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MATHEMATICAL MODELING OF THE OVERHEAD CONTACT LINE FOR THE PURPOSE OF DIAGNOSTICS OF PANTOGRAPHS

MODELOWANIE MATEMATYCZNE GÓRNEJ SIECI TRAKCYJNEJ DO POTRZEB DIAGNOSTYKI ODBIERAKÓW PRĄDU

Abstract

The overhead contact line (OCL) is the most effective way for supplying railway electric vehicles. The increase of the speed of vehicles increases power consumption and requires ensuring proper cooperation of pantographs with OCL. The paper describes the novel mathematical model of the OCL system and the simulation results. The primary objective is a more accurate analysis to increase the reliability of the evaluation of monitoring and diagnostics. The model was based on the Lagrange energy method. The paper presents the structure of the model and equations describing it, as well as the results of some laboratory tests that were performed to determine the model parameters. The selected results of simulations concerning the effects of force impact on the contact wire were carried out using the created model. The prepared program can be used for creating computer tools, which will support designers of OCL.

Keywords: electric traction, overhead catenary, mathematical modelling, energy method of Lagrange

Streszczenie

Sieć jezdna jest najbardziej efektywnym sposobem zasilania kolejowych pojazdów elektrycznych. Wzrost prędkości pojazdów zwiększa pobór mocy i wymaga zapewnienia właściwej współpracy odbieraków prądu z siecią jezdną. W artykule przedstawiono nowy model matematyczny sieci i wstępne wyniki symulacji. Głównym celem badań jest dokładniejsza analiza, zwiększająca wiarygodność oceny współpracy sieci z odbierakami prądu w celach monitoringu i diagnostyki. Model został oparty na metodzie energetycznej Lagrange'a. Przedstawiono strukturę modelu i równania go opisujące, a także wyniki niektórych pomiarów laboratoryjnych, które wykonano w celu określenia parametrów modelu. Wybrane wyniki symulacji dotyczące oddziaływania siły na przewód jezdny przeprowadzono w oparciu o utworzony model. Przygotowany program może być używany do tworzenia narzędzi informatycznych, które będą wspierać projektantów sieci trakcyjnych.

Słowa kluczowe: trakcja elektryczna, górna sieć trakcyjna, modelowanie matematyczne, metoda energetyczna Lagrange'a

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1. Introduction

The overhead contact line is currently the most effective way to power supply electric railway vehicles. The increase in vehicle speed increases the power consumption and requires ensuring the proper cooperation of the current collectors of the vehicle with the overhead contact line [11, 14–16]. Improperly designed or inaccurately adjusted overhead line – current collector system, in unfavourable local conditions, can lead to damage of the network or the pantograph. Analyses of failures in the line – pantograph power supply system carried out by railway managements indicate substantial costs incurred for technical repairs and resulting train delays [11, 17]. Therefore, a great importance is attributed to the line and pantograph design solutions, their current operational maintenance and diagnostic systems of their technical characteristics under operating conditions [4, 8, 10, 14].

Works on computer aided design of overhead contact lines are undertaken. Computer simulation tools are dynamically developed, especially those for the analysis of the interaction of the catenary construction cooperating with the current collector [1–3, 5, 9]. Validation of simulation results obtained using a particular dynamic model and numerical method is subject to standardisation in accordance with EN 50318 [7]. Carrying out such a simulation is required by Technical Specifications for Interoperability of the "Energy" subsystem [6].

The existing catenary designs are based on the so-called single chain catenaries (less frequently, double), in which the contact wires (single or double) are suspended by droppers to the messenger wire (single or double). In place of suspending the messenger wire to the support structure, different types of hardware are used, which allows to compensate the temperature effect on the stress in the contact wires. Therefore, the important elements in mathematical modelling of lines are: the contact wire, the droppers and the messenger wire. In the modern approach, the models of the messenger wire and the wire are replaced with elements of Bernoulli-Euler beams; the dropper is modelled as a spring element with a mass [1, 2, 5]. This allows the formulation of the contact wire and messenger wire model as a structural element implemented in the finite element method. The model of the collector is typically based on an appropriate combination of equivalent mass of the moving parts of the collector via the inertial, elastic and damping elements [1, 14].

In the dynamic three-dimensional analysis of an overhead line, the authors suggest, however, to use a mathematical model of the line based on dividing its components to lumped conservative and dissipative mechanical parts, appropriately distributed in 3D space and connected to each other in mechanical nodes. Constraint equations between lumped elements provide lateral stiffness of the suspension messenger wire and contact wire. The method has been successfully applied to static and dynamic simulations of the rail track [12, 13] in the issues of determining the axial forces in the track during the process of adjusting its geometries.

The work presents methodical aspects of modelling the critical components of a catenary construction, i.e. a contact wire, a messenger wire and a dropper. In the future, the models of these components will form the basis for the construction of a line reference model, first, and then of a complete model of the strain section of the line that is needed to diagnose the current collectors. The paper also presents the issues of identifying the contact wire model parameters based on measurements. At the same time, the complexity of the dynamics of the wire vibration in 3D space has been shown.

2. Mathematical modelling of selected elements of contact line

2.1. Assumptions

The modelling methodology adopted in this work is based on the Lagrangian energy method. The methodology has been applied only to those components that were included in the EN 50318 specification [7]. Specification of the reference model of the line according to EN 50318 defines only the geometrical arrangement, the set of input parameter values and the corresponding set of results for the impact at the collector-contact wire tangency point. It is therefore not a mathematical model, but a set of guidelines for its construction, which must be taken into account. The reference model is not intended to represent the mechanical structure of a particular line. Its purpose is to validate any method of simulation, especially dynamic states in the contact line-current collectors system. If the test simulation method will be positively verified on the basis of the reference model, it may be approved for experimental verification.

The geometric system of the reference line [7] consists of a series of 10 identical sections along the length of tension. Each section (Fig. 1) represents the span length and its length is 60 m. The reference line includes: a single contact wire, a messenger wire and droppers. The messenger wire is fixed to stationary brackets and restrained at both ends of the tension segment. The current collector of the vehicle moving on the railway line is modelled as a discrete system: masses-springs-dampers (Fig. 2) [7].

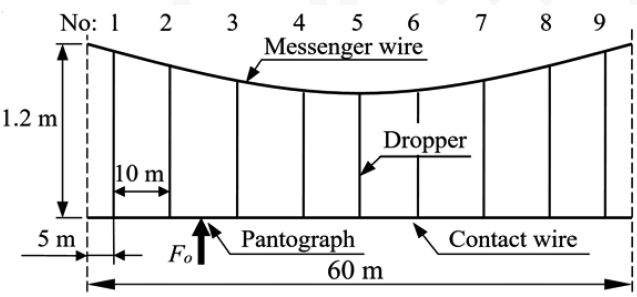

Fig. 1. Geometry of the reference catenary single suspension section

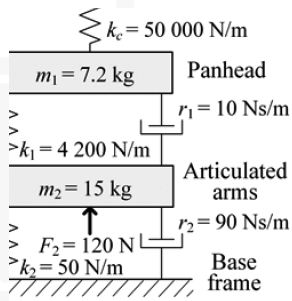

Fig. 2. Pantograph model

2.2. Modelling methodology adopted for the analysis of the catenary construction

According to the adopted method, the contact wire, the messenger wire and the droppers were divided into lumped conservative (kinetic and potential) elements and dissipative elements (dampers). The concept of division of the contact wire and the messenger wire is shown in Fig. 3a, whereas the division of the dropper is shown in Fig. 3b. The contact wire and the messenger wire are represented by a string of 8-node elements. In the particular mechanical nodes, lumped mass elements are arranged. Adjacent nodes are connected by lumped equivalent springs and dampers. This arrangement provides a longitudinal (axial) and lateral stiffness, and the distribution of nodes in a cross-section of the wire allows to

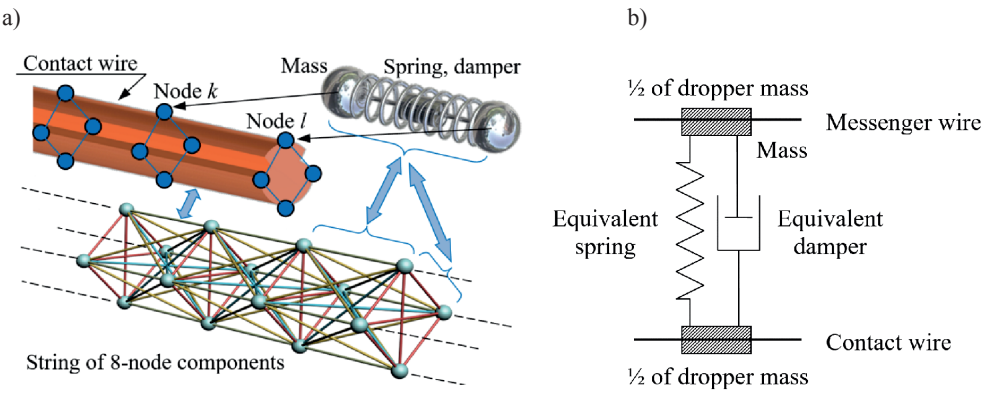

Fig. 3. The concept of dividing the elements of the catenary construction into a group of conservative and dissipative lumped elements: a) contact wire or messenger wire, b) dropper

simulate the effect of wear of the contact wire. The dropper is represented by two inertial elements placed in the nodes of the contact wire and the messenger wire, joined with an equivalent damper and spring with nonlinear characteristics.

The mathematical model is based on the Lagrangian formalism. This formalism requires defining: coenergy of kinetic inertial elements, energy of the potential elements and Rayleigh dissipation function for dissipative components. Kinetic coenergy of the inertial element a in translational motion expressed in coordinates associated with the element a is

$$E'_{k,a}(\dot{x}_a, \dot{y}_a, \dot{z}_a) = \frac{1}{2} m_a \left(\dot{x}_a^{\,2} + \dot{y}_a^{\,2} + \dot{z}_a^{\,2} \right) \tag{1}$$

where:

m_a – the mass of the element a,

$\dot{x}_a, \dot{y}_a, \dot{z}_a$ – the components of its velocity, respectively in the x, y and z directions.

The potential energy of a linear elastic element b in translational motion expressed in the coordinates associated with the b element is

$$E_{p,b}(x_b, y_b, z_b) = \frac{1}{2K_b} \left((x_b + X_b)^2 + (y_b + Y_b)^2 + (z_b + Z_b)^2 - 2R_b \sqrt{(x_b + X_b)^2 + (y_b + Y_b)^2 + (z_b + Z_b)^2} + R_b^{\,2} \right) \tag{2}$$

where:

K_b – the compliance of the spring b,

x_b, y_b, z_b – components of the relative spring terminals displacement respectively in the x, y and z directions

X_b, Y_b, Z_b – the dimensions of the spring in a state of zero potential energy, respectively in the x, y and z directions,

R_p – the length of the spring.

Rayleigh dissipation function of the linear viscous dissipative element c in translational motion expressed in the coordinates associated with the element c is

$$P_{m,c}(\dot{x}_c, \dot{y}_c, \dot{z}_c) = \frac{1}{2} D_c \left(\dot{x}_c^{\,2} + \dot{y}_c^{\,2} + \dot{z}_c^{\,2} \right) \tag{3}$$

where:

D_c – the damping coefficient of kinetic component c,

$\dot{x}_c, \dot{y}_c, \dot{z}_c$ – relative components of the damper terminals velocity, respectively in the x, y and z directions.

In the next step, a constraint equation is set, which defines the relationship between the coordinates of individual lumped elements with generalised coordinates (degrees of freedom of the system). On this basis, the Lagrangian was formulated: $L(\dot{\xi}, \xi)$, and its general form with generalised coordinates is as follows

$$L(\dot{\xi}, \xi) = E_k'(\dot{\xi}, \xi) - E_p(\xi), \quad \dot{\xi} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \\ \vdots \\ \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix}, \quad \xi = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ \vdots \\ x_n \\ y_n \\ z_n \end{bmatrix} \tag{4}$$

where:

$E_k'(\dot{\xi}, \xi)$ – the resultant coenergy of the kinetic system,

$E_p(\xi)$ – the resultant potential energy of the system,

$\dot{\xi}$ – the generalised velocity vector,

where in:

$\dot{x}_k, \dot{y}_k, \dot{z}_k$ – the components of the velocity of the node k,

ξ – the vector of generalised displacement,

where in:

x_k, y_k, z_k – components of displacement of the node k.

Then, an Euler-Lagrange equation was set, that, for the free node k, can be succinctly expressed by the formula:

$$\frac{d}{dt}\left[\frac{\partial L(\dot{\xi}, \xi, t)}{\partial \dot{\xi}_k} \right] - \frac{\partial L(\dot{\xi}, \xi, t)}{\partial \xi_k} + \frac{\partial P(\dot{\xi})}{\partial \dot{\xi}_k} = Q_k(t) \tag{5}$$

where:

$P(\dot{\xi})$ – a Rayleigh dissipation function of the system,

$Q_k(t)$ – a generalised force – the external force acting on the free node k of the system.

3. Identification of the mechanical parameters of the contact wire

An important element of the overhead line is the contact wire. Its irregular cross-section results in the need for detailed exploration of the selected mechanical parameters. For this purpose, it was necessary to measure the deflection and vibration for typical trolley wires used in the Polish State Railways, Inc., i.e. Djp 100 and Djp 150. The tests were performed on a simple experimental setup shown schematically in Fig. 4.

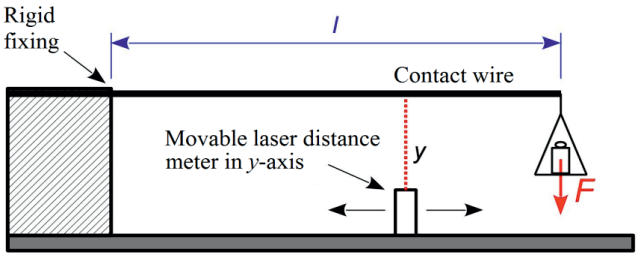

Fig. 4. The scheme of the experimental setup for testing mechanical parameters of contact wires, where: l – wire length (800 mm) F – force function; y – deflection of the wire

The wire is mounted rigidly on one side. In case of static measurements, different values of the force F were applied at the end of the wire, and by means of a laser rangefinder, the deflection y in the consecutive points spaced along the length l of the wire was measured. The results of static measurements for both types of wires are shown in Fig. 5. Measurements were carried out for the normal position of the wire for the indentation of the suspension and for the wire rotated by 90 degrees relative to this position.

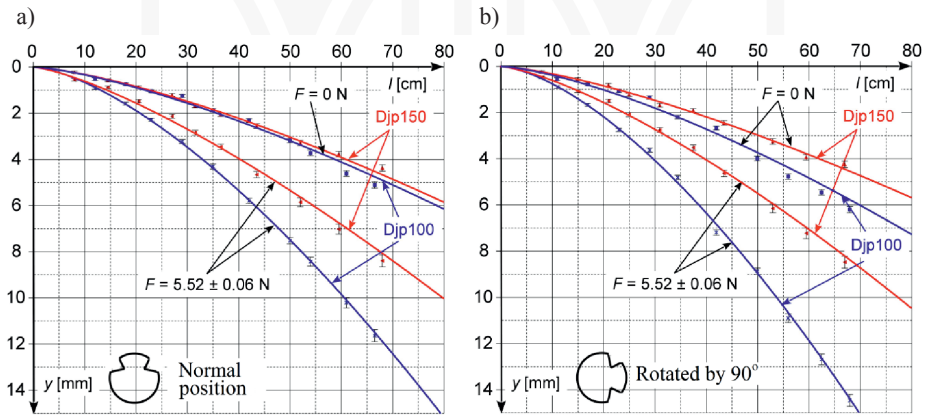

Fig. 5. The results of a static measurement, wherein: a) the wires in the normal position, b) wires rotated by 90° relative to the normal position

The results show more susceptibility of the wire Djp100 to the loading force, due to the smaller cross-sectional area of the wire. Measurements taken with a rotation by 90° show the effect of the indentation suspension on the stiffness of the wire. This effect is much larger for the wire Djp100. In case of the wire Djp 150, the difference between the measurements in both positions is virtually imperceptible.

When carrying out dynamic measurements, the applied loading force F was zeroed stepwise, and the displacement of the wire was measured at a point close to its end in both the direction of the force F (y-axis) and in the perpendicular direction (x-axis). Dynamic measurements were performed for different positions of the contact wire relative to the indentation of the dropper. Wire movements in the x and y axes were registered. Vibrations

of the wires in their normal position and when rotated by 45° are shown in Fig. 6. A similar vibration damping character as for the normal position is also appropriate for positions of 90°, 180° and 270°, and the damping of the character similar to the one for 45° takes place also for the rotation angle of 135°, 225° and 315°.

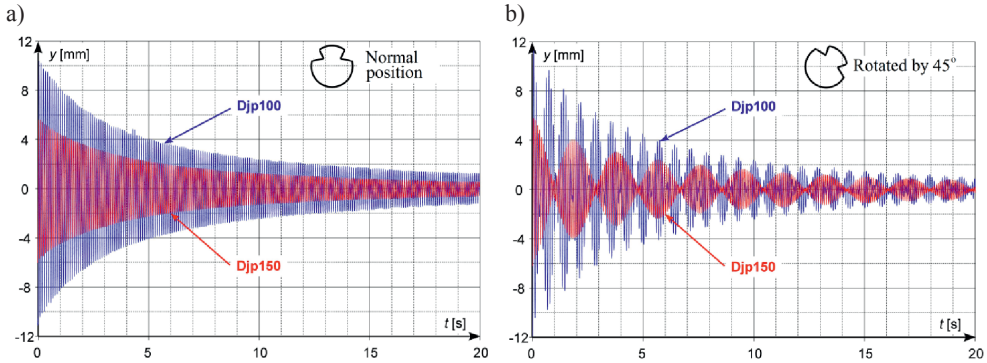

Fig. 6. Excessive vibrations at step reduction of the loading force: a) wires in the normal position; b) wires rotated by 45° relative to the normal position

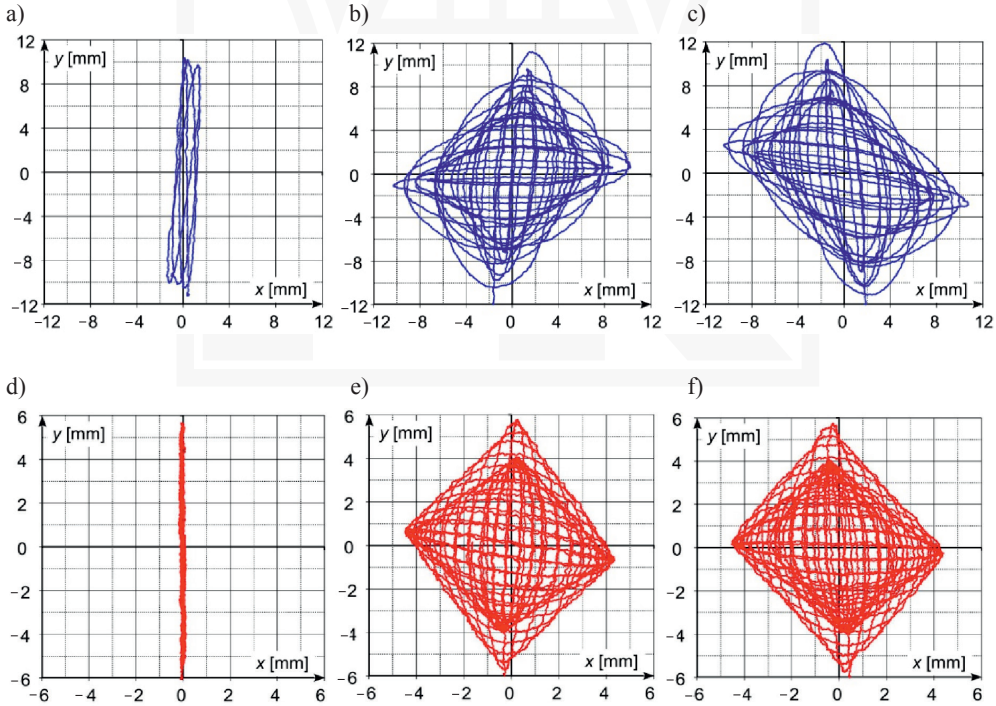

Fig. 7. The trajectory of vibrations at the end of the wire at step reduction of the loading force: a–c) wire Djp100 respectively in normal position (a), rotated by 45° (b), rotated by –45° (c); d–f) wire Djp150 respectively in the normal position (d), rotated by 45º (e), rotated by –45° (f)

On the basis of measurements and catalogue data of the contact wires, mechanical parameters of the concentrated elements were identified, i.e. inertial mass of the nodal elements, compliance of the substitute elastic elements and damping coefficients of kinetic equivalent dampers. In determining the susceptibility of equivalent springs in the contact wire, it was established at the outset that the characteristics of these springs are the same. It is also assumed that the characteristics of the equivalent dampers in the contact wire are the same. Similar assumptions were adopted for the springs and dampers representing the catenary construction.

The measurement results, which show the trajectory of the end of the wire, are shown in Fig. 7. When the axis of symmetry of the wire lies in the axis of force function F or is perpendicular to that surface, then the vibration occurring in the one axis (movements similar to those shown in Fig. 7a and 7d were obtained for wires rotated by 90 and –90°). If the axis of symmetry is not in the axis of the force function, we observe a complex vibration in the x and y axes (see Fig. 7b, 7c, 7e and 7f).

The resulting registrations of trajectory indicate that the simple models based on the reference model, which describes the movement of the system only in the vertical direction, may not fully reflect the complex nature of the phenomena for network sections with a special structure.

4. Examples of simulation results

Comparison of the results of simulations and measurements of the deformation of the section of unilaterally fixed wire Djp 150 is shown in Fig. 8. These results relate to a new wire in a normal position. The parameters of the lumped elements of the wire model are as follows: the mass of the inertial element in the node is $m = 0.032$ kg, the compliance of the equivalent spring is $K = 3.5\times10^{-5}$ m/N, the kinetic damping coefficient is $D = 0.038$ Ns/m. The K value is the same for all the springs in the wire model. The value of D is also constant for all dampers in the wire model.

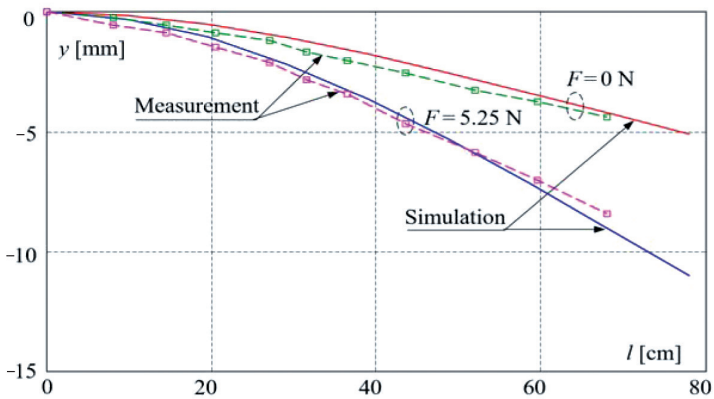

Fig. 8. Comparison of the results of simulations and measurements of deformation of a section of the wire Djp 150 in normal position, fixed unilaterally

Simulation of the dynamic state of the same section of the wire is shown in Fig. 9. The graph shows the dynamic deformation of the end of the wire Djp 150 in conditions similar to the conditions of measurement, the results of which are shown in Fig. 6a. In the model developed for the purposes of this work, the so-called viscous dampers were used. In further research, dry friction will also be analysed. As was demonstrated by a preliminary analysis of the envelope, dry friction may be a part of the dissipation of mechanical energy in the structure of the wire.

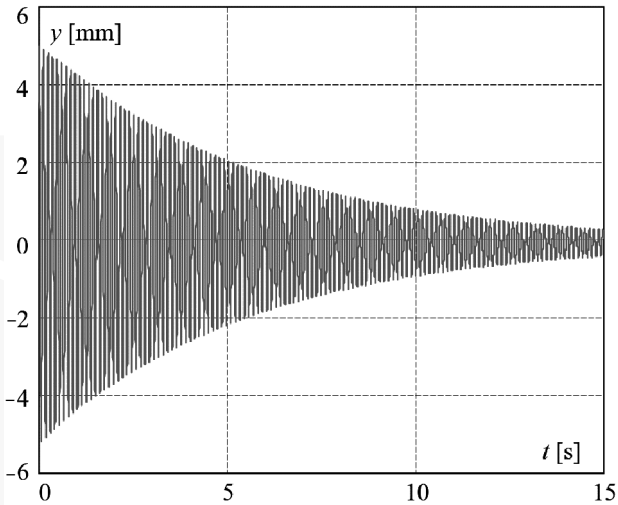

Fig. 9. The simulation result of the dynamic state of the section end free deformation for the wire Djp 150 in normal position, fixed unilaterally

From the comparison of simulation and measurements, a conclusion can be drawn that the proposed model of the division of wire into lumped elements, relatively correctly reproduces static and dynamic states. It also turned out that the use of equivalent elastic elements having the same characteristics in the structure of the contact wire and messenger is justified. This also applies the same characteristics of dissipative elements. This approach greatly facilitates the appointment of equivalent elastic and damping elements.

5. Conclusions

The paper presents a method of modelling catenary construction elements, especially continuous media (contact wire and messenger wire) using lumped conservative and dissipative elements. In order to determine the mathematical model of the line, the Lagrange's energy method was used. To model the contact wire and messenger wire, the authors propose to use a string of 8-node elements. The spatial arrangement of nodes provides a longitudinal and lateral stiffness of the wire. In the first approach, an assumption was made that the compliance of equivalent springs is the same in the model of the contact wire and the messenger wire. The same assumption was made with respect to the equivalent damper parameters. With

this approach, the influence of wear of the contact wire to its dynamics can be mapped by adjusting the relative position of the nodes in the 8-node element string.

The conducted experimental studies were used to determine the parameters of the concentrated elements of the model. They demonstrated the rather complicated dynamics of the vibration of the contact wire, especially at its different angular position.

Comparison of the results of the experiment and the simulation showed the validity of the approach for modelling the overhead line elements, especially the contact wire. Despite the linear characteristics of equivalent springs and equivalent dampers, a relatively good coherence of simulation results and measurements was achieved.

Currently, a software is developed that will be able to simulate the entire overhead contact line system taking into account elements that bring irregularities, for example, connectors, insulators, as well as heterogeneity of the cross-section of the wire, etc.

The developed program for the simulation of mechanical quantities of a catenary construction can be used for design purposes. It is expected to be used more in monitoring the current collectors on the railway in operating conditions in order to further refine the assessment of the technical condition of the collectors [10, 14].

Adding a software module for the calculation of electrical quantities, e.g. a simulation of useful voltage based on the distribution of substations with regard to vehicle motion, will develop useful tools for designing overhead lines.

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