

# Mathematical model defining volumetric losses of hydraulic oil compression in a variable capacity displacement pump

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## ABSTRACT

*The objective of the work is to develop the capability of evaluating the volumetric losses of hydraulic oil compression in the working chambers of high pressure variable capacity displacement pump. Volumetric losses of oil compression must be determined as functions of the same parameters, which the volumetric losses due to leakage, resulting from the quality of design solution of the pump, are evaluated as dependent on and also as function of the oil aeration coefficient  $\epsilon$ . A mathematical model has been developed describing the hydraulic oil compressibility coefficient  $k_{lc|\Delta p_{pi};b_p;\epsilon;v}$  as a relation to the ratio  $\Delta p_{pi}/p_n$  of indicated increase  $\Delta p_{pi}$  of pressure in the working chambers and the nominal pressure  $p_n$ , to the pump capacity coefficient  $b_p$ , to the oil aeration coefficient  $\epsilon$  and to the ratio  $v/v_n$  of oil viscosity  $v$  and reference viscosity  $v_n$ . A mathematical model is presented of volumetric losses  $q_{Pvc|\Delta p_{pi};b_p;\epsilon;v}$  of hydraulic oil compression in the pump working chambers in the form allowing to use it in the model of power of losses and energy efficiency*

**Keywords:** hydrostatic drive, variable capacity displacement pump, volumetric losses of oil compression, mathematical model

## Introduction

In references [1–4], this author attempted to evaluate the influence of working liquid compressibility on the picture of volumetric and mechanical losses in a high-pressure variable capacity displacement pump. The considerations were based on the assumptions made by the author in the developed theoretical and mathematical models of torque of mechanical losses in the pump used in the hydrostatic drive [5–7]. The models assume, that **increase  $\Delta M_{Pm|\Delta p_{pi};q_{Pgv}}$  of torque of mechanical losses in the pump „working chambers - shaft” assembly**, compared with torque of losses in that assembly in a no-load pump (when indicated increase

$\Delta p_{pi}$  of pressure in the working chambers equals to zero ( $\Delta p_{pi} = 0$ )), **is proportional to torque  $M_{Pi}$  indicated in the pump working chambers:**

$$\Delta M_{Pm|\Delta p_{pi};q_{Pgv}} \sim M_{Pi|\Delta p_{pi};q_{Pgv}} \quad (1)$$

In references [1–4], the author introduced also **the working liquid compressibility coefficient  $k_{lc|p_n}$** . It determines the degree of decrease of the active volume of working liquid displaced by the pump during one shaft revolution as an effect of increase  $\Delta p_{pi} = p_n$  of pressure in pump working chambers equal to the pump nominal pressure  $p_n$  (nominal pressure  $p_n$  of the hydrostatic drive system where the pump

operates). The pump active working volume  $q_{Pt|p_n}$  or  $q_{Pgv|p_n}$  is smaller compared with the active volume equal to theoretical working volume  $q_{p_i}$  or geometrical working volume  $q_{p_{gv}}$  (determined at the increase  $\Delta p_{p_i}$  of pressure equal to zero –  $\Delta p_{p_i} = 0$ ). It decreases then the indicated torque  $M_{p_i}$  and indicated power  $P_{p_i}$  in the pump working chambers, which can be generated (and calculated) with the increase  $\Delta p_{p_i}$  of pressure in the chambers, for example with  $\Delta p_{p_i} = p_n$ . In effect, decreases also torque  $M_p$  on the pump drive shaft and power  $P_{pc}$  consumed by the pump on shaft that the pump driving motor can be loaded with.

The  $k_{lc|p_n}$  coefficient is described by the expressions:

$$k_{lc|p_n} = \frac{q_{Pt} - q_{Pt|p_n}}{q_{Pt}} \quad (2)$$

and

$$k_{lc|p_n; b_p} = \frac{q_{Pgv} - q_{Pgv|p_n}}{q_{Pgv}} \quad (3)$$

Coefficient  $k_{lc|p_n}$  of the working liquid compressibility can also be described by the formulae:

$$k_{lc|p_n} = \frac{q_{Pvc|p_n}}{q_{Pt}} \quad (4)$$

and

$$k_{lc|p_n; b_p} = \frac{q_{Pvc|p_n}}{q_{Pgv}}, \quad (5)$$

where

**$q_{Pvc|p_n}$  are losses of pump capacity during one shaft revolution due to compression of non-aerated (or aerated) liquid (volumetric losses of liquid compression), the losses determined at indicated increase  $\Delta p_{p_i}$  of pressure in the pump working chambers equal to the nominal pressure  $p_n$  of pump in the hydrostatic drive system.**

Volumetric losses  $q_{Pvc}$  of liquid compression are an effect of not only the liquid compressibility but also of the variable capacity displacement pump operating principle. The change of geometrical working capacity  $q_{p_{gv}}$  of the pump is accompanied also with the change of the ratio of compressed liquid volume in the pump working chambers and volume  $q_{p_{gv}}$  and, in effect, with the change of ratio of losses  $q_{Pvc}$  due to liquid compressibility and the volume  $q_{p_{gv}}$ . Therefore, the compressibility coefficient  $k_{lc|p_n}$  of the same liquid increases in the pump with decreasing capacity  $q_{p_{gv}}$  per one shaft revolution.

In references [1 – 4] the author searched for value of the hydraulic oil compressibility coefficient  $k_{lc|p_n}$ , which, with increase  $\Delta p_{p_i}$  of pressure in the working chambers equal to nominal pressure  $p_n$ , will give the increase  $\Delta M_{Pm|p_n, q_{Pgv}}$  of torque of mechanical losses proportional to  $q_{p_{gv}}$ , i.e to indicated torque  $M_{p_i|p_n, q_{Pgv}}$ . The author determined, in the tested pump HYDROMATIK A7V.58.1.R.P.F.00 [8], an approximate value of oil compressibility coefficient during the pump test, equal to  $k_{lc|32MPa} = 0,030$ . Such value of the compressibility coefficient resulted also from aeration ( $\varepsilon > 0$ ) of oil in conditions of the test stand.

In references [11, 12] the author presents **the method of determining the value of liquid aeration coefficient  $\varepsilon$  during pump operation in a hydrostatic drive system or on a test stand**, consisting in finding such value of  $\varepsilon$ , with which calculated increase  $\Delta M_{Pm|p_n, q_{Pgv}}$  of torque of mechanical losses is proportional to indicated torque  $M_{p_i|p_n, q_{Pgv}}$  determined (calculated) at constant increase ( $\Delta p_{p_i} = cte$ ) of pressure in the pump working chambers. The constant value of  $\Delta p_{p_i}$ , assumed in searching for liquid aeration coefficient  $\varepsilon$ , equals to pump nominal pressure  $p_n$  ( $\Delta p_{p_i} = cte = p_n$ ).

During the considerations on compressibility of aerated liquid, the values of modulus B of volume elasticity of the hydraulic oils used in hydrostatic drive and control systems, were taken after M. Guillon's reference [10].

Increase  $\Delta M_{Pm|p_n, q_{Pgv}}$  of torque of mechanical losses in the pump „working chambers - shaft” assembly, at a constant value of  $\Delta p_{p_i}$  ( $\Delta p_{p_i} = cte$ ), is (in this author's opinion) proportional to the pump geometrical working capacity  $q_{p_{gv}}$ ; therefore:

**the (calculated) relation :**

$$\Delta M_{Pm|p_n, q_{Pgv}} \sim q_{Pgv} \quad (6)$$

**can be obtained only with taking into account the actual value of aeration coefficient  $\varepsilon$  of liquid displaced by the pump,**

At the same time, only with accounting for actual value of liquid aeration coefficient  $\varepsilon$  the calculated increase  $\Delta M_{Pm|p_n; q_{Pgv}; \varepsilon}$  of torque of mechanical losses tends to zero at the geometrical working capacity  $q_{p_{gv}}$  per one shaft revolution tending to zero:

$$\Delta M_{Pm|p_n; q_{Pgv} \rightarrow 0; \varepsilon} \rightarrow 0 \text{ when } q_{Pgv} \rightarrow 0. \quad (7)$$

The calculated aeration coefficient  $\varepsilon$  of hydraulic oil used during tests of A7V.58.1.R.P.F.00 HYDROMATIK pump, corresponding to the situation described by the expressions

(6) and (7) had the value  $\varepsilon = 0,0135$  [8, 11, 12].

The method, proposed by the author, of determining (calculating) the working liquid aeration coefficient  $\varepsilon$  was for the first time used in the research work (carried out by Jan Koralewski) concerning the influence of viscosity and compressibility of aerated hydraulic oil on volumetric and mechanical losses of a A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 9, 13, 14].

**The method of determining (calculating) of the aeration coefficient  $\varepsilon$  of working liquid displaced by displacement pump of variable capacity per one shaft revolution, allows to subdivide the volumetric losses per one shaft revolution in pump working chambers into volumetric losses  $q_{pvc}$  of compression of aerated (or non-aerated) liquid and volumetric losses  $q_{pvl}$  due to leakage.**

**The method allows also to evaluate the increase  $\Delta M_{Pm}|\Delta p_{Pi}, q_{Pgv}$  of torque of mechanical losses in the pump „working chambers - shaft” assembly being a function of torque  $M_{Pi}|\Delta p_{Pi}, q_{Pgv}$  indicated in the pump working chambers (by making more precise calculation of indicated torque  $M_{Pi}$ , possible).**

In this author's opinion, the above mentioned possibilities have not existed so far. They are important for evaluation of volumetric losses due to leakage of liquid in the working chambers and for evaluation of mechanical losses in the pump „working chambers - shaft” assembly, hence they are important for evaluation of a design solution of displacement pump generating those losses, particularly when operating in the conditions of high increase  $\Delta p_{Pi}$  of pressure in the chambers.

Objective of the work is to develop a capability of evaluating the volumetric losses resulting from hydraulic oil compression in the pump working chambers in function of the same parameters on which volumetric losses due leakage, resulting from the quality of design solution of the pump, are dependent and evaluated in function of the oil aeration coefficient  $\varepsilon$ . Therefore, it is necessary to develop a mathematical model describing the hydraulic oil compressibility coefficient  $k_{lc}|\Delta p_{Pi}; b_p; \varepsilon; \nu$  as a relation to:

- the ratio  $\Delta p_{Pi}/p_n$  of indicated increase  $\Delta p_{Pi}$  of pressure in the working chambers to the nominal pressure  $p_n$ ,
- the pump capacity coefficient  $b_p$ ,
- the oil aeration coefficient  $\varepsilon$ ,
- the ratio  $\nu/\nu_n$  of oil viscosity  $\nu$  and reference viscosity  $\nu_n$ .

It is also necessary to present a mathematical model of volumetric losses  $q_{pvc}|\Delta p_{Pi}; b_p; \varepsilon; \nu$  of hydraulic oil compression in the pump working chambers in the form allowing to use it in the model of power of oil compression in the pump and also in the model of power of losses and pump energy efficiency.

## 2. Mathematical model of hydraulic oil compressibility coefficient $k_{lc}|\Delta p_{Pi}; b_p; \varepsilon; \nu$ compatible with model of energy losses in a displacement pump

The knowledge of oil compressibility coefficient  $k_{lc}|p_n$  allows to make a numerical evaluation of subdivision of the volumetric losses in pump into losses due to oil leakage in the working chambers and losses of oil compression in chambers.

In a pump of variable capacity per one shaft revolution, operating with variable working capacity  $q_{pgv}$  (at the pump capacity coefficient  $b_p = q_{pgv}/q_{pt}$  changing in the  $0 < b_p \leq 1$  range), the  $k_{lc}|p_n$  liquid (hydraulic oil) compressibility coefficient is described (in reference to test conditions presented in [8,9]) by the formulae:

$$k_{lc}|p_n; b_p = \frac{q_{pvc}|\Delta p_{Pi} = p_n}{q_{pgv}} = \frac{0,5q_{Pt} + 0,5q_{Pgv}}{q_{pgv}} \cdot \left[ \frac{1}{B|_{p_{Plia}=0,15MPa, \vartheta=20^\circ C} (1 + a_p p_n + a_\vartheta \Delta \vartheta)} + \frac{\varepsilon}{p_{Plia} + p_n} \right] p_n \quad (8)$$

or

$$k_{lc}|p_n; b_p = \frac{1 + b_p}{2b_p} \cdot \left[ \frac{1}{B|_{p_{Plia}=0,15MPa, \vartheta=20^\circ C} (1 + a_p p_n + a_\vartheta \Delta \vartheta)} + \frac{\varepsilon}{p_{Plia} + p_n} \right] p_n, \quad (9)$$

and with geometrical variable working capacity  $q_{pgv}$  equal to the theoretical capacity  $q_{pt}$  per one shaft revolution ( $q_{pgv} = q_{pt}$ ) (at  $b_p = 1$ ), by the formula:

$$k_{lc}|p_n = \frac{q_{pvc}|\Delta p_{Pi} = p_n}{q_{Pt}} = \left[ \frac{1}{B|_{p_{Plia}=0,15MPa, \vartheta=20^\circ C} (1 + a_p p_n + a_\vartheta \Delta \vartheta)} + \frac{\varepsilon}{p_{Plia} + p_n} \right] p_n. \quad (10)$$

Therefore, in a variable capacity displacement pump operating at the theoretical working capacity  $q_{pt}$  per one shaft revolution, the oil compressibility coefficient  $k_{lc}|p_n$  (formula (10)) results from:

- the oil volume elasticity modulus  $B$  (value  $B = 1500MPa$  [10] assumed at absolute pressure  $p_{Plia} = 0,15MPa$  and temperature  $\vartheta = 20^\circ C$ ),
- coefficient  $a_p = 0,005/1MPa$  of the increase of modulus

- B along with increase of pressure p,
- coefficient  $a_g = -0,005/1^\circ\text{C}$  of the decrease of modulus B along with increase of temperature  $\vartheta$ ,
- oil temperature  $\vartheta$  (temperature increase  $\Delta\vartheta$  in relation to the reference temperature  $\vartheta = 20^\circ\text{C}$ ),
- oil aeration coefficient  $\varepsilon$  determined at absolute pressure  $P_{p1ia}$ ,
- absolute pressure  $p_{p1ia}$  in the working chambers during their connection with the inlet channel (during tests presented in [8, 9], pressure in the working chambers was  $p_{1ia} \approx 0,15\text{MPa}$ ),
- the system nominal pressure  $p_n$ .

In the same displacement pump operating with the variable geometrical working capacity  $q_{p_{gv}}$  per one shaft revolution, smaller than  $q_{p_t}$ , the value of oil compressibility coefficient  $k_{lc|p_n; b_p}$  (formulae (8) and (9)) increases in comparison with the value  $k_{lc|p_n}$  during pump operation at theoretical working capacity  $q_{p_t}$ . This is an effect of an increase of initial oil volume subjected to compression, i.e. volume  $(0,5q_{p_t} + 0,5q_{p_{gv}})$  to the set working capacity  $q_{p_{gv}}$ .

**Therefore, the decreasing of the pump capacity setting  $q_{p_{gv}}$  ( $b_p$  coefficient in the  $0 < b_p \leq 1$  range) causes, in a displacement pump with variable capacity per one shaft revolution, an increase of the working liquid compressibility coefficient  $k_{lc|p_n; b_p}$  (formulae (8) and (9)).**

**The value  $k_{lc|p_n}$  of liquid (oil) compressibility coefficient must be determined in the same conditions as the coefficient  $k_l$  of volumetric losses due to leakage in the pump working chambers, used in the model of losses and energy efficiency of the pump, i.e.:**

- system nominal pressure  $p_n$ ,
- theoretical capacity  $q_{p_t}$  per one shaft revolution ( $b_p = 1$ ),
- oil temperature  $\vartheta_n$  corresponding to the oil viscosity coefficient  $v/v_n = 1$ , i.e. at the viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$  (during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], ( the temperature corresponding to the oil reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$  was  $\vartheta_n = 43^\circ\text{C}$ ),
- and also with the actual oil aeration coefficient  $\varepsilon$  determined at oil temperature  $\vartheta_n$  corresponding to the oil viscosity coefficient  $v/v_n = 1$ .

Fig. 1 presents an example of relations of the hydraulic oil compressibility coefficient  $k_{lc|p_n; b_p}$  (determined by formulae (8) and (9)) to the oil aeration coefficient  $\varepsilon$  at different values of the pump capacity coefficient  $b_p$ , determined at the nominal pressure  $p_n = 32\text{MPa}$ , at temperature  $\vartheta_n = 43^\circ\text{C}$  of oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], the temperature corresponding to oil viscosity ratio  $v/v_n = 1$  (reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$ ).

Fig. 2 presents relations of hydraulic oil compressibility coefficient  $k_{lc|p_n; b_p}$  (determined by formulae (8) and (9)) to the pump capacity coefficient  $b_p$  at different values of oil

aeration coefficient  $\varepsilon$  determined at the nominal pressure  $p_n = 32\text{MPa}$ , hydraulic oil temperature  $\vartheta_n = 43^\circ\text{C}$  during tests of A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], i.e. the temperature corresponding to oil viscosity coefficient  $v/v_n = 1$  (at the reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$ ).

Relation of hydraulic oil compressibility coefficient  $k_{lc|p_n; b_p}$  to the pump capacity coefficient  $b_p$  is based on the displacement pump operation principle and described (in the  $0 < b_p \leq 1$  range) by the formula:

$$k_{lc|p_n; b_p} = \frac{1+b_p}{2b_p} k_{lc|p_n}, \quad (11)$$

where  $k_{lc|p_n}$  is a liquid compressibility coefficient during pump operation at the theoretical working capacity  $q_{p_t}$  per one shaft revolution (at  $b_p = 1$ ) and at liquid (oil) temperature  $\vartheta_n$  corresponding to the oil viscosity ratio  $v/v_n = 1$ , i.e. at the reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$ .

Fig. 3 presents examples of the relations of hydraulic oil compressibility coefficient  $k_{lc|\Delta p_{p_i}; b_p}$  and  $k_{lc|\Delta p_{p_i}}$  (described by formulae (8), (9) and (10)) to ratio  $\Delta p_{p_i}/p_n$  of indicated increase  $\Delta p_{p_i}$  of pressure in the pump working chambers and nominal pressure  $p_n = 32\text{MPa}$ , at pump capacity coefficient  $b_p = 0,229$  and  $b_p = 1$ , at different values of aeration coefficient  $\varepsilon$ , at temperature  $\vartheta_n = 43^\circ\text{C}$  of hydraulic oil used in the tests of A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], the temperature corresponding to oil viscosity ratio  $v/v_n = 1$  (reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$ ).

The proposed relations (in the  $0 < b_p \leq 1$  range):

$$k_{lc|\Delta p_{p_i}; b_p; \varepsilon} = \frac{1+b_p}{2b_p} \left[ k_{lc|p_n; \varepsilon=0} (\Delta p_{p_i} / p_n)^{a_{pc}} + \varepsilon \right] \quad (12)$$

and

$$k_{lc|\Delta p_{p_i}; \varepsilon} = k_{lc|p_n; \varepsilon=0} (\Delta p_{p_i} / p_n)^{a_{pc}} + \varepsilon \quad (13)$$

allow to evaluate, with satisfying precision, the liquid compressibility coefficient  $k_{lc|\Delta p_{p_i}; b_p}$  and  $k_{lc|\Delta p_{p_i}}$  in a range of indicated increase  $\Delta p_{p_i}$  of pressure in the pump working chambers:

$$\Delta p_{p_i} > 3,2\text{MPa}. \quad (14)$$

Exponent  $a_{pc}$  in Eq. (12) and (13), describing the relation of coefficient  $k_{lc|\Delta p_{p_i}; b_p; \varepsilon}$  and  $k_{lc|\Delta p_{p_i}; \varepsilon}$  of oil compressibility to the expression  $(\Delta p_{p_i} / p_n)^{a_{pc}}$  is independent of the pump capacity coefficient  $b_p$  and the oil aeration coefficient  $\varepsilon$ .



The value of exponent  $a_{pc}$  is:

$$a_{pc} = 0,89.$$

**Expressions (12) and (13) (Fig. 3) have a form allowing to use them in equations describing energy losses due to liquid compressibility in pump.**

Fig. 4 shows examples of relations of hydraulic oil compressibility coefficient  $k_{lc|p_n; b_p}$  and  $k_{lc|p_n}$  (determined by formulae (8), (9) and (10)) to oil temperature  $\vartheta$ , at pump capacity coefficient  $b_p = 0,229$  and  $b_p = 1$ , with different values of aeration coefficient  $\varepsilon$  used in the tests of A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12]. Temperature  $\vartheta_n$ , corresponding to the recommended reference viscosity  $\nu_n = 35\text{mm}^2\text{s}^{-1}$  (oil viscosity ratio  $\nu/\nu_n = 1$ ) was equal to  $\vartheta_n = 43^\circ\text{C}$ . The tests were carried out in the temperature range  $20^\circ\text{C} \leq \vartheta \leq 68^\circ\text{C}$  (at  $b_p = 1$  – in the  $24^\circ\text{C} \leq \vartheta \leq 68^\circ\text{C}$  range)

**In the description of volumetric losses in the pump due to oil compression in the pump working chambers, it was decided to replace the relations of oil compressibility coefficient  $k_{lc|p_n; b_p}$  and  $k_{lc|p_n}$  to oil temperature  $\vartheta$  by relation to oil viscosity ratio  $\nu/\nu_n$ .** This is justified on one hand by dependence of oil viscosity  $\nu$  on temperature  $\vartheta$  and on the other hand on the fact, that the energy losses: volumetric losses due to leakage in the working chambers, pressure losses in the pump channels, and mechanical losses in the pump „working chambers - shaft” assembly are determined in function of oil viscosity  $\nu$ , and exactly in function of oil viscosity ratio  $\nu/\nu_n$ .

Fig. 5 shows examples of relations of the hydraulic oil compressibility coefficient  $k_{lc|p_n; b_p}$  and  $k_{lc|p_n}$  to oil viscosity  $\nu$  described by the expressions:

$$k_{lc|p_n; b_p; \nu} = k_{lc|p_n; b_p} (\nu / \nu_n)^{a_{vc}} \quad (15)$$

and

$$k_{lc|p_n; \nu} = k_{lc|p_n} (\nu / \nu_n)^{a_{vc}} \quad (16)$$

allowing to evaluate the dependence of oil compressibility coefficient  $k_{lc|p_n; b_p; \nu}$  and  $k_{lc|p_n; \nu}$  on viscosity  $\nu$  in the temperature  $20^\circ\text{C} \leq \vartheta \leq 68^\circ\text{C}$  range, at pump capacity coefficient  $b_p = 0,229$  and  $b_p = 1$ , at different values of aeration coefficient  $\varepsilon$  of oil during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12].

Exponent  $a_{vc}$  in Eq. (15) and (16), describing the relations of coefficient  $k_{lc|p_n; b_p}$  and  $k_{lc|p_n}$  of oil compressibility to oil viscosity  $\nu$  by means of expression  $(\nu / \nu_n)^{a_{vc}}$  replacing the relation to oil temperature  $\vartheta$  by relation to oil viscosity  $\nu$ , is independent (in the  $0 < b_p \leq 1$  range) of pump capacity coefficient  $b_p$ .

However, exponent  $a_{vc}$  changes with the change of oil

aeration coefficient  $\varepsilon$ , i.e.:

with:

$\varepsilon = 0$	–	$a_{vc} = -0,12,$
$\varepsilon = 0,004$	–	$a_{vc} = -0,1,$
$\varepsilon = 0,008$	–	$a_{vc} = -0,086,$
$\varepsilon = 0,012$	–	$a_{vc} = -0,076,$
$\varepsilon = 0,0135$	–	$a_{vc} = -0,072,$
$\varepsilon = 0,016$	–	$a_{vc} = -0,067.$

**The influence, in the  $3,5 \geq \nu/\nu_n \geq 0,4$  range, of ratio  $\nu/\nu_n$  of hydraulic oil viscosity (influence of change of oil temperature  $\vartheta$  in the  $20^\circ\text{C} \leq \vartheta \leq 68^\circ\text{C}$  range) on the change of oil compressibility coefficient  $k_{lc|\Delta p_{pi}; b_p; \varepsilon; \nu}$  is small in comparison with the influence of ratio  $\Delta p_{pi}/p_n$  of indicated increase  $\Delta p_{pi}$  of pressure in the working chambers and nominal pressure  $p_n$ , with the influence of pump capacity coefficient  $b_p$  and with the influence of oil aeration coefficient  $\varepsilon$ . Therefore, in order to simplify the expression describing the mathematical model of oil compressibility coefficient  $k_{lc|\Delta p_{pi}; b_p; \varepsilon; \nu}$  (and also model of volumetric losses  $q_{pvc}$  of oil compression and model of power  $\Delta P_{pvc}$  of volumetric losses of oil compression), it was decided to adopt a single value of  $a_{vc}$  exponent equal:**

$$a_{vc} = -0,12,$$

corresponding to non-aerated ( $\varepsilon = 0$ ) oil condition.

Mathematical model of relation of the hydraulic oil compressibility coefficient  $k_{lc|\Delta p_{pi}; b_p; \varepsilon; \nu}$  to the non-aerated oil compressibility coefficient  $k_{lc|p_n; \varepsilon=0}$  (formula (10) at  $\varepsilon = 0$ ) in a variable capacity displacement pump and:

- to ratio  $\Delta p_{pi}/p_n$  of indicated increase  $\Delta p_{pi}$  of pressure in the pump working chambers and nominal pressure  $p_n$  (in the  $\Delta p_{pi} > 3,2\text{MPa}$  range),
- to pump capacity coefficient  $b_p$  (in the  $0 < b_p \leq 1$  range),
- to oil aeration coefficient  $\varepsilon$ ,
- to oil viscosity relation  $\nu/\nu_n$

takes, in reference to formulae (11) ÷ (16), the form:

$$k_{lc|\Delta p_{pi}; b_p; \varepsilon; \nu} = \frac{1 + b_p}{2b_p} \left[ k_{lc|p_n; \varepsilon=0} (\Delta p_{pi} / p_n)^{a_{pc}} + \varepsilon \right] (\nu / \nu_n)^{a_{vc}}, \quad (17)$$

with exponent  $a_{pc} = 0,89$

and with exponent  $a_{vc} = -0,12$ .

The form of expression (17) allows to determine the impact of hydraulic oil compression in mathematical model of the power of oil compression in pump operating in the conditions produced by change of  $\Delta p_{pi}$ ,  $b_p$ ,  $\varepsilon$  and  $\nu$ .

$\vartheta_n=43^\circ\text{C}$  ( $\nu/\nu_n=1,000$ )  $p_n=32\text{MPa}$   $\nu_n=35\text{mm}^2\text{s}^{-1}$

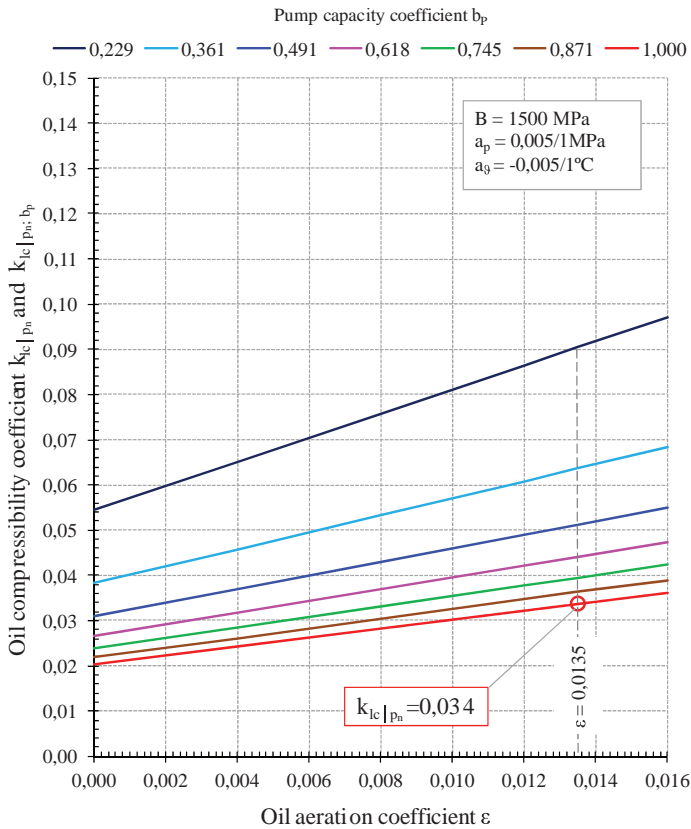


Fig.1 Example of relation of the oil compressibility coefficient  $k_{lc|p_n; b_p}$  to the oil aeration coefficient  $\varepsilon$  at different values of the pump capacity coefficient  $b_p$ , determined at nominal pressure  $p_n = 32\text{MPa}$ , at temperature  $\vartheta_n = 43^\circ\text{C}$  of hydraulic oil used in tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], temperature corresponding to the oil viscosity ratio  $\nu/\nu_n = 1$  (reference viscosity  $\nu_n = 35\text{mm}^2\text{s}^{-1}$ ); the value of oil compressibility coefficient  $k_{lc|p_n}$  used in the model of losses and energy efficiency is determined in the same conditions as coefficient  $k_l$  of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity  $q_{pt}$  per one shaft revolution (at the pump capacity coefficient  $b_p = 1$ ); the actual oil aeration coefficient determined during the pump tests was  $\varepsilon = 0,0135$  and the corresponding value of oil compressibility coefficient  $k_{lc|p_n} = 0,034$

$\vartheta_n=43^\circ\text{C}$  ( $\nu/\nu_n=1,000$ )  $p_n=32\text{MPa}$   $\nu_n=35\text{mm}^2\text{s}^{-1}$

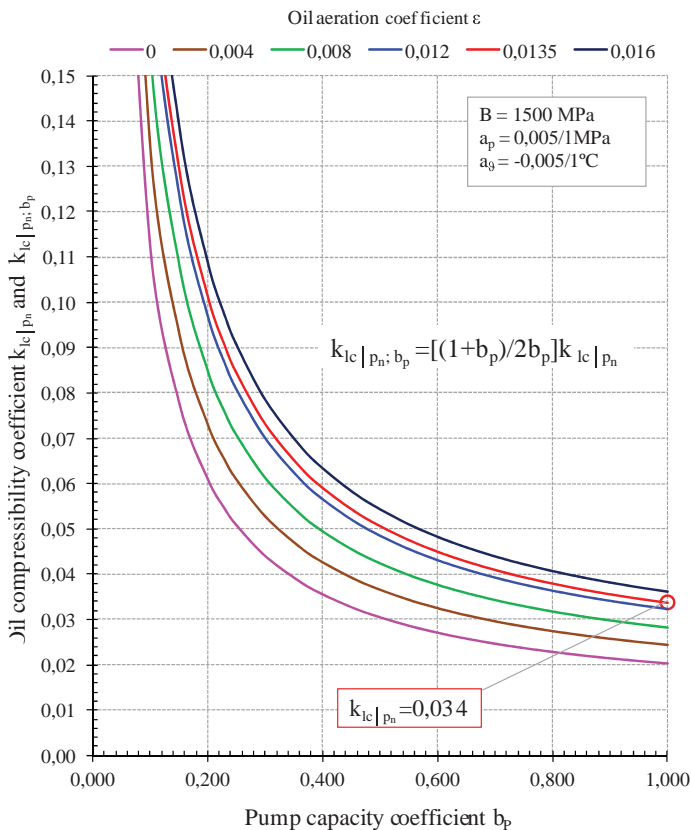


Fig.2 Example of relation of the oil compressibility coefficient  $k_{lc|p_n; b_p}$  to pump capacity coefficient  $b_p$  at different values of oil aeration coefficient  $\varepsilon$ , determined at nominal pressure  $p_n = 32\text{MPa}$ , at temperature  $\vartheta_n = 43^\circ\text{C}$  of hydraulic oil used in tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], temperature corresponding to the oil viscosity ratio  $\nu/\nu_n = 1$  (reference viscosity  $\nu_n = 35\text{mm}^2\text{s}^{-1}$ ); the value of oil compressibility coefficient  $k_{lc|p_n}$  used in the model of losses and energy efficiency is determined in the same conditions as coefficient  $k_l$  of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity  $q_{pt}$  per one shaft revolution (at the pump capacity coefficient  $b_p = 1$ ); the actual oil aeration coefficient determined during the pump tests was  $\varepsilon = 0,0135$  and the corresponding value of oil compressibility coefficient  $k_{lc|p_n} = 0,034$ ; the value of liquid compressibility coefficient  $k_{lc|p_n; b_p}$  increases with the decreasing value of pump capacity coefficient  $b_p$  (in the  $0 < b_p \leq 1$  range) in accordance with the formula  $k_{lc|p_n; b_p} = [(1+b_p)/2b_p]k_{lc|p_n}$

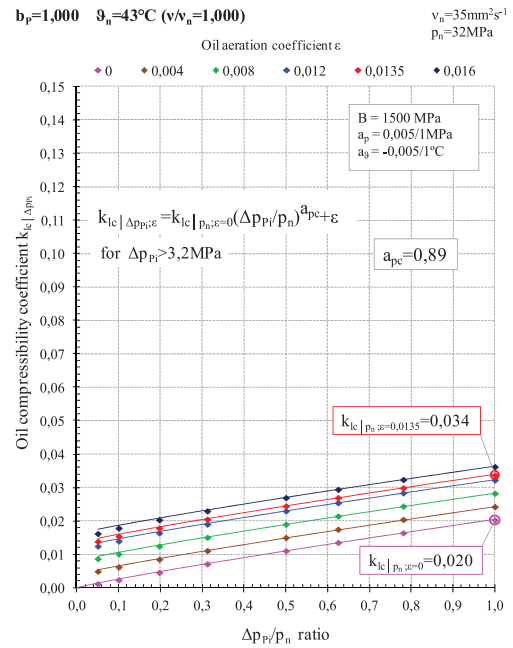
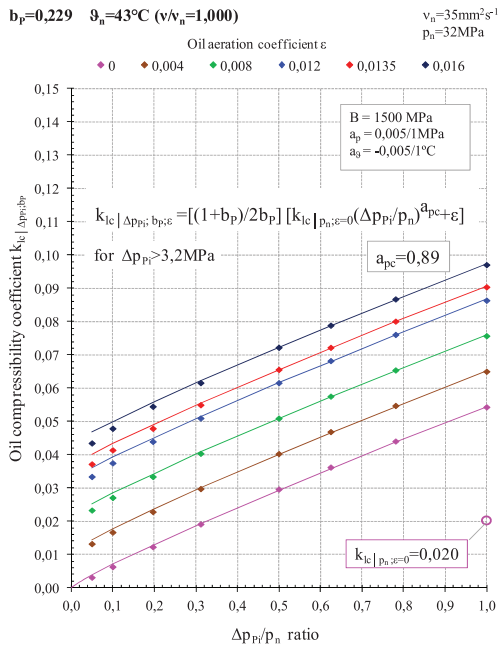


Fig.3 Examples of relation of the hydraulic oil compressibility coefficient  $k_{lc|\Delta p_{pi}; b_p}$  and  $k_{lc|\Delta p_{pi}}$  to the ratio  $\Delta p_{pi}/p_n$  of indicated increase  $\Delta p_{pi}$  of pressure in the pump working chambers to nominal pressure  $p_n = 32\text{MPa}$  at the pump capacity coefficient  $b_p = 0,229$  and  $b_p = 1$ , at different values of oil aeration coefficient  $\epsilon$ , at temperature  $\vartheta_n = 43^\circ\text{C}$  of hydraulic oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12], temperature corresponding to oil viscosity ratio  $v/v_n = 1$  (reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$ ); value of the oil compressibility coefficient  $k_{lc|p_n}$  used in the model of losses and energy efficiency is determined in the same conditions as coefficient  $k_l$  of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity  $q_{pt}$  per one shaft revolution (at pump capacity coefficient  $b_p = 1$ ); the actual oil aeration coefficient determined during the pump tests was  $\epsilon = 0,0135$  and the corresponding value of oil compressibility coefficient  $k_{lc|p_n} = 0,034$

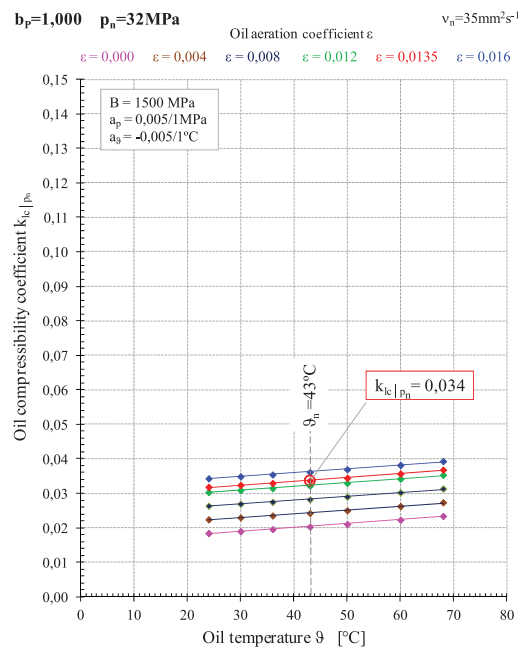
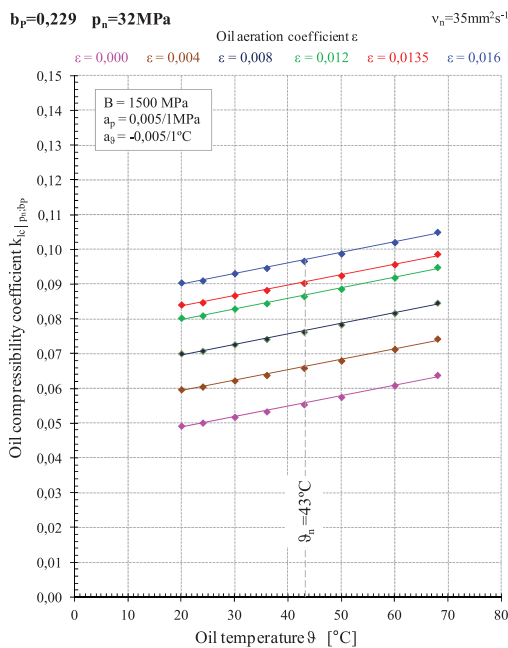


Fig.4 Examples of relation of hydraulic oil compressibility coefficient  $k_{lc|p_n; b_p}$  and  $k_{lc|p_n}$  to oil temperature  $\vartheta$  at pump capacity coefficient  $b_p = 0,229$  and  $b_p = 1$ , at different values of aeration coefficient  $\epsilon$  of oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12]; value of oil compressibility coefficient  $k_{lc|p_n}$  used in the model of losses and energy efficiency is determined in the same conditions as coefficient  $k_l$  of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity  $q_{pt}$  per one shaft revolution (at pump capacity coefficient  $b_p = 1$ ) and at oil temperature  $\vartheta_n$  corresponding to oil viscosity ratio  $v/v_n = 1$  (reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$ ); temperature corresponding to oil viscosity ratio  $v/v_n = 1$  was  $\vartheta_n = 43^\circ\text{C}$ ; actual value of oil aeration coefficient was  $\epsilon = 0,0135$  and the corresponding value of oil compressibility coefficient  $k_{lc|p_n} = 0,034$

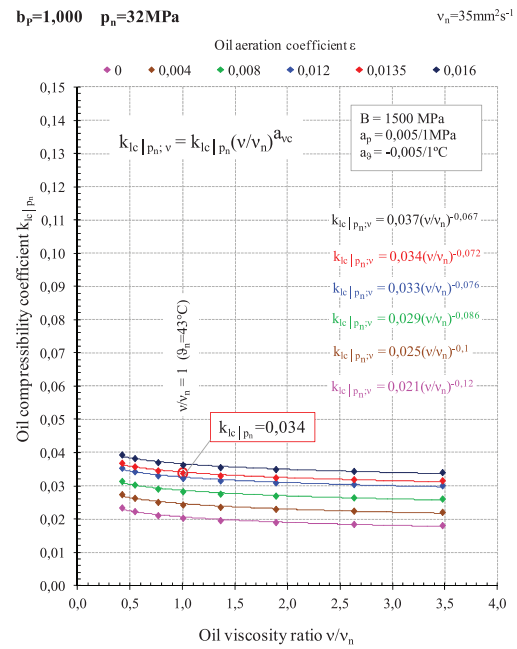
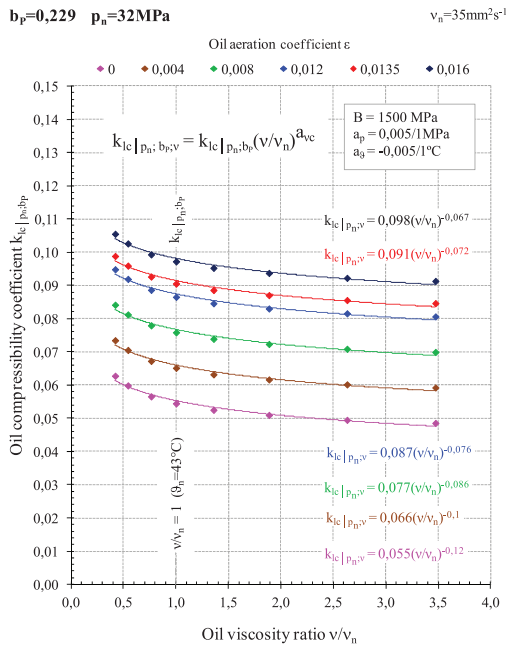


Fig.5 Examples of relation of hydraulic oil compressibility coefficient  $k_{lc|p_n; b_p}$  and  $k_{lc|p_n}$  to oil viscosity ratio  $v/v_n$  corresponding to oil temperature  $\vartheta$  at pump capacity coefficient  $b_p = 0,229$  and  $b_p = 1$ , at different values of aeration coefficient  $\varepsilon$  of oil used during tests of the A7V.58.1.R.P.F.00 HYDROMATIK pump [8, 11, 12]; value of oil compressibility coefficient  $k_{lc|p_n}$  used in the model of losses and energy efficiency is determined in the same conditions as coefficient  $k_l$  of volumetric losses due to leakage in the pump working chambers, i.e. at the theoretical capacity  $q_{p_t}$  per one shaft revolution (at pump capacity coefficient  $b_p = 1$ ) and at oil temperature  $\vartheta_n$  corresponding to oil viscosity ratio  $v/v_n = 1$  (reference viscosity  $v_n = 35\text{mm}^2\text{s}^{-1}$ ); temperature corresponding to oil viscosity ratio  $v/v_n = 1$  was  $\vartheta_n = 43^\circ\text{C}$ ; actual value of oil aeration coefficient was  $\varepsilon = 0,0135$  and the corresponding value of oil compressibility coefficient  $k_{lc|p_n} = 0,034$

### 3. Mathematical model of volumetric losses $q_{pvc}$ of hydraulic oil compression in the pump working chambers

Volumetric losses  $q_{pvc}$  determined per one shaft revolution of compression of liquid pressed in the pump working chambers are not attributable to the pump design solution. They result from the liquid compressibility itself and from aeration of the liquid. Main reason of working liquid aeration is air dissolved in the liquid (not having in such form any influence on its compressibility) and getting out of the liquid (in the form of bubbles) in the conditions of local (in the system conduits or in pump working chambers during their connection with pump inlet channel) drop of pressure below the atmospheric pressure. One of the reasons may be admitting by the hydrostatic system designer or user too low pressure in the pump inlet conduit, which may cause cavitation in the pump working chambers during their connection with the inlet channel.

Mathematical model of volumetric losses  $q_{pvc}|\Delta p_{p_i}; b_p; \varepsilon; v$  per one shaft revolution resulting from compression of

non-aerated ( $\varepsilon = 0$ ) or aerated ( $\varepsilon > 0$ ) hydraulic oil, determined at indicated increase  $\Delta p_{p_i}$  of pressure in the pump working chambers, at pump capacity coefficient  $b_p$  (in the  $0 < b_p \leq 1$  range) and at the ratio  $v/v_n$  of oil viscosity  $v$  and reference viscosity  $v_n$  is described (with  $\Delta p_{p_i} > 3,2\text{MPa}$ ) by the formula:

$$\begin{aligned}
 q_{pvc}|\Delta p_{p_i}; b_p; \varepsilon; v &= k_{lc}|\Delta p_{p_i}; b_p; \varepsilon; v \cdot q_{p_gv} = k_{lc}|\Delta p_{p_i}; b_p; \varepsilon; v \cdot b_p q_{p_t} = \\
 &= \frac{1 + b_p}{2b_p} \left[ k_{lc|p_n; \varepsilon=0} (\Delta p_{p_i} / p_n)^{a_{pc}} + \varepsilon \right] (v / v_n)^{a_{vc}} b_p q_{p_t} = \\
 &= \frac{1 + b_p}{2} \left[ k_{lc|p_n; \varepsilon=0} (\Delta p_{p_i} / p_n)^{a_{pc}} + \varepsilon \right] (v / v_n)^{a_{vc}} q_{p_t}.
 \end{aligned} \tag{18}$$

with exponent  $a_{pc} = 0,89$

and with exponent  $a_{vc} = -0,12$ .

**Coefficient  $k_{lc|p_n; \varepsilon=0}$  of hydraulic oil compressibility is a coefficient of non-aerated oil compressibility. At pressure  $p_n = 32\text{MP}$ , this coefficient is of an order  $k_{lc|32\text{MPa}; \varepsilon=0} = 0,020$ .**

In reference to formulae (13), (17) and (18), a conclusion may be made that coefficient  $k_{lc|p_n; \varepsilon} > 0$  of aerated ( $\varepsilon > 0$ ) oil compressibility may be replaced, with sufficient accuracy, (in the  $\Delta p_{p_i} > 3,2\text{MPa}$  range), by the expression:



$$k_{lc|p_n; \varepsilon} = k_{lc|p_n; \varepsilon=0} + \varepsilon. \quad (19)$$

Formula (18) describing volumetric losses  $Q_{PVC|\Delta p_{P_i}; b_p; \varepsilon; \nu}$  of hydraulic oil compression in the working chambers should be used in the model of power of oil compression in the pump.

#### 4. Conclusions

1. The objective of this work was to develop the capability of evaluating the volumetric losses of hydraulic oil compression in the working chambers of high pressure variable capacity displacement pump. Volumetric losses of oil compression must be determined in function of the same parameters, on which the volumetric losses due to leakage, resulting from the quality of design solution of the pump, are dependent and evaluated also in function of the oil aeration coefficient  $\varepsilon$ .
2. A mathematical model has been developed for describing the coefficient  $k_{lc|\Delta p_{P_i}; b_p; \varepsilon; \nu}$  of hydraulic oil compressibility in function of:
  - the ratio  $\Delta p_{P_i}/p_n$  of indicated increase  $\Delta p_{P_i}$  of pressure in the working chambers and nominal pressure  $p_n$ ,
  - the pump capacity coefficient  $b_p$ ,
  - the oil aeration coefficient  $\varepsilon$ ,
  - the ratio  $\nu/\nu_n$  of oil viscosity  $\nu$  and reference viscosity  $\nu_n$ .
3. A mathematical model of volumetric losses  $Q_{PVC|\Delta p_{P_i}; b_p; \varepsilon; \nu}$  of hydraulic oil compression has been presented in the form allowing to use it in the model of power of oil compression in the pump.

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