Data Descriptor

Minimal Sets of Lefschetz Periods for Morse-Smale Diffeomorphisms of a Connected Sum of g Real Projective Planes

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Abstract

The dataset titled Database of the minimal sets of Lefschetz periods for Morse-Smale diffeomorphisms of a connected sum of g real projective planes contains all of the values of the topological invariant called the minimal set of Lefschetz periods, computed for Morse-Smale diffeomorphisms of a non-orientable compact surface without boundary of genus g (i.e. a connected sum of g real projective planes), where g varies from 1 to 54.

Keywords: periodic points; Morse-Smale diffeomorphisms; minimal sets of Lefschetz periods; Lefschetz number

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Specification table (data records)

Subject area	Dynamical systems
More specific subject area	Topological invariants, Periodic points theory
Type of data	Text
How the data was acquired	The data was generated using the Mathematica program, based on the algorithm published in (Graff and Myszkowski, 2019)
Data format	The lists are in the .txt format
Experimental factors	The data contained in the dataset was not processed
Experimental features	Values of the invariant

Data source location	MOST Wiedzy Open Research Catalog, Gdańsk University of Technology, Gdańsk, Poland
Data accessibility	The dataset is accessible and is publicly and freely available for any research or educational purposes

Background

Since 2007, the minimal sets of Lefschetz periods have been considered for Morse-Smale diffeomorphisms f of several classes of spaces, such as orientable and non-orientable compact surfaces without boundary, Cartesian product of spheres and n-dimensional tori. The importance of the set results from the fact that it constitutes a subset of minimal periods for the map f. The definition of is based on the analysis of possible forms of Lefschetz zeta functions for f whose form is rather complicated and thus manual computation of the invariant is usually extremely difficult.

In (Graff and Myszkowski, 2019), the definition of the minimal set of Lefschetz periods was reformulated by the use of so-called periodic expansion (cf. (Jezierski and Marzantowicz, 2006)), and the following theorem was proven: for a given Morse-Smale diffeomorphism f with a given periodic expansion of the Lefschetz numbers , we have:

This statement allows us to build an algorithm for finding minimal sets of Lefschetz periods within the given setting. Let us note that the number of periods grows exponentially as the genus g of the surface increases. As a consequence, the calculations could only be conducted for relatively small values of g.

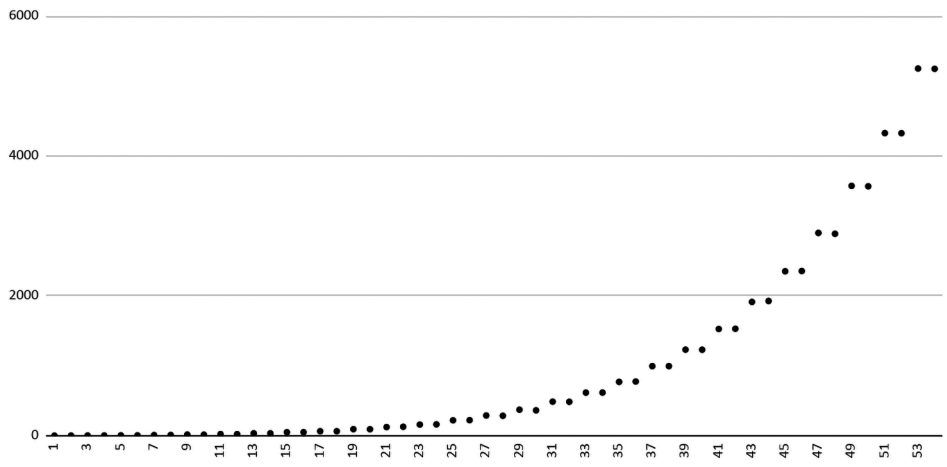

Fig. 37.1. The graph shows the number of all of the different minimal sets of Lefschetz periods for a connected sum of g real projective planes, g = 1, …, 54

Our dataset, Database of the minimal sets of Lefschetz periods for Morse-Smale diffeomorphisms of a connected sum of g real projective planes, may be useful for under-

standing the dynamics of Morse-Smale diffeomorphisms. It can also be used as a tool for checking hypotheses about the behaviour of periods of Morse-Smale diffeomorphisms. Moreover, the datasets allow the omission in (Llibre and Sirvent, 2013) to be eliminated in the calculation of the minimal sets of Lefschetz periods in the case of $g = 9$. Our results are a typical example of generating useful mathematical data: they include a list of some important mathematical quantities that could not be determined in a straightforward way, by manual calculations; however, application of a clever algorithm made it possible to compute them.

Methods

The results were obtained by a program in Mathematica based on the algorithm introduced in (Graff and Myszkowski, 2019). In order to obtain the results in a reasonable amount of computing time, it was necessary to express abstract mathematical theory by means of a computationally efficient framework. The design of the algorithm took into account many numerical properties of Lefschetz numbers and Möbius functions, which allowed the number of necessary calculations to be significantly reduced.

Data quality and availability

Dataset DOI
10.34808/9aj1-1977

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References

Graff, G., Lebiedź, M. and Myszkowski, A. (2019) 'Periodic expansion in determining minimal sets of Lefschetz periods for Morse–Smale diffeomorphisms', *Journal of Fixed Point Theory and Applications*, 21, 47. DOI: 10.1007/s11784-019-0680-4.

Jezierski, J. and Marzantowicz, W. (2006) '*Homotopy methods in topological fixed and periodic points theory. Topological fixed point theory and its applications*', 3, Springer, Dordrecht. DOI: 10.1007/1-4020-3931-X.

Llibre, J. and Sirvent, V.F. (2013) 'Minimal sets of periods for Morse–Smale diffeomorphisms on non-orientable compact surfaces without boundary', *Journal of Difference Equations and Applications,* 19(3), pp. 402–417. DOI: 10.1080/10236198.2011.647006