

## Mode vibrations of plates – experimental analysis

KACZMAREK A.<sup>1)</sup>, JAVOREK L.<sup>2)</sup>, ORŁOWSKI K.<sup>1)</sup>

<sup>1)</sup> Department of Manufacturing Engineering and Automation, Mechanical Engineering Faculty, Gdansk University of Technology, Poland

<sup>2)</sup> Department of Woodworking Machines and Equipment, Faculty of Environmental and Manufacturing Technology, Technical University in Zvolen, Slovakia

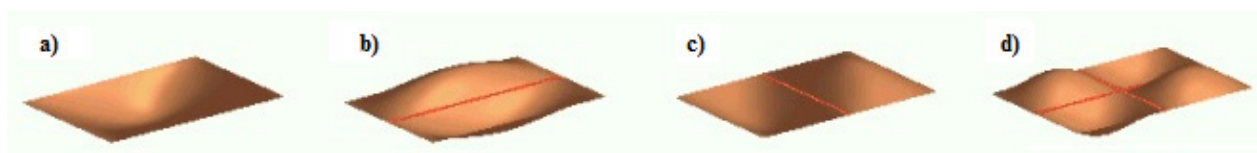
**Abstract:** *Mode vibrations of plates - experimental analyses.* In this paper results of the square MDF plate and circular saw blades examination with the use of the harmonic method to static identification of resonant frequencies and shapes of mode vibrations are presented. Obtained results revealed that for circular saws with large holes in a saw blade appeared two resonant frequencies for the same nodal number.

**Keywords:** harmonic method, resonant frequencies, circular saw blades

### INTRODUCTION

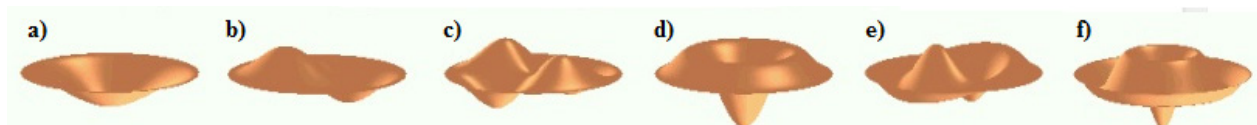
A harmonic method is based on the classical Chladni patterns method of identity the modal shape of resonant of plates. Ernst Chladni has started this field of science in frame of acoustic and more specifically for examination standing waves. It was helping him to achieving the desired tone of instruments which were made all by himself. In fact it has become to be the way for violin makers of testing their products. This method is used as well for examination violins and guitars or other instruments and also as another objects with more or less complicated shapes – for example circular saw blades (Orłowski and Javorek 2009).

According to Chladni's method it has been proved that plates with different shapes have different ways of the modes appearing, therefore their modes should be named diversely. For a rectangular plate modes are identified by two numbers  $(n, m)$  – where  $n$  is corresponding to number of the lines running parallel to the longest axis of the plate and  $m$  is the number of lines being in perpendicular direction to this axis (Fig. 1.). For a circular plate, that modes are also called  $(n, m)$ , but in this case  $n$  represents the number of lines which are located on diameters, while  $m$  is the number of lines which have radial shape (Fig. 2.) [9, 10].



**Figure 1.** Models of vibration modes of rectangular plate:

a) mode  $(n=1, m=1)$ , b) mode  $(n=1, m=2)$ , c) mode  $(n=2, m=1)$ , d) mode  $(n=2, m=2)$  [10]



**Figure 2.** Models of vibration modes of circular plate: a) mode  $(n=0, m=1)$ , b) mode  $(n=1, m=1)$ , c) mode  $(n=2, m=1)$ , d) mode  $(n=0, m=2)$ , e) mode  $(n=1, m=2)$ , f) mode  $(n=0, m=3)$  [10]

The frequency of modes for thin rectangular or square plate should be determined as [9]:

$$f = \sqrt{\left(\frac{n}{l}\right)^2 + \left(\frac{m}{w}\right)^2} \quad [\text{Hz}] \quad (1)$$

where:  $l$  – is length of the plate  
 $w$  – is width of the plate

For circular saw blades there exists theory, which says that resonance of circular plates is a result of the interference two component waves in which the first is traveling forward and the second is traveling backward. Because of that it gives two equations – one for each wave (Schajer 1986; Orłowski et al. 2007; Šteuček 1971):

$$f_f = f_{s(N)} + \frac{n * N}{60} \quad [\text{Hz}] \quad (2)$$

$$f_b = f_{s(N)} - \frac{n * N}{60} \quad [\text{Hz}] \quad (3)$$

where:  $N$  – is rotational speed of saw [rpm]  
 $n$  – is the number of nodal diameter [-]  
 $f_s$  – is the natural frequency of saw:

$$f_{s(0)}^2 = f_{(N=0)}^2 + \lambda * \left(\frac{N}{60}\right)^2 \quad [\text{Hz}] \quad (4)$$

where:  $f_{(N=0)}$  – is the natural frequency of non-rotating saw ( $n = 0$ ) [Hz]  
 $\lambda$  – coefficient of centrifugal force and in the work by Šteuček (1971) is defined by:

$$\lambda = \frac{m_p - 1}{4 * m_p} * n^2 + \frac{3 * m_p + 1}{60} * n \quad [-] \quad (5)$$

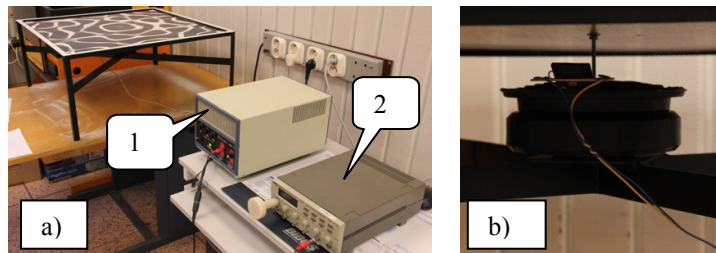
where:  $m_p$  – is constant value of Poisson's process [-]

Knowledge of the frequency of the circular saw blade vibration is necessary to determine its critical rotational speed. It is important because the critical rotational speed of the circular saw defines the maximum speed for which we can be sure that saw stability is guaranteed (Orłowski et al. 2007; Stakhiev 1998, 2000, 2003). Therefore, the value of critical rotational speed may be calculated from the following equation:

$$n_{cr} = \frac{60 * f_{(N=0)}}{\sqrt{n^2 - \lambda}} \quad [\text{rpm}] \quad (6)$$

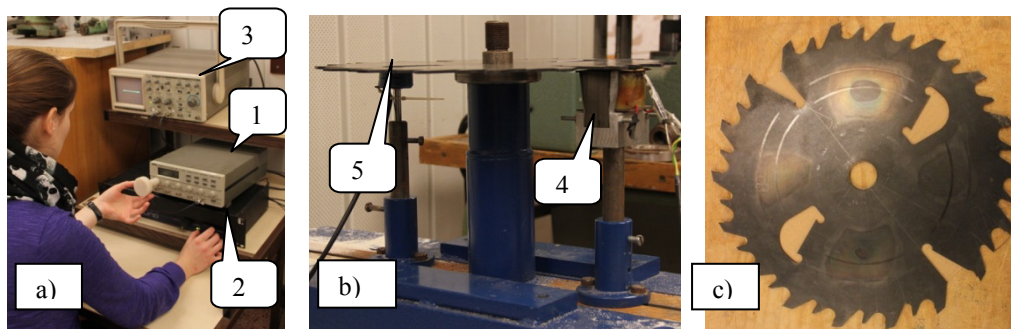
## MATERIALS AND METHODS

Experiments were carried out at the laboratory of the Department of Woodworking Machines and Equipment of the Technical University in Zvolen. The first object which resonant frequencies were examined was a square thin plate made of MDF with dimensions: 660 × 660 mm and thickness 3 mm. The laboratory stand on which the MDF plate was examined is shown in Fig. 3. It should be emphasized that this plate was inducted centrically, so expected shapes of modes ought to be in some way symmetrical and centrically spaced.



**Figure 3.** Laboratory stand for testing MDF plate by method of harmonic test: a) view of stand, b) inductor, where: 1 – input function generator, 2 – oscilloscope

The second kind of tested object was the circular saw blade type “Multix” made by ASPI TECH Sp. z o.o. from Suwałki (PL). The circular saw blade data is as follows: outside diameter  $D = 350$  mm, internal diameter  $d = 30$  mm, saw blade thickness  $s = 2.8$  mm, teeth number  $z = 18$ . The examined circular saw blade was clamped with collars (of the external diameters in parentheses  $d_z$  [mm]) **K1**(90)). The laboratory stand for testing circular saw blades at the Laboratory of TU in Zvolen is presented in Fig. 4.



**Figure 4.** Laboratory stand for testing circular saws with harmonic method: a) view of stand, b) tested saw on stand, c) tested saw, where: 1 – input function generator, 2 – signal amplifier, 3 – oscilloscope, 4 – inductor, 5 – vibration detector

## RESULTS

In Tab. 1. examples of vibration modes with corresponding values of resonant frequencies of the examined MDF plate are shown. Since the examined MDF plate was excited centrally, so as it was expected, the obtained shapes of modes were in some way symmetrical and centrically spaced. Moreover, the simple kinds of modes were not observed as in could be for this rectangular plate because of different dynamical behavior of the thicker plate, which is additionally stiffened by its restraint.

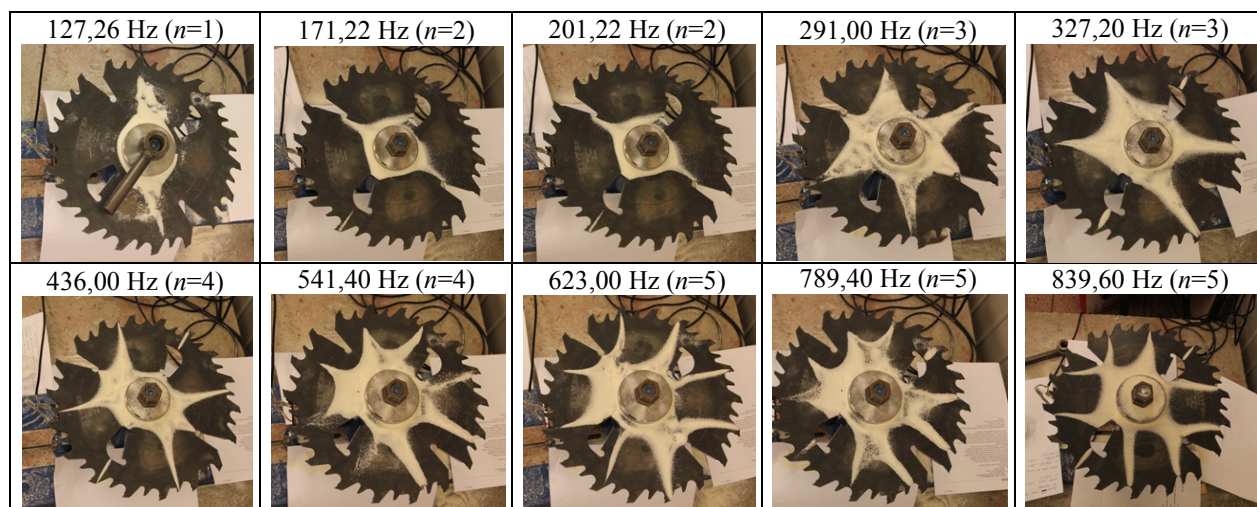
In Tab. 2. The examples of obtained vibration modes of the examined circular saw blade are presented. The conducted experiments revealed that for that kind of the circular saw blades with large holes in the saw blade simultaneously with deep notches for cleaning (shaving) knives the same sorts of modes were observed for different frequencies. It is proposed to call these frequencies as a twin frequencies. This phenomenon ought be thoroughly examined in the further investigations. There are some doubts connected with determination of the centrifugal coefficient for that kind of the circular saw blade design.

**Table 1.** Examples of vibration modes of square MDF plate with its resonant frequencies

285 Hz 	505 Hz 	539,4 Hz 	563,2 Hz 	1417,8 Hz 
1722,8 Hz 	1851,8 Hz 	2018 Hz 	2194 Hz 	2556 Hz 

**Table 2.** Vibration modes of testes saw blade clamped with collars K1, its resonant frequencies and nodal number





## CONCLUSIONS

1 – vibration modes for square plates, which are built-in corners and inducted centrally, do not corresponds to vibration modes for plates with free edges, which are mostly presented in literature;

2 – for nodal number from  $n = 2$  and higher, for examined circular saw blade was observed the phenomenon of the same vibration modes appearing, however, with different resonant frequencies.

**Acknowledgements:** It is kindly acknowledged that the experimental part of this work has been carried out in the frame of the CEEPUS III Program the net SK310 at the Technical University in Zvolen (SK). The authors would like also to acknowledge the firm PPH ASPITECH Sp. z o.o. (Suwalki, PL) for donation of the circular saw blades.

## REFERENCES

- ORŁOWSKI K., JAVOREK L., 2009: *Limitations of the Chladni patterns method of the determination of circular saw blade critical rotational speed*. In.: Annals Warsaw University of Life Science – Forestry and Wood Technology No 69. pp.152-157, Warsaw.
- ORŁOWSKI K., SANDAK J., TANAKA Ch., 2007: *The critical rotational speed of circular saw; simple measurement method and its practical implementations*. J Wood Sci. October 2007, vol. 53, Issue 5, pp. 388-393.
- SCHAJER S.G., 1986: *Simple formulas for natural frequencies and critical speeds of circular saws*. Forest Products Journal, vol. 36, No. 2. pp. 37-43.
- STAKHIEV Y.M., 1998: *Research on circular saws vibration in Russia: from theory and experiment to the needs of industry*. Holz als Roh- und Werkstoff, vol. 56. pp.131-137. Springer-Verlag
- STAKHIEV Y.M., 2000: *Today and tomorrow circular sawblades: Russian version*. Holz als Roh- und Werkstoff, vol. 58. pp.229-240. Springer-Verlag.
- STAKHIEV Y.M., 2003: *Research on circular saw disc problems: several of results*. Holz als Roh- und Werkstoff, vol. 61. pp.13-22. Springer-Verlag.
- ŠTEUČEK D., 1971: *Zisťovanie kritických obrátok pilových kotúčov*. Bezpečná práca, num. 5, vol. 2. pp.7-11. Bratislava.
- SVOREŇ J., 2012: *The analysis of the effects of the number of teeth of circular-saws blade on the critical rotation speed*. Acta Facultatis Technicae vol. XVII(2):109-117. [http://www.tuzvo.sk/files/FEVT/fakulta\\_fevt/akta\\_fevt-2-2012-svoren.pdf](http://www.tuzvo.sk/files/FEVT/fakulta_fevt/akta_fevt-2-2012-svoren.pdf)
- DAVIDSON PHYSICS: <http://www.phy.davidson.edu/stuhome/derekk/resonance/pages/plates.htm> (access on August 24, 2014)
- INSTITUTE OF SOUND: [http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial\\_files/Web-standing-membrane.htm](http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-standing-membrane.htm) (access on August 24, 2014)



**Streszczenie:** *Postaci drgań płyt – analiza doświadczalna.* W pracy przedstawiono wyniki badań kwadratowej płyty MDF oraz płyty tarczowej z zastosowaniem metody testu harmonicznego służącego do statycznego wyznaczania częstotliwości rezonansowych oraz towarzyszącym im postaciom drgań. Zaobserwowane wyniki pokazują, że dla płyt tarczowych o dużych otworach w korpusie płyty pojawiają się dwie częstotliwości rezonansowe dla tej samej liczby węzłowej.

Corresponding author:

Kazimierz A. Orlowski,  
Department of Manufacturing Engineering  
and Automation,  
Mechanical Engineering Faculty,  
Gdansk University of Technology,  
Narutowicza 11/12,  
80-233 Gdansk, Poland  
e-mail: [korlowski@pg.gda.pl](mailto:korlowski@pg.gda.pl)

Lubomír Javorek,  
Department of Woodworking Machines and Equipment,  
Faculty of Environmental and Manufacturing  
Technology,  
Technical University in Zvolen,  
26 Študentská str.,  
960 53 Zvolen, Slovakia  
e-mail: [lubomir.javorek@vsld.tuzvo.sk](mailto:lubomir.javorek@vsld.tuzvo.sk)