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Multipath Complex Root Tracing

Sebastian Dziedziewicz, Malgorzata Warecka, Rafal Lech, Piotr Kowalczyk
Department of Microwave and Antenna Engineering,
Faculty of Electronics, Telecommunications and Informatics
Gdansk University of Technology,
Narutowicza 11/12, 80-233 Gdansk, Poland,
e-mail: sebastian.dziedziewicz@pg.edu.pl

***Abstract*—The problem of multipath root tracing is being addressed in this communication. The self-adaptive complex root tracing algorithm, which was previously utilized for the investigation of various propagation and radiation problems, is analyzed here for the cases when the traced characteristic bifurcates. A procedure of multiroute detection is proposed and demonstrated on the coaxially loaded cylindrical waveguide example.**

***Index Terms*—Complex modes, complex root, iterative algorithms, resonant structure, root tracing**

I. Introduction

The problem of complex root calculation can be found in many fields of science. In electromagnetic and RF engineering, it is commonly related to the calculation of the propagation coefficient of various waveguides [1]–[6], resonant frequencies [7]–[9], antenna input impedances [10], or material characterization [11] to name but a few. In modern physics in the research of exotic particles [12], for solving of nonlinear Schrödinger equation [13], spectroscopy [14], in acoustics [15], in optics [16], [17], and in automatic stability analysis [18].

When a single root is found, it is often of interest to find its behaviour in a function of another parameter such as frequency or structure dimension. Instead of utilizing the root finding algorithm, which can be called several times for each discretized value of the additional parameter, a root tracing algorithm can be used. Recently, the authors developed a self-adaptive complex root tracing algorithm which was successfully utilized to the analysis of various electromagnetic problems [19]. The algorithm was a modified and enhanced version of the algorithm [20]. This approach creates a curve in $\mathbb{C} \times \mathbb{R}$ space representing a path of the root as a function of another parameter, and it is generally more efficient and produces results in a fraction of the time the finding algorithm takes to run. The only drawback of this solution is the possibility of tracking a single root. In the case when the traced characteristic splits or when two separate characteristics are closely located or cross, the algorithm may loose the correct path or mix the characteristics. Thus, the obtained characteristic (from this critical point) is the only one of the possible solutions that we get as a result of the algorithm's operation and the user has no influence on the choice of the specific solution in the current version of the algorithm.

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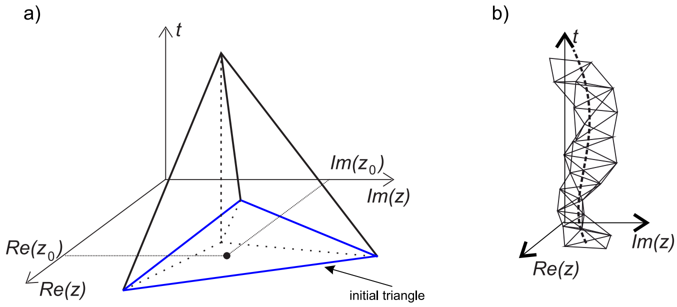

Fig. 1. Regular tetrahedron in $\mathbb{C} \times \mathbb{R}$ space and chain of regular tetrahedrons representing the searched curve.

In this communication, we show that the resultant traced path, which is shaped by the existence of critical points (such as splits or path crossing), depends on the parameters of the tracing algorithm and their choice can be used to obtain all possible solutions.

II. Formulation of the problem

In order to track a single root of the function $F(z, t) = 0$, its initial value (for initial value of parameter t) needs to be found. One can use several different root finding algorithms for this task. Below, a brief summary of the utilized root tracing algorithm [19] is presented.

When the initial root value is calculated, the root tracing algorithm [19] establishes a triangle on the complex plane in which the root is located. The size of this base triangle is depended on the assumed accuracy. Next, taking into account the assumed step of the parameter for which the root is traced, three additional triangles are built, which form an initial tetrahedron (see Fig. 1a). In each iterative step, a new outgoing root value is searched on each new face of the tetrahedron. A face, on which a new root value is found, becomes the next base triangle and three new triangles are built forming a new tetrahedron. The procedure is repeated until the entire root characteristic is obtained in the assumed range of the additional parameter producing the tetrahedron chain (see Fig. 1b).

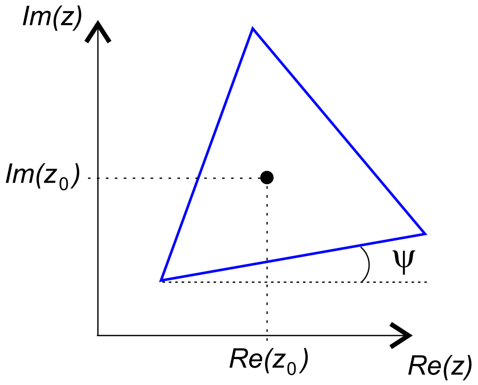

Fig. 2. The orientation of the initial triangle on the complex plane.

The algorithm works very efficiently when the traced characteristic has a single path form. However, when the characteristic bifurcates, a second root appears in the area surrounded by a tetrahedron, and the algorithm needs to be modified to take account of this change. Multiple roots may be localized on the same face or be on separate faces of the tetrahedron.

In the original algorithm, Cauchy argument principle involving the evaluated points at the edges is used. The function values (at the edges) are iteratively evaluated until a single root is detected and then the algorithm automatically advances to the next tetrahedron. It is then possible to detect multiple roots on any face of a tetrahedron. However, it is not obvious that these roots can be found simultaneously from the evaluated set of points. If multiple values of the root are found, the algorithm can remember the detection, proceed with a single root tracing, and return to the second root after completing the first path. However, the algorithm is not suited to look for multiple values of roots in a single tetrahedron due to the lack of termination condition - a single outgoing root value in the tetrahedron is guaranteed and the search of another would require the addition of an infinitely large number of evaluated points without the knowledge of its existence.

In this communication, we show that the introduction of the additional algorithm parameter can influence the shape of the root path. In the algorithm [19] the user defines the starting point (z_0, t_0) and the length of the initial triangle edge Δr as a single analysis parameter, which corresponds to the accuracy of the obtained characteristic. The starting point is surrounded by the initial triangle, which forms the basis of the initial tetrahedron. The orientation of this triangle on the complex plane was fixed (one of the edges was parallel to the real axis for $t = t_0$). This assumption was justified because any modification of the triangle orientation does not change the shape of the obtained characteristic in the case where no critical points are present – only the configuration of the tetrahedrons inside the chain can be slightly different. However, this orientation can be crucial when the critical points appear along the path. In such a case, this may cause the algorithm to take a different path after passing the critical point.

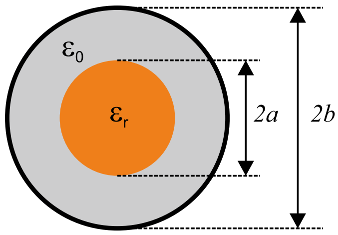

Fig. 3. Cross-section of coaxially loaded cylindrical waveguide.

Therefore, the introduction of the additional algorithm parameter ψ (see Fig. 2)., which represents the rotation of the initial triangle in the range from $0°$ to $120°$, allows one to obtain all possible root paths. However, the determination of all possible paths requires multiple calls of the algorithm for different values of angle ψ. The number of required algorithm calls for different values of ψ depends on the path complexity and the range of the analysis. The conducted analyzes showed that the bisection method may be useful in sequence selection of ψ angle: $0°$, $60°$, $\{30°, 90°\}$, $\{15°, 45°, 75°, 105°\}$ etc. In order to confirm the detection of all possible paths, the additional root searching analysis [21] can be performed, for a few cross-sections (for chosen values of $t = const$).

III. Results

The analyzed example considers tracing of the propagation coefficient of a mode in a coaxially loaded cylindrical waveguide. This example has already been discussed in [19] (see Fig. 3). The inner rod relative permittivity is $\varepsilon_r = 10$ and the structure dimensions: inner radius $a = 6.35$ mm and outer radius $b = 10$ mm. The structure is modeled analytically and the mode matching technique is utilized to build the function $F(z, t) = 0$, where z is the propagation coefficient and t represents the frequency. The analysis was performed in the frequency range from 1 to 7 GHz. The value of the initial root, which corresponds to the propagation coefficient $\gamma = 2.172$, was calculated by the standard root finding algorithm [21] at $f = 7$ GHz with accuracy $\delta = 10^{-12}$ (the region $\Omega = \{z \in \mathbb{C} : -0.5 < \mathrm{Re}(z) < 0.5 \wedge 1.8 < \mathrm{Im}(z) < 2.6\}$ was considered - see. Fig. 4a). The tracing routine was performed with a resolution step $\Delta r = 0.01$ with frequency normalization to 1 GHz.

By changing value of ψ of the initial triangle (see Fig. 4b), we obtain four separate mode characteristics as presented in Fig. 5a. The final combined characteristic presented in Figs. 5b-d is a result of four separate tracing routines.

IV. Conclusions

The introduction of the additional algorithm parameter can influence the shape of the traced root path. Its proper choice allows for the evaluation of all possible characteristics, which are the result of the presence of critical points. The proposed modification extends the capabilities of the algorithm.

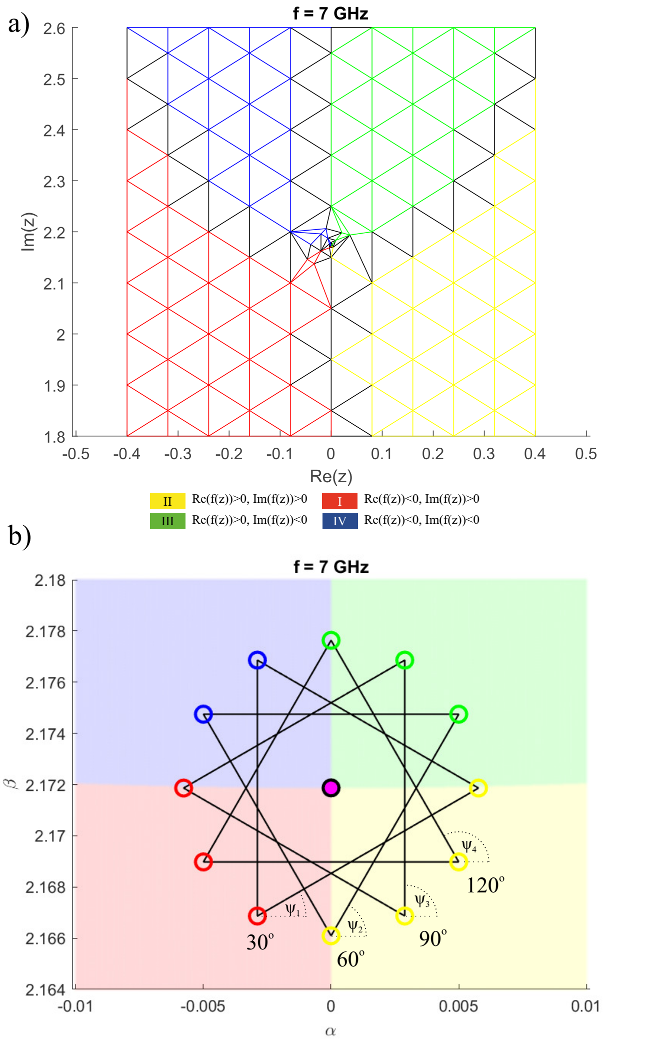

Fig. 4. A phase portrait of the function is placed in the background to clarify the idea of triangle arrangement depending on the initial angle.

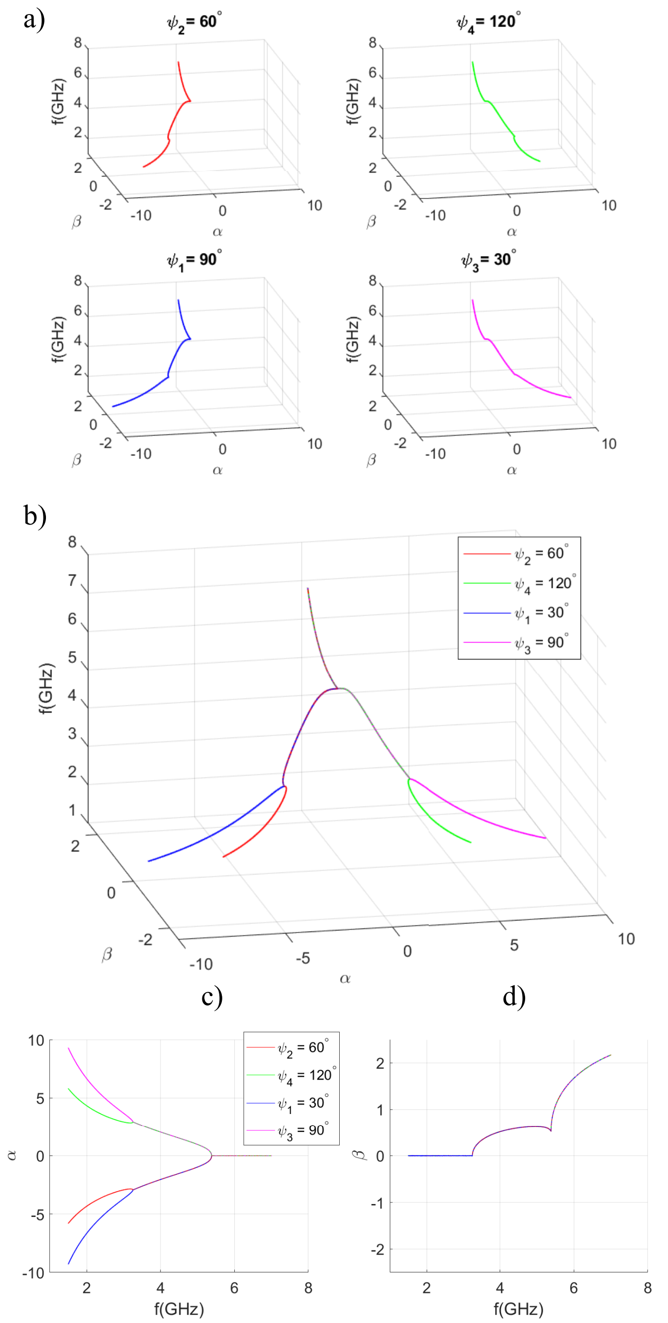

Fig. 5. Propagation coefficient characteristics of the analyzed example: a) separate characteristic components, b) combined characteristic, c) real axis projection and d) imaginary axis projection.

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