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Nonlinear generation of non-acoustic modes by low-frequency sound in a vibrationally relaxing gas

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Summary

Two dynamic equations referring to a weakly nonlinear and weakly dispersive flow of a gas where molecular vibrational relaxation takes place, are derived. The first one governs an excess temperature associating with the thermal mode, and the second one describes variation in vibrational energy. Both quantities refer to the non-wave types of gas motion. These variations are caused by the nonlinear transfer of acoustic energy into that of thermal mode and internal vibrational degrees of freedom of a relaxing gas. The final dynamic equations are instantaneous, they include quadratic nonlinear acoustic source reflecting the nonlinear character of interaction of low-frequency acoustic and non-acoustic motions of fluid. All types of sound, periodic or aperiodic, may serve as an acoustic source of both phenomena. The low-frequency sound is considered in this study. Some conclusions about temporal behavior of non-acoustic modes caused by periodic and aperiodic sound, are made. Under certain conditions, acoustic cooling instead of heating takes place.

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1 Introduction. Basic equations and starting points

The attention of physicists to non-equilibrium phenomena, particularly to non-equilibrium gases, was attracted by the fundamental results leading to laser revolution [1-3]. The establishment of non-equilibrium molecular physics began in the sixties due to the progress in laser technique and following it researches in physics and chemistry. Non-equilibrium gases include not only the active gases used in lasers, but the discharge plasma, the rarified levels of upper atmosphere, the interstellar media, and so on.

Interest in non-equilibrium phenomena in the physics of gases was firstly connected with studies of anomalous dispersion and absorption of ultrasonics waves. The reason for these anomalies is the mechanism of retarded energy exchange between the internal and translational degrees of freedom of the molecules [1]. The non-equilibrium energy distribution alters the adiabatic compressibility, and hence the sound velocity, leading to anomalous dispersion of sound [3-5]. The general theory of thermodynamic relaxation was created by Mandelshtam and Leontovich [6]. The hydrodynamics of non-equilibrium fluids is a quickly developing area of hydrodynamics passing now through the stage of formulating the fundamental equations and for revealing new physical effects. It is based on the advances in molecular physics, which itself studies kinetic processes in fluids, in spite of hydrodynamics considers fluid as a continuum medium.

This paper is devoted to the secondary phenomena, caused by the low-frequency sound. The first is nonlinear interaction of sound with the thermal mode. Losses in acoustic energy in the standard thermoviscous fluid always increase the background temperature, and this phenomenon is called acoustic heating [7,8]. It was firstly discovered by Molevich that the nonlinear exchange of energy between sound and the thermal mode may lead to a cooling instead of heating in a gas where non-equilibrium relaxation takes place [9]. The second phenomenon also originates from the vibrational relaxation and refers to nonlinear generation of non-wave part of vibrational energy by the dominative sound. As far as the author knows, it is a new subject to study. The choice of the low-frequency sound in the role of origin of both secondary phenomena is stipulated by its insignificant standard attenuation during propagation. On the other hand, in view of small characteristic times of vibrational relaxation, the low-frequency approach allows to consider wide domains of sound frequencies, including those transmitted usually by acoustic transducers. Some estimations are made in the Concluding Remarks below.

We start from the linear determination of modes as specific types of gas motion in a simple case of motions in a gas whose steady but non-equilibrium state is maintained by pumping energy into the vibrational degrees of freedom by power I and heat withdrawal from the translational degrees of freedom of power Q, both I and Q refer to unit mass (Sec.2).

The relaxation equation for the vibrational energy per unit mass completes the system of conservation equations in the differential form. It takes the form:

$$\frac{d\varepsilon}{dt} = -\frac{\varepsilon - \varepsilon_{eq}(T)}{\tau} + I. \tag{1}$$

The equilibrium value for the vibrational energy at the given temperature T is denoted by $\varepsilon_{eq}(T)$, and $\tau(\rho, T)$ is the vibrational relaxation time. For a system of harmonic oscillators,

$$\varepsilon_{eq}(T) = \frac{\hbar\Omega}{m\left(exp(\hbar\Omega/k_B T) - 1\right)}, \tag{2}$$

where m is a molecule mass, $\hbar\Omega$ is the magnitude of the vibrational quantum, and k_B is the Boltzmann constant. Eq.(2) is valid over the temperatures, where one can neglect anharmonic effects, i.e., below the characteristic temperatures, which are fairly high for most molecules [1-3].

The mass, momentum and energy equations governing thermoviscous flow in a vibrationally relaxing gas read:

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}(\rho\vec{v}) = 0,$$

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\vec{\nabla})\vec{v}\right] = -\vec{\nabla}p + \eta\Delta\vec{v} + \frac{\eta}{3}\vec{\nabla}(\vec{\nabla}\vec{v}), \tag{3}$$

$$\rho\left[\frac{\partial(e+\varepsilon)}{\partial t}+(\vec{v}\vec{\nabla})(e+\varepsilon)\right]+p\vec{\nabla}\vec{v}=\chi\Delta T+\rho(I-Q)+\frac{\eta}{2}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right)^2.$$

Here, $\vec{v}$ denotes velocity of fluid, ρ, p are density and pressure, e marks internal energy per unit mass of translation motion of molecules, η is the shear viscosity, χ denotes the thermal conductivity (all supposed to be constants), x_i - space coordinates. Besides Eq.(2), two thermodynamic functions $e(p,\rho), T(p,\rho)$ complete the system (3). Thermodynamics of ideal gases gives :

$$e(p,\rho)=\frac{R}{\mu(\gamma-1)}T(p,\rho)=\frac{p}{(\gamma-1)\rho}, \tag{4}$$

where $\gamma=C_{P,\infty}/C_{V,\infty}$ is the isentropic exponent without account for vibrational degrees of freedom ($C_{P,\infty}$ and $C_{V,\infty}$ denote "frozen" heat capacities correspondent to very quick processes), R is the universal gas constant, and μ is a molar mass of a gas.

2 Dispersion relations. One-dimensional motions of infinitely small amplitude and their decomposing

Let start to consider a motion of infinitely small amplitude of a gas in the case $\eta=0,\ \chi=0,\ I=Q$. Every quantity q is represented as a sum of unperturbed value q_0 (in absence of the background flows, $v_0=0$) and its variation q'. Sec. 4.3 accounts for standard attenuation and weak influence of perturbations of I and Q on temperature ($I_0=Q_0$). The flow is supposed to be one-dimensional along axis Ox. Going out one dimension needs considering of the vorticity mode. Its generation in the field of dominative sound is briefly discussed in the Concluding Remarks. Firstly, the relations of excess perturbations specific for every mode should be established. These relations will be fixed going to a weakly nonlinear flow. That allow to decompose equations governing sound and non-wave modes accounting for their interactions correctly (Sec.3). Following Molevich [10], we consider weak transversal pumping, which may vary the background quantities in the transversal direction of axis Ox. It is assumed that the background stationary quantities are constant along axis Ox. That is essential for the low-frequency sound.

The governing equations of momentum, energy balance and continuity may be easily rearranged in the following ones:

$$\begin{gathered}\frac{\partial v'}{\partial t}+\frac{1}{\rho_0}\frac{\partial p'}{\partial x}=0,\\ \frac{\partial p'}{\partial t}+\gamma p_0\frac{\partial v'}{\partial x}-(\gamma-1)\rho_0\frac{\varepsilon'}{\tau}+(\gamma-1)\rho_0T_0\Phi_1\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)=0,\\ \frac{\partial\rho'}{\partial t}+\rho_0\frac{\partial v'}{\partial x}=0,\\ \frac{\partial\varepsilon'}{\partial t}+\frac{\varepsilon'}{\tau}-T_0\Phi_1\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)=0,\end{gathered} \tag{5}$$

where

$$\Phi_1=\left(\frac{C_{V,eq}}{\tau}+\frac{\varepsilon-\varepsilon_{eq}}{\tau^2}\frac{d\tau}{dT}\right)_0,\quad C_{V,eq}=\left(\frac{d\varepsilon_{eq}}{dT}\right)_0. \tag{6}$$

The expansions in series of equations of state (4) was used in the second and fourth equations from (5):

$$e' = \frac{R}{\mu(\gamma-1)}T' = \frac{p_0}{(\gamma-1)\rho_0}\left(\frac{p'}{p_0} - \frac{\rho'}{\rho}\right). \qquad (7)$$

The last equation in the set (5) follows from Eqs (1),(7):

$$\frac{\partial \varepsilon'}{\partial t} + \frac{\varepsilon'}{\tau} = \left(\frac{C_{V,eq}}{\tau} + \frac{\varepsilon - \varepsilon_{eq}}{\tau^2}\frac{d\tau}{dT}\right)_0 T' = T_0\left(\frac{C_{V,eq}}{\tau} + \frac{\varepsilon - \varepsilon_{eq}}{\tau^2}\frac{d\tau}{dT}\right)_0 \left(\frac{p'}{p_0} - \frac{\rho'}{\rho_0}\right). \qquad (8)$$

The relaxation time in the most important cases may be thought as a function of temperature accordingly to Landau and Teller, $\tau(T) = A\exp(BT^{-1/3})$, which gives negative values of $d\tau/dT$ [1-3]. It is the most simple but physically justified model [1]. This study does not account for dependence of the relaxation time on density. As regards to the sound properties (sound velocity, attenuation and parameter of nonlinearity), dependence of τ on both T and ρ was considered in details in [10-12].

Studies of motions of infinitely-small amplitudes start with representing of all perturbations as a sum of planar waves, where $\tilde{q}$ is the Fourier-transform of any perturbation q':

$$q'(x,t) = \int_{-\infty}^{\infty} \tilde{q}(k)\cdot \exp i(\omega t - kx)dk + cc. \qquad (9)$$

The dispersion equation follows from Eqs (5):

$$\omega\left(i\Phi_1(\gamma-1)T_0\tau(c_\infty^2 k^2 - \gamma\omega^2) + c_\infty^2(c_\infty^2 k^2 - \omega^2)(i - \omega\tau)\right) = 0, \qquad (10)$$

where $c_\infty = \sqrt{\gamma p_0/\rho_0}$ is the "frozen" sound velocity. The approximate roots of dispersion equation for both acoustic branches, progressive in the positive or negative directions of axis Ox in the low-frequency domain $\omega\tau << 1$, are as follows (see also [5,10,11]):

$$\omega_1 = c_0 k, \quad \omega_2 = -c_0 k, \qquad (11)$$

where $c_0 = c_\infty - \frac{(\gamma-1)^2 T_0 \tau}{2c_\infty}\Phi_1$ is the low-frequency sound velocity. In this study, we consider weak dispersion: $|\Phi_1|\tau << C_{V,\infty}$, but Φ_1 may be positive or not. All formulae below are written on in the leading order with respect to powers of small parameters, one of them being $|\Phi_1|\tau/C_{V,\infty}$. The relationship between the low-frequency and high-frequency sound velocities follows from the expressions for the heat capacities:

$$C_P = C_{P,\infty} + \left(\frac{\partial \varepsilon'}{\partial T}\right)_P, \quad C_V = C_{V,\infty} + \left(\frac{\partial \varepsilon'}{\partial T}\right)_V, \qquad (12)$$

and a perturbation in vibrational energy which follows from the last equation from (5) in the low-frequency limit:

$$\varepsilon' = \Phi_1 \tau T', \qquad (13)$$

which along with Eqs (12) results in the leading order to the following equality

$$c_0^2 = \frac{C_{P,0}}{C_{V,0}}\frac{p_0}{\rho_0} = \left(\frac{C_{P,\infty} + \Phi_1\tau}{C_{V,\infty} + \Phi_1\tau}\right)\frac{p_0}{\rho_0} \approx \left(\frac{C_{P,\infty}}{C_{V,\infty}} - \frac{(\gamma-1)\Phi_1\tau}{C_{V,\infty}}\right)\frac{p_0}{\rho_0} = c_\infty^2 - (\gamma-1)^2\Phi_1\tau T_0. \qquad (14)$$

Both c_0 and c_∞ denote the infinitely-small signal velocities of sound. The linear sound velocity and the parameter of nonlinearity in dependence of sound frequency in the relaxing gases were studied in details in important papers by Molevich and co-authors (among other, [5,12]).

The non-equilibrium regime presupposes $\Phi_1 < 0$ and inverse dispersion, $c_0 > c_\infty$. These two last roots of the dispersion equation, estimated without limitation $\omega\tau << 1$, sound:

$$\omega_3 = i\left(\frac{1}{\tau} + \frac{(\gamma-1)(\gamma + c_0^2k^2\tau^2)T_0}{c_0^2(1+c_0^2k^2\tau^2)}\Phi_1\right), \quad \omega_4 = 0. \tag{15}$$

The last two roots represent slow, non-wave motions of a gas. They are of importance, among other applications (problem of stability of slow perturbations etc), in studies of the variations in the background after the sound passing. The root $\omega_4 = 0$ relates to the thermal (or entropy) mode. This type of non-wave motion in an equilibrium gas means an isobaric increase in the background temperature and correspondent variation in the density. The third root ω_3 is responsible for the non-wave variations in the vibrational energy and appears as one of possible motions in a gas with excited vibrationally degrees of molecular energy.

In accordance to the roots (11),(15), the Fourier-transforms of dynamic variables may be represented as a sum of four specific Fourier-transforms of excess density $\tilde\rho_1,\ \tilde\rho_2, \tilde\rho_3, \tilde\rho_4$ as follows:

$$\tilde v = \sum_{n=1}^{4}\omega_n\tilde\rho_n/(k\rho_0), \quad \tilde p = \sum_{n=1}^{4}\omega_n^2\tilde\rho_n/k^2, \quad \tilde\rho = \sum_{n=1}^{4}\tilde\rho_n, \tag{16}$$

$$\tilde\epsilon = \frac{T_0\Phi_1}{\rho_0c_\infty^2}\sum_{n=1}^{2}\tilde\rho_n\left(\frac{\gamma\omega_n^2}{k^2} - c_\infty^2\right)/(i\omega_n) + \frac{T_0\Phi_1}{\rho_0c_\infty^2}\tilde\rho_3\left(\frac{\gamma\omega_3^2}{k^2} - c_\infty^2\right)/(i\omega_3+1/\tau) + \frac{\tau T_0\Phi_1}{\rho_0c_\infty^2}\tilde\rho_4\left(\frac{\gamma\omega_4^2}{k^2} - c_\infty^2\right).$$

So that, a perturbation of every dynamic variable may be expressed in terms of specific excess densities. The correspondent formulas in the (x,t) space follow from Eqs (16) and the roots of the dispersion equation, Eqs (11),(15), taking in mind, that the overall excess velocity, pressure, density and internal energy are sums of specific parts:

$$v'(x,t) = \sum_{n=1}^{4}v_n'(x,t), p'(x,t) = \sum_{n=1}^{4}p_n'(x,t), \rho'(x,t) = \sum_{n=1}^{4}\rho_n'(x,t), \varepsilon'(x,t) = \sum_{n=1}^{4}\varepsilon_n'(x,t). \tag{17}$$

Relations of acoustic rightwards and leftwards progressive waves in the low-frequency regime ($c_0k\tau << 1$) follow from the dispersion relations $\omega_1(k)$, $\omega_2(k)$ and Eqs (16):

$$v_1'(x,t) = \frac{c_0}{\rho_0}\rho_1'(x,t), \quad p_1'(x,t) = c_0^2\rho_1'(x,t), \quad \varepsilon_1'(x,t) = -\frac{(\gamma-1)T_0\Phi_1}{c_0\rho_0}\int dx\ \rho_1'(x,t), \tag{18}$$

$$v_2'(x,t) = -\frac{c_0}{\rho_0}\rho_2'(x,t), \quad p_2'(x,t) = c_0^2\rho_2'(x,t), \quad \varepsilon_2'(x,t) = \frac{(\gamma-1)T_0\Phi_1}{c_0\rho_0}\int dx\ \rho_2'(x,t).$$

The third mode in the limit $c_0k\tau << 1$ possesses the leading-order relationships

$$v_3'(x,t) = \frac{1}{\rho_0\tau}\left(1 + \frac{(\gamma-1)\gamma T_0\Phi_1\tau}{c_0^2}\right)\int dx\ \rho_3'(x,t),$$

$$p_3'(x,t) = \frac{1}{\tau^2}\left(1 + \frac{2(\gamma-1)\gamma T_0\Phi_1\tau}{c_0^2}\right)\int dx\int dx\ \rho_3'(x,t),$$

$$\varepsilon_3'(x,t) = -\frac{1}{(\gamma-1)\rho_0\tau^2}\left(1+\frac{2(\gamma-1)\gamma T_0\Phi_1\tau}{c_0^2}\right)\int dx \int dx\ \rho_3'(x,t). \tag{19}$$

Equalities defining the thermal mode correspond to an isobaric motion:

$$v_4'(x,t)=0, \quad p_4'(x,t)=0, \quad \varepsilon_4'(x,t) = -\frac{T_0\Phi_1\tau}{\rho_0}\rho_4'(x,t). \tag{20}$$

Relations (18)-(20) along with linear property of superposition, Eq.(17), points out a way of combination of four equations from (5) in order to get dynamic equations for perturbation of only one specific mode. The correspondent decomposing of specific mode is actually acting of matrix operator at the vector of overall perturbations $(v' \quad p' \quad \rho' \quad \varepsilon')$:

$$P_n \begin{pmatrix} v'(x,t) \\ p'(x,t) \\ \rho'(x,t) \\ \varepsilon'(x,t) \end{pmatrix} = \begin{pmatrix} v_n'(x,t) \\ p_n'(x,t) \\ \rho_n'(x,t) \\ \varepsilon_n'(x,t) \end{pmatrix}, \quad n=1,\dots 4. \tag{21}$$

The projecting in regard to problems of weakly nonlinear flows, was worked out and applied by the author in studies of acoustic streaming and heating in the standard thermoviscous flows [13,14] as well as in the dispersive flows. Within accuracy up to the terms of order $(c_0k\tau)^1$, $(|\Phi_1|\tau/C_{V,\infty})^1$, projectors take the form as follows:

$$P_1 = \begin{pmatrix}
\frac{1}{2}+\frac{\Phi_1(\gamma-1)^2T_0\tau}{2c_0^2} + \frac{\Phi_1(\gamma-1)^2T_0}{2c_0^3}\int dx & \frac{1}{2c_0\rho_0}-\frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^3\rho_0} & \frac{\Phi_1(\gamma-1)T_0\tau}{2c_0\rho_0} & \frac{(\gamma-1)}{2c_0}+\frac{(\gamma-1)\tau}{2}\frac{\partial}{\partial x} + \frac{\Phi_1(\gamma-1)^2(\gamma-2)T_0\tau}{2c_0^3} \\
\frac{c_0\rho_0}{2}+\frac{\Phi_1(\gamma-1)^2T_0\tau}{2} + \frac{\Phi_1(\gamma-1)^2T_0}{2c_0}\int dx & \frac{1}{2}-\frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^2} & \frac{\Phi_1(\gamma-1)T_0\tau}{2} & \frac{(\gamma-1)\rho_0}{2}+\frac{(\gamma-1)\rho_0\tau c_0}{2}\frac{\partial}{\partial x} + \frac{\Phi_1(\gamma-1)^2(\gamma-2)\rho_0T_0\tau}{2c_0^2} \\
\frac{\rho_0}{2c_0}+\frac{\Phi_1(\gamma-1)^2\rho_0T_0\tau}{2c_0^3} + \frac{\Phi_1(\gamma-1)^2\rho_0T_0}{2c_0^4}\int dx & \frac{1}{2c_0^2}-\frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^4} & \frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^2} & \frac{(\gamma-1)\rho_0}{2c_0^2}+\frac{(\gamma-1)\rho_0\tau}{2c_0}\frac{\partial}{\partial x} + \frac{\Phi_1(\gamma-1)^2(\gamma-2)\rho_0T_0\tau}{2c_0^4} \\
-\frac{\Phi_1(\gamma-1)T_0}{2c_0^2}\int dx & -\frac{\Phi_1(\gamma-1)T_0}{2c_0^3\rho_0}\int dx & 0 & -\frac{\Phi_1(\gamma-1)^2T_0\tau}{2c_0^2}-\frac{\Phi_1(\gamma-1)^2T_0}{2c_0^3}\int dx
\end{pmatrix},$$

$$P_2 = \begin{pmatrix} \begin{array}{c}\frac{1}{2} + \frac{\Phi_1(\gamma-1)^2 T_0\tau}{2c_0^2} \\ -\frac{\Phi_1(\gamma-1)^2 T_0}{2c_0^3}\int dx\end{array} & -\frac{1}{2c_0\rho_0} + \frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^3\rho_0} & -\frac{\Phi_1(\gamma-1)T_0\tau}{2c_0\rho_0} & \begin{array}{c}-\frac{(\gamma-1)}{2c_0} + \frac{(\gamma-1)\tau}{2}\frac{\partial}{\partial x} \\ -\frac{\Phi_1(\gamma-1)^2(\gamma-2)T_0\tau}{2c_0^3}\end{array} \\ \begin{array}{c}-\frac{c_0\rho_0}{2} - \frac{\Phi_1(\gamma-1)^2 T_0\tau}{2} \\ +\frac{\Phi_1(\gamma-1)^2 T_0}{2c_0}\int dx\end{array} & \frac{1}{2} - \frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^2} & \frac{\Phi_1(\gamma-1)T_0\tau}{2} & \begin{array}{c}\frac{(\gamma-1)\rho_0}{2} - \frac{(\gamma-1)\rho_0\tau c_0}{2}\frac{\partial}{\partial x} \\ +\frac{\Phi_1(\gamma-1)^2(\gamma-2)\rho_0 T_0\tau}{2c_0^2}\end{array} \\ \begin{array}{c}-\frac{\rho_0}{2c_0} - \frac{\Phi_1(\gamma-1)^2\rho_0 T_0\tau}{2c_0^3} \\ +\frac{\Phi_1(\gamma-1)^2\rho_0 T_0}{2c_0^4}\int dx\end{array} & \frac{1}{2c_0^2} - \frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^4} & \frac{\Phi_1(\gamma-1)T_0\tau}{2c_0^2} & \begin{array}{c}\frac{(\gamma-1)\rho_0}{2c_0^2} - \frac{(\gamma-1)\rho_0\tau}{2c_0}\frac{\partial}{\partial x} \\ +\frac{\Phi_1(\gamma-1)^2(\gamma-2)\rho_0 T_0\tau}{2c_0^4}\end{array} \\ -\frac{\Phi_1(\gamma-1)T_0}{2c_0^2}\int dx & \frac{\Phi_1(\gamma-1)T_0}{2c_0^3\rho_0}\int dx & 0 & -\frac{\Phi_1(\gamma-1)^2 T_0\tau}{2c_0^2} + \frac{\Phi_1(\gamma-1)^2 T_0}{2c_0^3}\int dx \end{pmatrix},$$

$$P_3 = \begin{pmatrix} -\frac{\Phi_1(\gamma-1)^2 T_0\tau}{c_0^2} & 0 & 0 & -(\gamma-1)\tau\frac{\partial}{\partial x} \\ -\frac{\Phi_1(\gamma-1)^2\rho_0 T_0}{c_0^2}\int dx & \frac{\Phi_1(\gamma-1)T_0\tau}{c_0^2} & -\Phi_1(\gamma-1)T_0\tau & -(\gamma-1)\rho_0 - \frac{\Phi_1(\gamma-1)^2(\gamma-2)\rho_0 T_0\tau}{c_0^2} \\ 0 & 0 & 0 & 0 \\ \frac{\Phi_1(\gamma-1)T_0}{c_0^2}\int dx & -\frac{\Phi_1 T_0\tau}{c_0^2\rho_0} & \frac{\Phi_1 T_0\tau}{\rho_0} & 1 + \frac{\Phi_1(\gamma-1)(\gamma-2)T_0\tau}{c_0^2} \end{pmatrix}, \tag{22}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\Phi_1(\gamma-1)^2 T_0\rho_0}{c_0^4}\int dx & -\frac{1}{c_0^2} + \frac{\Phi_1(\gamma-1)T_0\tau}{c_0^4} & 1 - \frac{\Phi_1(\gamma-1)T_0\tau}{c_0^2} & -\frac{(\gamma-1)\rho_0}{c_0^2} + \frac{\Phi_1(\gamma-1)^2(\gamma-2)T_0\rho_0\tau}{c_0^4} \\ 0 & \frac{\Phi_1 T_0\tau}{c_0^2\rho_0} & -\frac{\Phi_1 T_0\tau}{\rho_0} & \frac{\Phi_1(\gamma-1)T_0\tau}{c_0^2} \end{pmatrix}.$$

The projectors $P_1, \dots P_4$ form a full orthogonal set of projectors:

$$P_n \cdot P_n = P_n, \quad P_n \cdot P_m = \overline{0} \quad (n \neq m), \quad \sum_{n=1}^{n=4} P_n = \overline{E}, \quad n, m = 1, \dots 4, \tag{23}$$

where $\overline{0}$ and $\overline{E}$ denote zero and unit matrix operators, correspondingly. The remarkable property of projectors is to decompose the dynamic equations governing the correspondent mode by immediate acting at the linear system (5):

$$P_n\left(\frac{\partial}{\partial t}\overline{E}\begin{pmatrix}v'\\p'\\\rho'\\\varepsilon'\end{pmatrix}+\overline{L}\begin{pmatrix}v'\\p'\\\rho'\\\varepsilon'\end{pmatrix}\right)=\frac{\partial}{\partial t}\overline{E}\begin{pmatrix}v'_n\\p'_n\\\rho'_n\\\varepsilon'_n\end{pmatrix}+\overline{L}\begin{pmatrix}v'_n\\p'_n\\\rho'_n\\\varepsilon'_n\end{pmatrix}=0. \tag{24}$$

where $\overline{L}$ is the matrix operator correspondent to the system (5). Acting of the second rows of P_1 or P_2 at the system (5) distinguish the governing equations for the specific acoustic pressures p'_1 or p'_2, respectively:

$$\frac{\partial p'_1}{\partial t}+c_0\frac{\partial p'_1}{\partial x}=0,\quad \frac{\partial p'_2}{\partial t}-c_0\frac{\partial p'_2}{\partial x}=0, \tag{25}$$

which obviously coincide to the roots of dispersion relation ω_1 and ω_2 from Eqs (11). Acting of the last rows of P_3 or P_4 at Eqs (5) decompose the equations for excess specific energies as follow, correspondingly:

$$\frac{\partial \varepsilon'_3}{\partial t}+\left(\frac{1}{\tau}+\frac{(\gamma-1)\gamma T_0\Phi_1}{c_0^2}\right)\varepsilon'_3=0,\quad \frac{\partial \varepsilon'_4}{\partial t}=0. \tag{26}$$

These equations coincide with ω_3 and ω_4 established by Eqs (15).

3 Interaction of the dominative sound and non-acoustic types of motion in a weakly nonlinear flow

3.1 Decomposing of dynamic equations in a weakly nonlinear flow

Account for the nonlinear terms of the second order in the relaxation equation (1) and the state equations (4) yields the leading order equalities:

$$T'=T_0\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}+\frac{\rho'^2}{\rho_0^2}-\frac{p'\rho'}{p_0\rho_0}\right), \tag{27}$$

$$\frac{d\varepsilon'}{dt}=-\frac{\varepsilon'}{\tau}+T_0\left(\frac{1}{\tau^2}\frac{d\tau}{dT}\right)_0\varepsilon'\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)+T_0\Phi_1\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}+\frac{\rho'^2}{\rho_0^2}-\frac{p'\rho'}{p_0\rho_0}\right)+T_0\Phi_2\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)^2,$$

$$\Phi_2=T_0\left(-\frac{1}{\tau^2}C_V\frac{d\tau}{dT}-\frac{(\varepsilon_0-\varepsilon_{eq})}{\tau^3}\left(\frac{d\tau}{dT}\right)^2+\frac{1}{2\tau}\frac{dC_V}{dT}+\frac{(\varepsilon_0-\varepsilon_{eq})}{2\tau^2}\frac{d^2\tau}{dT^2}\right)_0.$$

The governing dynamic system with account for quadratic nonlinear terms differs from (5) by the quadratic right-hand side:

$$\frac{\partial v'}{\partial t}+\frac{1}{\rho_0}\frac{\partial p'}{\partial x}=-v'\frac{\partial v'}{\partial x}+\frac{\rho'}{\rho_0^2}\frac{\partial p'}{\partial x},$$

$$\frac{\partial p'}{\partial t}+\gamma p_0\frac{\partial v'}{\partial x}-(\gamma-1)\rho_0\frac{\varepsilon'}{\tau}+(\gamma-1)\rho_0T_0\Phi_1\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)=$$
$$-v'\frac{\partial p'}{\partial x}-\gamma p'\frac{\partial v'}{\partial x}+(\gamma-1)\rho'\left(\frac{\varepsilon'}{\tau}-T_0\Phi_1\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)\right)- \tag{28}$$

$$(\gamma-1)\rho_0\left(T_0\left(\frac{1}{\tau^2}\frac{d\tau}{dT}\right)_0\varepsilon'\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)+T_0\Phi_1\left(\frac{\rho'^2}{\rho_0^2}-\frac{p'\rho'}{p_0\rho_0}\right)+T_0\Phi_2\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)^2\right),$$

$$\frac{\partial\rho'}{\partial t}+\rho_0\frac{\partial v'}{\partial x}=-v'\frac{\partial\rho'}{\partial x}-\rho'\frac{\partial v'}{\partial x},$$

$$\frac{\partial\varepsilon'}{\partial t}+\frac{\varepsilon'}{\tau}-T_0\Phi_1\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)=T_0\left(\frac{1}{\tau^2}\frac{d\tau}{dT}\right)_0\varepsilon'\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)+T_0\Phi_1\left(\frac{\rho'^2}{\rho_0^2}-\frac{p'\rho'}{p_0\rho_0}\right)+$$

$$T_0\Phi_2\left(\frac{p'}{p_0}-\frac{\rho'}{\rho_0}\right)^2-v'\frac{\partial\varepsilon'}{\partial x}.$$

The left-hand sides of Eqs (28) may be successfully decomposed into specific parts by projecting. The right-hand nonlinear terms become distributed between specific dynamic equations by projecting in the correct way as well: a sum of all projectors is the unit operator. The nonlinear right-hand parts of Eqs (28) include terms of all modes, and the further analysis depends on input of every mode there.

The most important problems consider dominative sound. From the physical point of view, the mode is dominative, when amplitudes of its perturbations are much larger than that of other modes. It is well-established, that nonlinear loss in acoustic energy and momentum of dominative sound makes magnitudes of thermal and vorticity modes to grow in the standard thermoviscous nonlinear flow. Weak nonlinearity presupposes a slow growth of the secondary modes. Order of their magnitude is no higher, than $O(M^2)$, where $M = max|v'|/c_0$ is the acoustic Mach number. An accurate account for the quadratic corrections in the dominative sound, which are of the same order, are necessary in the studies of nonlinear phenomena caused by dominative sound [13,14].

3.2 Weakly nonlinear equation of the dominative sound

Let the progressive in the positive direction of axis Ox sound be dominative. Accurate estimates need involving of the corrections in the linear definition of sound by account for terms specific for the Riemann wave [7]. In the lossless flow, they support adiabaticity of sound with accuracy up to the terms of order M^2. The corrected relationships for the rightwards progressive sound in the leading order are

$$\begin{pmatrix} v_1' \\ p_1' \\ \rho_1' \\ \varepsilon_1' \end{pmatrix}=\begin{pmatrix} \frac{c_0}{\rho_0}+\frac{(\gamma-3)c_0}{4\rho_0^2}\rho_1' \\ c_0^2+\frac{(\gamma-1)c_0^2}{2\rho_0}\rho_1' \\ 1 \\ -\frac{(\gamma-1)T_0\Phi_1}{c_0\rho_0}\int dx \end{pmatrix}\rho_1'=\begin{pmatrix} \frac{1}{c_0\rho_0}-\frac{(\gamma+1)}{4c_0^3\rho_0^2}p_1 \\ 1 \\ \frac{1}{c_0^2}-\frac{(\gamma-1)}{2\rho_0c_0^4}p_1' \\ -\frac{(\gamma-1)T_0\Phi_1}{c_0^3\rho_0}\int dx \end{pmatrix}p_1'. \qquad (29)$$

Acting by the second row of P_1 at both sides of Eqs (28), accounting for Eq.(29), and keeping only leading order acoustic terms, yield an equation governing acoustic pressure of the progressive in the positive direction of axis Ox sound:

$$\frac{\partial p_1'}{\partial t}+c_0\frac{\partial p_1'}{\partial x}+\frac{\gamma+1}{2}\frac{p_1'}{c_0\rho_0}\frac{\partial p_1'}{\partial x}=0. \qquad (30)$$

Eq.(30) is the leading-order equation governing the low-frequency sound in a weakly nonlinear and weakly dispersive flow. The detailed data on sound velocity and parameter of nonlinearity in flows not only weakly dispersive, a reader may find in [5,11,12].

3.3 Governing equations for secondary non-acoustic types of motion

Let still consider only the rightwards propagating sound with corrected links (29). At this point, we make routine manipulations to decompose the dynamic equation for the specific excess density of the thermal mode by acting at the system (28) by the third row of projector P_4 and collecting together terms of the leading order. Only dominative acoustic terms are hold in the right-hand non-linear part, which may be expressed in terms of acoustic excess pressure in view of Eqs (29):

$$\frac{\partial \rho_4'}{\partial t} = -\frac{\rho_0}{T_0}\frac{\partial T_4'}{\partial t} = \frac{(\gamma-1)^2 T_0 \Phi_1}{\rho_0 c_0^7 \tau} p_1' \int dx p_1'. \tag{31}$$

An acoustic pressure in the right-hand side should itself satisfy the dynamic equation (30). Acting of the last row of P_3 at the system (28) decomposes the dynamic equation for the part of excess vibrational energy correspondent to the third mode:

$$\frac{\partial \varepsilon_3'}{\partial t} + \left(\frac{1}{\tau} + \frac{(\gamma-1)\gamma T_0 \Phi_1}{c_0^2}\right)\varepsilon_3' =$$

$$\frac{(\gamma-1)^2 T_0 (\Phi_2 - \Phi_1/4)}{\rho_0^2 c_0^4} p_1'^2 - \frac{(\gamma-1)^2 T_0^2 \Phi_1}{\rho_0^2 c_0^5}\frac{1}{\tau^2}\left(\frac{d\tau}{dT}\right)_0 p_1' \int dx p_1'. \tag{32}$$

Deriving of dynamic equations (30)-(32) is based on the consequent decomposing of the equations for different types of motions by use of their properties, when sound progressive in the positive direction of axis Ox is dominative. They are instantaneous and apply to periodic or aperiodic low-frequency sound in the role of the origin of the non-wave modes. They are also the leading-order equations accounting for a weak nonlinearity and dispersion. The sign of the right-hand acoustic source depends on the sign of Φ_1 in the case of thermal mode and on balance of Φ_1 and Φ_2 as for equation governing the third type of motion caused by relaxation in vibrational energy. The subsection below includes the brief discussion and examples of generation of both non-acoustic types of motion.

4 Non-acoustic modes caused by periodic and impulse sound

4.1 Periodic sound

In the role of acoustic pressure, the solution of linear wave equation may be preliminary considered:

$$p_1'(x,t) = P_0 \sin(\omega_1(t - x/c_0)). \tag{33}$$

Eqs (31), (32) after averaging over the sound period rearrange in the leading order into (square brackets denote averaging over the sound period $2\pi/\omega_1$)

$$\left\langle \frac{\partial T_4'}{\partial t} \right\rangle = 0,$$

$$\frac{\partial \langle \varepsilon_3' \rangle}{\partial t} + \left(\frac{1}{\tau} + \frac{(\gamma-1)\gamma T_0 \Phi_1}{c_0^2}\right) \langle \varepsilon_3' \rangle = \frac{(\gamma-1)^2 T_0 (\Phi_2 - \Phi_1/4)}{2\rho_0^2 c_0^4} P_0^2. \tag{34}$$

So that, an efficiency of acoustic heating or cooling generated by the low-frequency periodic sound, is insignificant. After integrating with initial condition $\langle \varepsilon_3' \rangle (t=0) = 0$, a variation in vibrational energy of the third mode takes a form:

$$\langle \varepsilon_3' \rangle \approx \varepsilon_3' = \frac{(\gamma-1)^2 T_0 (\Phi_2 - \Phi_1/4)}{2\alpha \rho_0^2 c_0^4} P_0^2 (1 - e^{-\alpha t}), \quad \alpha = \frac{1}{\tau} + \frac{(\gamma-1)\gamma T_0 \Phi_1}{c_0^2}, \tag{35}$$

if $\alpha \neq 0$. For small αt, $\varepsilon_3' \approx \frac{(\gamma-1)^2 T_0 (\Phi_2 - \Phi_1/4)}{2\rho_0^2 c_0^4} P_0^2 t$. The sign of α is positive in view of weak dispersion. The sign of $\langle \varepsilon_3' \rangle$ depends on sign of $\Phi_2 - \Phi_1/4$. Anyway, Eq.(35) is valid over the temporal domain, where sound keeps still dominative:

$$Max\, |\varepsilon_1(x,t)| = Max \left| \frac{(\gamma-1) T_0 \Phi_1}{c_0 \rho_0} \int dx p_1' \right| = \left| \frac{(\gamma-1) T_0 \Phi_1 P_0}{\omega_1 \rho_0} \right| >> |\langle \varepsilon_3(x,t) \rangle| . \tag{36}$$

If $\alpha = 0$, Eqs (35),(36) rearrange into the following ones:

$$\langle \varepsilon_3' \rangle \approx \varepsilon_3' = \frac{(\gamma-1)^2 T_0 (\Phi_2 - \Phi_1/4)}{2\rho_0^2 c_0^4} P_0^2 t, \quad Max\, |\varepsilon_1(x,t)| >> |\langle \varepsilon_3(x,t) \rangle| . \tag{37}$$

Sound is dominative comparatively to the non-acoustic modes, but cased by it an excess pressure must be small itself in view of a weak nonlinearity of a flow: $P_0 << p_0$.

4.2 Impulse sound

In the case of sound being a solution of linear wave equation in a role of acoustic source, $p_1' = p_1'(\eta = (t - x/c_0)/\theta)$, where θ is a characteristic duration of a pulse, $\theta >> \tau$, Eqs (31),(32) are readily integrated:

$$T_4'(\eta) = \frac{(\gamma-1)^2 T_0^2 \Phi_1}{2\rho_0^2 c_0^6 \tau} \left(\int_{-\infty}^{\eta} d\eta p_1' \right)^2, \tag{38}$$

$$\varepsilon_3'(\eta) \approx \frac{(\gamma-1)^2 T_0^2 \Phi_1}{\rho_0^2 c_0^4} \frac{1}{\tau^2} \left(\frac{d\tau}{dT}\right)_0 \exp(-\theta\alpha\eta) \int_{-\infty}^{\eta} d\eta \left(\left(p_1' \int_{-\infty}^{\eta} d\eta p_1' \right) \exp(\theta\alpha\eta) \right). \tag{39}$$

Eq. (39) for small $\theta\alpha\eta$ easily rearranges into

$$\varepsilon_3' \approx \frac{(\gamma-1)^2 T_0^2 \Phi_1}{2\rho_0^2 c_0^4} \frac{1}{\tau^2} \left(\frac{d\tau}{dT}\right)_0 \left(\int_{-\infty}^{\eta} d\eta\, p_1' \right)^2. \tag{40}$$

An excess temperature correspondent to the thermal mode, is positive in the equilibrium acoustic regime ($\Phi_1 > 0$) and negative otherwise. The trace after a pulse passing are namely $T_4'(\eta \to \infty)$ and $\varepsilon_3'(\eta \to \infty)$. If we consider two pulses of the same energy, the efficiency of heating is much greater for mono-polar pulses, independently on its polarity. In the non-equilibrium regime, the acoustic cooling instead of heating takes place.

The part of vibrational energy belonging to the third mode, behaves differently, as follows from Eq.(40), in view of negative values of $d\tau/dT$: it is negative in equilibrium regime, and positive otherwise. Conclusions are valid if sound keeps dominative.

4.3 Including of viscosity, thermal conductivity and heat withdrawal $Q'(T)$ and power of energy pumping $I'(T)$

Involving of thermal conductivity and viscosity leads to the corrections in the roots of dispersion relation. Two acoustic ones, under condition $\omega\tau << 1$, keep unchanged, as well as the third root. In contrast to the high-frequency sound, there is no attenuation in the leading order nor due to the standard viscosity, not to the non-zero $Q_T = dQ'/dT$ or $I_T = dI'/dT$ because damping effects are described by the terms of order $(\omega\tau)^2$.

The thermal root in the leading order rearranges into

$$\omega_4 = i\frac{\chi k^2}{C_{P,\infty}\rho_0} + i\frac{(Q_T - I_T)}{C_{P,\infty}}, \tag{41}$$

see also [16]. We consider a weak dependence of Q' and I' on temperature: $|Q_T| << Q_0/T_0$, $|I_T| << I_0/T_0$ and hold in Eq.(41) terms proportional to the first powers of Q_T and I_T only. Finally, the governing equation for sound (30) keeps unchanged, and this one governing the thermal mode, takes the form:

$$\frac{\partial\rho_4'}{\partial t} - \frac{\chi}{C_{P,\infty}\rho_0}\frac{\partial^2\rho_4'}{\partial x^2} + \frac{(Q_T - I_T)}{C_{P,\infty}}\rho_4' = \frac{(\gamma-1)^2 T_0\Phi_1}{\rho_0 c_0^7\tau}p_1'\int dx p_1'. \tag{42}$$

The further analysis depends on balance of two last terms in the linear part of Eq.(42) (in spite of standard attenuation also results in damping, enough large positive $I_T - Q_T$ may cause growth in time of the magnitude of excess density correspondent to the thermal mode), and on the sign of Φ_1.

5 Concluding remarks

The standard attenuation and attenuation due to relaxation only weakly influence the low-frequency sound, but play an essential role in dynamics of the non-acoustic modes in field of the dominative sound. The domain of frequencies, satisfying condition $\omega\tau << 1$, for the typical laser mixture $CO_2 : N_2 : He = 1 : 2 : 3$ at pressure $p_0 = 1atm$ and temperature $T_0 = 300K^o$ is $\omega << 10^5 s^{-1}$ [2,12]. The standard thermoviscosity always leads to sound attenuation and nonlinear growth of excess temperature belonging to the thermal mode. This excess temperature may decrease if the non-equilibrium relaxation takes place. The behavior of a non-equilibrium gas over a wide range of variations of the parameters needs taking account of influence of pumping and heat removal [3,15]. With increasing of relaxation time, the amplification coefficient declines; however, a larger magnitude of pumping I is required to maintain the same degree of non-equilibrium, since $\varepsilon - \varepsilon_{eq} \approx I\tau$. That makes the non-equilibrium media inhomogeneous [3,15,17]. The conclusions above are not longer valid, because the linearization should be proceeded with respect to the background with non-zero spatial gradients of pressure and density. That alter the very definition of modes and further analysis, making it fairly complex mathematically. The investigation devoted to the amplification of sound in the flat layer of a non-equilibrium gas reveals some new properties compared to the case of the uniform gas [15]. In particular, the area of instability at the plane pumping intensity - an inverse time of relaxation becomes smaller. The features of non-acoustic modes and governing equations for them may alter essentially. Unfortunately, the mathematical difficulties do not allow to consider the problem in general.

This investigation is devoted to the nonlinear generation of the thermal and relaxation modes by the low-frequency dominative sound, periodic or aperiodic. The analysis is based on the method of successful decomposition of weakly nonlinear equations worked out by the author. The main results are instantaneous dynamic equations (31),(32). Among other conclusions from them, the thermal mode is not efficiently generated by the low-frequency periodic sound. The aperiodic sound produces positive excess temperature in the equilibrium regime ($\Phi_1 > 0$) and negative excess temperature otherwise.

The non-equilibrium processes in gases include also those due to establishment of equilibrium between translational, rotational, electronic degrees of freedom of a molecule, and those taking place in chemically reacting gases . The molecular physics studies hierarchy of relaxation processes. Difference in relaxation times follow from difference of probabilities of different elementary events. In light of hydrodynamics, relaxation due to chemical reactions or vibrational one may be included at the level of macroscopic description. Features of sound propagation over gases were one of relaxation processes listed before takes place, are fairly similar [10], as well as description of nonlinear interaction of sound and non-acoustic types of motion.

Going out of one dimension leads to the new modal field, the vorticity mode (one branch in 2D flows and two branches in 3D ones) with relations between specific quantities

$$\vec{\nabla}\vec{v}_{vort} = 0,\ p'_{vort} = 0,\ \rho'_{vort} = 0. \qquad (43)$$

The equation governing vorticity mode (43) in terms of vorticity $\vec{\Omega} = \vec{\nabla} \times \vec{v}_{vort}$ in the field of the dominative sound, is following:

$$\frac{\partial\vec{\Omega}}{\partial t} - \frac{\eta}{\rho_0}\Delta\vec{\Omega} = \vec{\nabla} \times \left(-\rho_a\frac{\partial}{\partial t}\vec{v}_a\right). \qquad (44)$$

The leading-order relations for velocity associated with sound and acoustic pressure, yield zero right-hand acoustic source relating to the low-frequency sound. It includes terms of order $(|\Phi_1|\tau/C_{V,\infty})^2M^2$ which are out of frames of the present study. So that, acoustic streaming caused by the low-frequency sound, is ignorable.

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