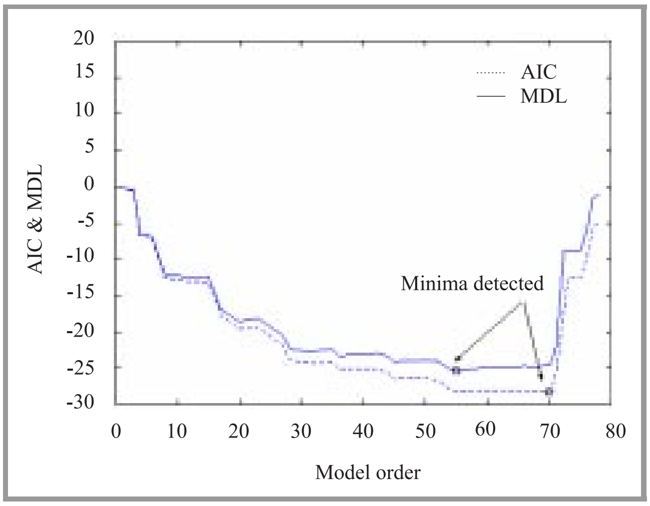

Fig. 5. Determination of the model order for the dual-mode waveguide filter.

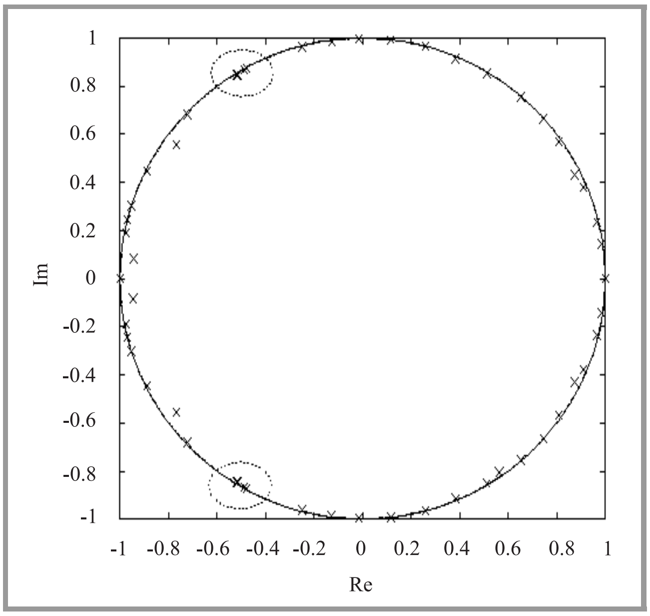

Fig. 6. Pole locations in complex plane for the dual-mode waveguide filter (poles in band of interest are circled).

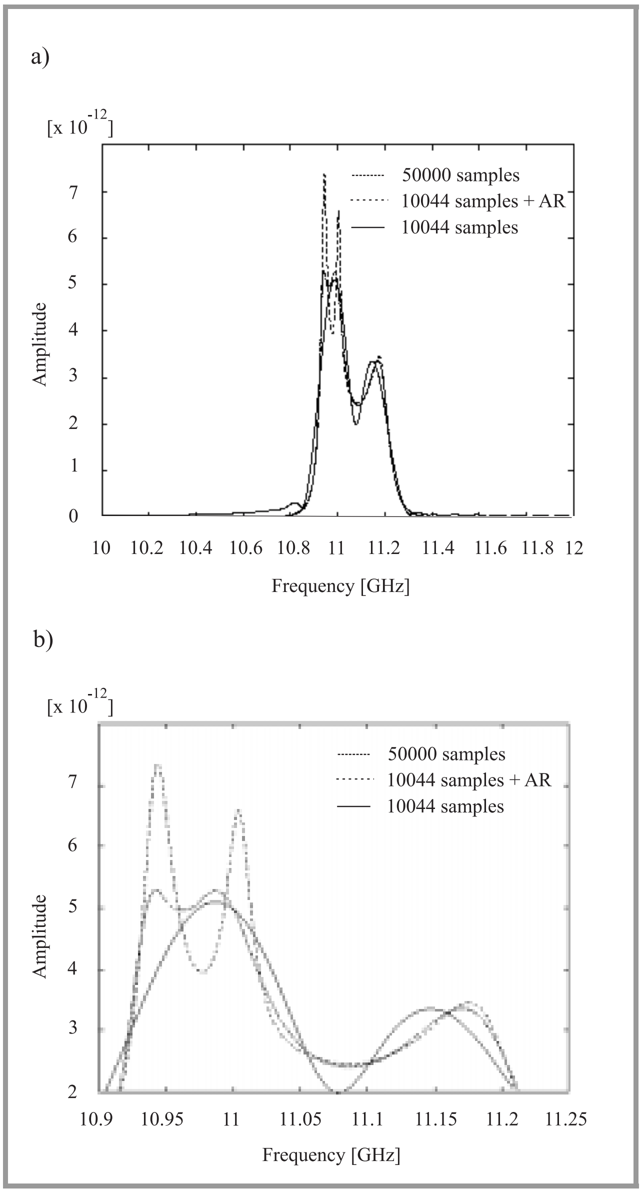

Fig. 7. (a) Amplitude spectrum of the dual-mode waveguide filter; (b) exploded view of the amplitude spectrum of the dual-mode waveguide filter.

tra were still predicted quite well in the cases of the low and medium Q circuits. The error norms, given in the last row of the table, are almost equal to those computed using the models with higher orders. For the low and medium Q structures the AR method together with the proposed parameters selection techniques yields superior results to those reported in [3, 6], where the model orders were established on the level of 50 for both the microstrip band-pass filter and the three-section waveguide filter. However, for the case of the high Q filter, the comparison of original spectrum with the estimated one showed some discrepancy (Fig. 7). The error norm is much higher compared to the case of the model of the 56 order. Thus only for low and medium Q structures with reduced model orders we were able to achieve good agreement between spectra.

4. Conclusions

The contribution demonstrated that using some simple techniques of the model parameters selection it is possible to considerably reduce model orders for low and medium Q circuits. The numerical results showed that, for the considered structures, extrapolation of signal signatures using the forward-backward AR with afore-mentioned parameters selection techniques yielded superior results to those obtained by other authors [3, 6]. Furthermore, in our case, the frequency spectra were calculated with an analytical formula involving the digital filter coefficients. This approach eliminates the need of the use of the Fourier transform and therefore it further reduces the numerical costs of the method as a whole.

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Piotr Kozakowski
e-mail: piotek@task.gda.pl
Department of Electronics, Telecommunications
and Informatics
Technical University of Gdańsk
Narutowicza st 11/12
80-952 Gdańsk, Poland

Michał Mrozowski
e-mail: mim@pg.gda.pl
Department of Electronics, Telecommunications
and Informatics
Technical University of Gdańsk
Narutowicza st 11/12
80-952 Gdańsk, Poland