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NONLINEAR VIBRATION ANALYSIS OF BEAM AND PLATE WITH CLOSED CRACK: A REVIEW

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Abstract: The effect of nonlinearity is high sensitivity in damage detection, especially for closed cracks and delamination. This review illustrates the results of several researchers dealing with nonlinear effects caused by the closure of cracks in the structure, i.e., beam and plate structures. Early detection of damage is an important aspect for the structure and, therefore, continuous progress is being made in developing new and effective methods that use nonlinear effects for early detection of damage and barely visible cracks, i.e., closed cracks and delamination, as well as for the determination of crack size and location. After analysing various methods, the merits, drawbacks and prospects of a number of nonlinear vibration methods for structural damage detection are discussed, and recommendations are made for future researchers.

Key words: closed crack, closed delamination, nonlinearity, crack detection methods

1. INTRODUCTION

Safe structures with high efficiency are a prerequisite for various applications, such as sporting goods, automotive, construction and aerospace industries. Nowadays, structural damage poses a challenging problem in most applications. There are two types of cracks that commonly occur in structural systems: open cracks and closed cracks. Open cracks cause a decrease in physical stiffness, resulting in a largely linear structure with lower load carrying capacity and lower vibration frequencies [1, 2]. Owing to the bilinear stiffness characteristics associated with open and closed circumstances, the formation of breathing cracks not only lead to a reduction in stiffness of the structure, but also causes the linear structure to become nonlinear. The most common cause of structural failure is cracking due to fatigue and corrosion [3, 4].

A closing crack or breathing is a change in the stiffness of the body in the process of cyclic deformation by stretching and compression with opening and closing, respectively. A closing crack is an immediate change in the stiffness structure. This property leads to the nonlinear dynamic behaviour of mechanical systems and nonlinear effects such as the occurrence of subharmonic and superharmonic resonances and the nonlinearity of the vibration response (which includes displacement, velocity, acceleration and deformation) and the main resonance [5-7].

To avoid catastrophic failure in a variety of practical applications, the development of diagnostic techniques that are sufficiently sensitive to incipient fractures in structures and machines is a critical issue [8]. The diagnostics employ vibration-based and wave propagation based approaches; the most important potential advantage of vibration diagnostics based on nonlinear effects is its relatively high sensitivity to closing crack damage. Many vibration-based diagnostic techniques are used and studied by different researchers, as well as wave propagation-based approaches. Conventional spectral analysis is an approach based on the existing vibration-based diagnostic system. However, spectrum analysis has the crucial disadvantage that the amplitudes of nonlinear harmonics are extremely susceptible to measurement noise, which may distort the applied damage detection procedure employed [9, 10]. Researchers are still searching for newer methods (for detecting internal cracks in structures based on vibration feature extraction) that have higher sensitivity to the presence of damage, while being simple to use. Vibration characteristics include changes in damping, subharmonic and superharmonic vibrations, nonlinear distortions of vibration at the principal resonance, changes in the transfer function, distortion of phase trajectories and so on [11-13].

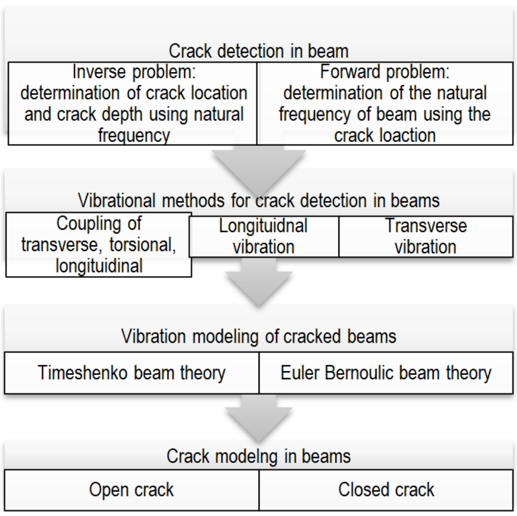

Fig. 1. Crack detection in beam-like structures [14]

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The primary objective of this review is to highlight the major accomplishments of numerous researchers who have investigated the nonlinear effects caused by a closing crack in beam and plate structural elements, with the goal of determining the possibility and future prospects of using nonlinear behaviour to detect damage. After a review of current methods, various methods are analysed, the merits, drawbacks and prospects of a number of nonlinear vibration methods for structural damage detection are discussed and a recommendation is made for future researchers.

The rest of the paper is organised as follows. Section 2 discusses various methods for detecting closed cracks in structures using nonlinear vibration methods. Section 3 presents numerical and experimental studies for closed crack detection in beams. Section 4 analyses various methods in plates using a nonlinear effect in the structure. Section 5 concludes the paper.

2. METHODS OF CLOSED CRACK DETECTION IN STRUCTURE

Non-destructive testing of structures and diverse technical components can be performed effectively, economically and quickly using vibration measurements. The features of the linear response of the open crack model can only detect the crack at an advanced level, but the effects of the nonlinearity of the closed crack model can be used to detect the crack at an early stage [15]. Potential structural damage mechanisms in engineering structures are associated with structural nonlinearities, which require extensive measurements and analysis of nonlinear vibration properties to detect and evaluate. The difficulty in finding breathing cracks is that the dynamic properties of a structure with a breathing fracture change significantly more slowly. Researchers have found that a structure with a closing crack behaves more like a nonlinear system, comparable to a bilinear oscillator, and that the nonlinear response characteristics can be used to detect the crack [12]. Early researchers used methods to evaluate changes in natural frequencies and natural modes to detect damage [16]. Changes in natural frequencies and natural modes, according to M. Dilena et al. [17] and Vimal M. et al. [18], are insufficient to detect minor cracks in the structure, i.e. closed cracks.

Cracked structures with one degree of freedom have been simplified as oscillators with single degree of freedoms (DOFs) [19]. In early research on breathing cracks, cracked beams were simplified as single-DOF oscillators whose stiffness in half cycles for tension and compression was represented by asymmetric linear or nonlinear functions. This approach was extended to cracked structures with multiple degrees of freedom, while, periodic changes in crack locations were incorporated into dynamic stiffness matrices to model closing cracks [20]. For continuously cracked beams with infinite DOFs, ‘breathing’ cracks were modelled analytically [21].

A. Chatterjee [22] applies the nonlinear dynamic model of the cracked structure using higher-order frequency response functions and a bilinear restoring force using a polynomial series. The effect of crack severity on the amplitudes of the harmonic response is investigated and a new method for estimating crack severity by measuring the amplitudes of the first and second harmonics is proposed [1, 23]. Most researchers model closing cracks for beam structures and closing delaminations for plate-like structures to simulate their closing motions during vibration. The crack is modelled as a local flexibility [24] in the structure. The size of the crack and its location are studied by changing the modal parameters such as natural frequency [25-26] and damping factor [27]. The detection of cracks by the vibro-acoustic method has been performed elsewhere in the literature [28-31]. A well-known approach for vibro-acoustic diagnosis of bilinearity of a mechanical system is the free vibration method [32]. In this method vibro-acoustic signals are used to evaluate the dependencies of the free vibration decrement and the natural frequency as a function of the crack size. Z. Kıral et al. [33] state that the change in decrement is a more sensitive indicator of damage compared to the natural frequency. Some effective and accurate measurement and analysis techniques have been developed and effectively used to detect and identify structural nonlinearities, the effects of which are readily apparent in the observed conventional first-order frequency response functions (FRFs) [34]. The other structural nonlinearities caused by closing cracks do not contribute to the first-order FRFs. Therefore, Riccardo Cappello et al. [35] developed a method to identify the physical parameters of breathing cracks using second-order FRFs, and R.M. Lin and T.Y. Ng [36] report that higher harmonics are efficient indicators of closing cracks. Although higher harmonics may indicate the occurrence of closing cracks [33], the amplification effect of higher harmonics due to differentiation is quantitatively investigated using several scenarios.

The nonlinear behaviour of harmonics generated by breathing cracks is well explained by the nonlinear pseudo-force (NPF) and experimentally validated on a beam with a fatigue crack by non-contact vibration measurement [38]; the same evidence was also provided by M. Cao et al. [39] for the localisation of ‘breathing’ delamination by using deflection forms at nonlinear harmonics. W. Xu et al. [40] analytically formulated the nonlinear pseudo-force by rearranging the equation of transverse motion of the cracked beam. The concept can be used to localise cracks and elucidate the mechanism needed for the generation of higher harmonics by ‘breathing’ cracks. Cui et al. [41] used bispectrum analysis to detect ‘breathing’ cracks by studying the nonlinear dynamic properties of cracked beams. K. Wang et al. [42] developed a two-dimensional analytical model to interpret the modulation mechanism of a breathing crack to guide ultrasonic waves. Semperlotti F. et al. [43] explicitly reveal that the harmonic generation mechanism is a fundamental issue in the study of breathing cracks.

3. NUMERICAL AND EXPERIMENTAL DETECTION OF CLOSED CRACK IN BEAM

3.1. Model of crack beam

The use of nonlinear models for cracked beams is crucial in a closed crack of the structure to detect the crack and fully understand the vibration characteristics generated by the breathing crack. The modelling of the breathing crack is an important step that lays the groundwork for crack analysis and identification.

According to A. Rivola and PR. White et al. [44], the bispectrum can be used to examine the system response and the bilinear oscillator model to mimic the nonlinear behaviour of a beam with a closing crack. As seen in Fig. 2, the breathing crack can have a bilinear stiffness effect. Then, in Figs. 3–5, a single, two, and many cracks in the structure are depicted, respectively, where a is crack depth, *b* is beam width, *h* is beam height and *L* is beam length.

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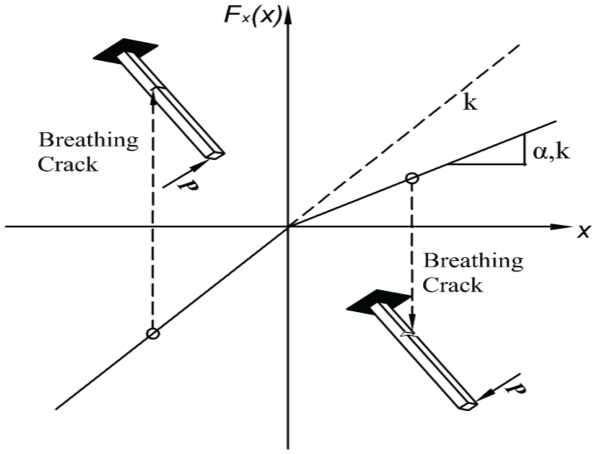

Fig. 2. Cantilever beam with breathing crack [45]

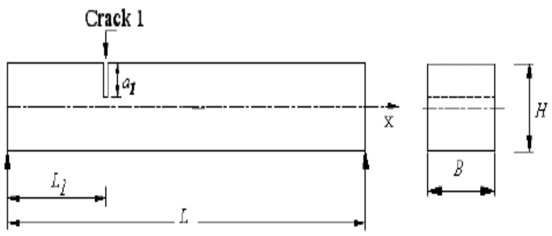

Fig. 3. Schematic diagram of a cracked simply supported beam with single cracks [46]

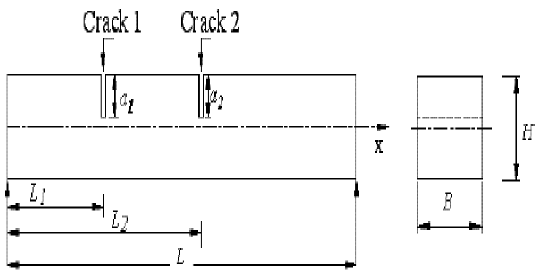

Fig. 4. Schematic diagram of a cracked simply supported beam with two cracks [46]

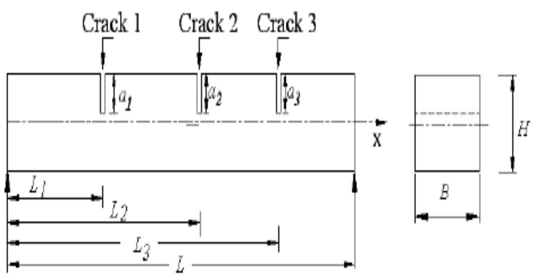

Fig. 5. Schematic diagram of a cracked simply supported beam with three cracks [46]

To model stiffness change in bilinear oscillators, Chu and Shen [47] proposed a replacement closed-form solution for bilinear oscillators under low-frequency excitation utilising two square wave functions. Their study proposes a simple polynomial model for modelling the beam with a breathing crack. For a cracked cantilever beam with a transverse one-edge non-propagating closing crack, the polynomial model is confirmed. This amplitude-dependent polynomial model can be used to simulate nonlinear systems with bilinear behaviour. According to Caddemi and Caliò' [48], the closing crack was approximated using Dirac's deltas, which enabled closed-form evaluation of beam mode forms for a generic crack configuration. It should also be mentioned that the nonlinear effect of the system is stronger if the superharmonic components include more energy in the output spectrum.

3.2. Subharmonic and superharmonic resonance vibration

When a system with a breathing crack is activated by a single harmonic force, the response shows different nonlinear properties, according to F.E. Dotti et al. [49] the excitation causes the crack to open and close, resulting in harmonics that are integer multiples or fractional multiples of the driving frequency. Superharmonic and subharmonic are the terms used to describe these harmonics.

According to Bovsunovskii et al. [50], the vibration of a cantilever beam with a closing crack under the action of a harmonic concentrated force was characterised by superharmonic resonances of order *j/i* (the symbol *j/i* denotes the ordinal number of the nonlinear regime, which indicates how many natural periods of vibration *j* fall at *i* periods of the external harmonic excitation), with very small amplitudes.

The displacement time history response of the actual system and the corresponding Fourier spectrum of the approximated polynomial system are given in Figs. 6a and 6b, respectively, as depicted in the study of Prawin and Rao [45]. The nonlinear effect of the system is increased if the superharmonic components include more energy in the output spectrum.

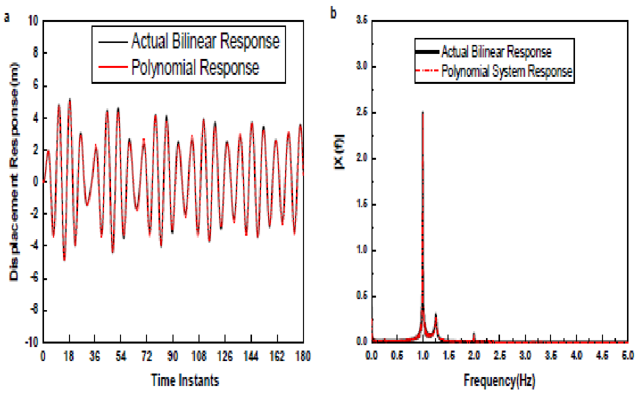

Fig. 6. The actual bilinear response and polynomial response [45]

Nonlinear vibration characteristics, according to Broda et al. [51], are more sensitive to breathing cracks, especially when subharmonic and superharmonic resonances are present. Additionally, Andreaus et al. [52] investigated the cracked beam's forced harmonic response by treating the closing crack interfaces as a contact problem and used finite analysis software. The results of the simulations showed that subharmonic and superharmonic resonances exist in the broken beam. However, the phenomena that caused these several harmonics to appear in the experiment have yet to be fully explained. The fundamental reason for this is that the nonlinear vibration of a beam with a breath-

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ing crack is poorly understood, and the intrinsic link between the subharmonic and superharmonic resonance responses and the beam parameters is not well understood, necessitating further research into cracked beams [53]. Pugno et al. [54] created a novel stiffness model that takes partial crack closure into account. The finite element approach was used to generate a nonlinear dynamic equation for a beam with a breathing crack, and the multiple-scales method was used to assess the approximate steady-state response of the cracked beam to a harmonic excitation. According to their findings, the crack parameters, as well as a combination of the crack parameters and excited force parameters, influence the nonlinear vibration of a cracked beam.

Under some circumstances, this system presents third-order superharmonic resonance, second-order superharmonic resonance, 1/2th-order subharmonic resonance, combination resonance and multiple resonances. The most used combinations are the second-order subharmonic when the system is excited at twice its natural frequency; the second-order superharmonic, when the system is excited at half of its natural frequency; and the third-order superharmonic, when the system is excited at one-third of its natural frequency [55, 56].

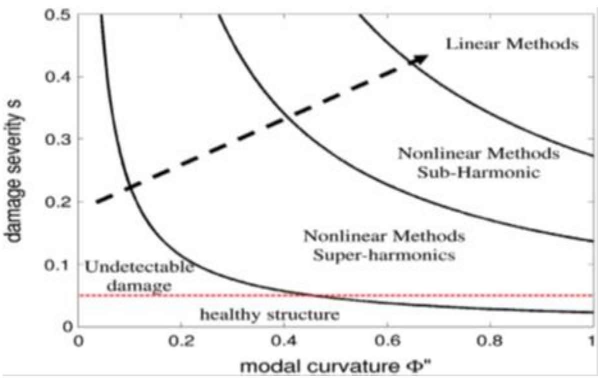

Fig. 7. damage severity with modal curvature [57]

This is especially important when dealing with a more sophisticated structure. In fact, if the chosen forcing frequency only excites modes with curvature-nodes close to the damage position, even huge cracks may go undetected by harmonics approaches. Figure 7 shows how this problem is qualitatively expressed. As the modal-effective damage increases, one moves from a low-damage zone where no identification is possible to a zone where only superharmonic are sensitive to damage; as the damage increases further, one reaches a zone where subharmonic are sensitive and preferable; and finally, a zone is reached where more simple linear methods are effective.

3.3 Forced vibration

3.3.1. Governing equation of forced vibration

The Euler–Bernoulli equation for the beam elements without crack is written as

$$M\ddot{u} + C\dot{u} + Ku = f(t) \tag{1}$$

where f(t) is an external force that can be harmonic, non-harmonic and periodic; M is mass; C is dampness; and K is stiffness.

$$M\ddot{u} + c\dot{u} + ku = fosin\omega t \tag{2}$$

$$\ddot{u} + 2\xi\omega\dot{u} + \omega n^2 u = fosin\omega t \tag{3}$$

If $\omega = \omega n$ resonance will occur, and that means large amplitude can be expected.

Zhao et al. [58] used Green's functions approach and the superposition principle to produce closed-form explicit formulations of the steady-state forced vibration of a multi-cracked Euler–Bernoulli beam subjected to a time-harmonic load. The numerical results revealed that crack geometries (depth and location) have a substantial impact on the system's dynamic behaviour.

X. Zhao et al. [59] built on Zhao et al.'s work [58] to investigate the steady-state forced vibrations of multi-cracked Timoshenko beams, revealing that the effects of shear deformation and rotating inertia on the deflection pattern for a thick beam could not be ignored. Crack beams also infrequently show the mechanical mechanisms of crack geometries, particularly crack positions, in their dynamic behaviours. Also, by examining the dynamic behaviours of a general cracked double-beam system interconnected by a viscoelastic layer sitting on the Winkler–Pasternak elastic foundation, Chen et al. [60] try to fill these gaps. They obtain closed-form solutions for forced vibrations of a cracked beam connected viscoelastic layer, taking into account axial stresses and the Winkler–Pasternak elastic basis.

3.4 Wave propagation

Small or substantial portions of the beams may be damaged. The consequences of nonlinearity of a closed crack in a beam structure can be studied using a variety of methodologies. The Volterra series response representation is used to examine the nonlinear response of a beam with a closed crack, according to Surace et al. [61]. A polynomial series is used to approximate the bilinear restoring force due to closed crack modes, and the first- and second-order frequency response functions are created in terms of bilinear parameter. However, as the selected stimulation frequency is quite distant when the crack is very deep, the response amplitude is not observable [62]. Also, Wang et al. [40, 63] describe the development of a two-dimensional (2D) analytical model to analyse the modulation process of a 'breathing' crack to guide ultrasonic pulses (GUWs). The model can analyse the breathing behaviour of the crack as the probing GUWs cross it and reflect the breathing generated by the propagating and decaying waves, allowing for the extraction of nonlinear signal characteristics in contact with acoustic nonlinearity (CAN). Based on an analytical prediction of CAN formation as a function of crack severity, a correlation between CAN in GUW signals and crack parameters is established.

Based on the numerical and experimental detection of high-order harmonic waves or subharmonic waves caused by the interaction of ultrasound with closed cracks, Taizo et al. [64] and Koskinen et al. [65] use piezoelectric contact transducers, which are also studied by Lee and Hong [66]. Since the contact transducer requires a liquid coupling agent, the nonlinearity induced by the transducer into the measurement process could be much greater than the material's nonlinearity, making it unsuitable for detecting complex structures with curved surfaces.

In complex shapes and hazardous environments, Wu TC et al. [67] employ fully non-contact NUT based on laser ultrasound (LUT) on closed surface cracks. The result reveals a weak second harmonic, which is caused by the material's or measuring system's nonlinearity. Higher harmonics are also used to locate closed cracks that are dominated by the initial harmonics, which

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appear to be hidden [68, 69]. As a result, these hidden higher harmonics encounter difficulty in detecting the crack. To solve this problem, Cao et al. [69] developed an energy modulation effect phenomenon (EME) based on the concept of quadratic-Teager–Kaiser energy (Q-TKE) to clearly detect hidden higher harmonics in a closed crack. This was accomplished both numerically and experimentally using finite element methods and a Doppler laser vibrometer (DLV). However, noise causes false peaks while using this method. As a result, the stimulation must be carefully chosen, and noise suppression is also necessary.

Based on the vibration concept and the modal decomposition method, X. Sun Xu et al. [70] use the second harmonic Lamb wave modulated by the crack-induced acoustic nonlinearity of the contact. Non-zero energy flux and phase velocity are required due to the dispersion and the multimodal character of Lamb waves. As a result, different researchers [71] have chosen different mode pairs for different damages, although the amplitude of the required second harmonic Lamb mode was always altered by other Lamb waves with double frequency [72]. For Lamb waves in partially closed cracks, W. Zhu et al. [73] use a static component. In the research of Chen B. et al. [74], they use the hypothesised and experimentally studied approach of vibro-acoustic modulation (VAM) based on electromagnetic loading, which concentrates the loading energy around the closed crack. However, in the structure, it is critical to locate the sample's closed crack, after which additional work is required.

Carneiro S. and Inman D. [75] developed a continuous model based on Timoshenko beam theory to consider the nonlinear behaviour of bending and shear vibration in a breathing cracked beam, assuming that stiffness decreases as the distance from the crack position increases. A closed crack's nonlinear vibration inhibits the correct determination of the natural frequency [76]. As a result, crack nonlinear behaviour is included in the models. The closed crack behaviour is shown in the transverse vibration analysis of beam [77], axial beam [78] and longitudinal beam [79] with a single closed crack using the Perturbation method. A beam with numerous cracks may have a more nonlinear structural dynamic response than a beam with a single crack. The singularity of the concentrated cracks along the beam may impact the structure's continuity and geometric aspects. M. Kharazan et al. [80] use Euler–Bernoulli theory to simulate the transverse vibrations of a beam with several breathing cracks.

4. ANALYSING DIFFERENT METHODS IN PLATE USING NONLINEAR EFFECT

4.1. Natural frequency

Due to sinusoidal loading, delamination experiences tensile and compressive cycles. The delamination opens when the delamination surfaces are in tension, modifying the amplitude signal. Since the delamination surfaces come into contact with one another and behave similar to an intact surface during compression, the amplitude does not vary [81]. The area of damage was mapped in nonlinear acoustics by measuring the amplitude of nonlinear harmonic and subharmonic frequencies in the frequency spectrum of all nodes, according to Sun et al. [82]. Near the defect, the ratio between the amplitude of the second harmonic and the amplitude of the linear frequency is more pronounced, giving a qualitative indicator of the damage site. Likewise, the subharmonic amplitude to the excitation frequency amplitude ratio is bigger at the defect zone. Experiments with the nonlinear harmonic imaging approach for defect identification have already been conducted. However, there are some places in both ratios that are not above the delamination but have a bigger ratio value, giving a false sense of the damage in that region. This is because these spots are on the modal pattern, which has very small vibration amplitudes. As a result, doing the experiments at many frequencies could be an option. Also, in the research of Tabatabaeipour et al. [83], it was assumed that nonlinear spring dampers that have been installed between contact states represent a nonlinear interaction of delamination surfaces. The spring stiffness and damping coefficient parameters are not available in the nonlinear spring-damper model; thus, they must be adjusted when the magnitude of the parameters varies sufficiently.

C. Andreades et al. [84] propose an ultrasonic phased array technique based on nonlinear modulation of dual frequency excitation to increase the sensitivity and accuracy of contact detection. The work has shown that the nonlinear array method can detect multiple contact defects in samples and has a higher localisation accuracy than the traditional linear phase array method. However, the work only looks at the bottom surface; phase signals at multiple points on the sample need to be investigated further.

4.2 Velocity/displacement relation

Delamination can also occur in a more complicated instance, where only some zones within the delaminated region are in touch while the rest are not, as shown in Fig. 8.

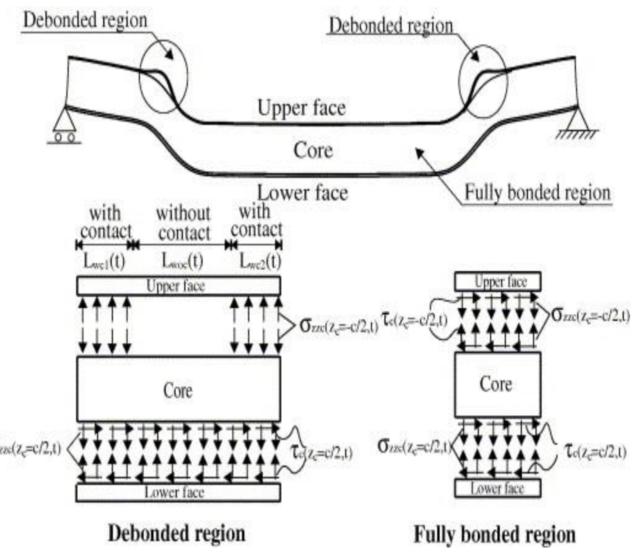

Fig. 8. A delaminated sandwich panel with partial contact conditions [85]

To define the dynamic behaviour of delaminated plates, Schwarts-Givli et al. [85] investigated nonlinear effects associated with time-dependent contact features of delaminated surfaces and large displacements of the face-sheets, at high-order displacement, velocity and acceleration fields in the core. The data clearly demonstrate the nonlinear behaviour's impacts. To characterise the effect of delamination on smart laminated composite plate structures, Shankar et al. [86] performed a numerical analysis using the finite element approach. They employed the time integration technique in various boundary conditions and locations of active fibre composite (AFC). According to their findings, the amplitude of dynamic responses is higher in the event of delamination and AFC patch locations, with less dynamic displacement when the patches are close to the delamination. The major shift in dynamic displacement response with respect to time history in the simply supported and clamped–clamped boundary is shown in Figs. 9 and 10. The velocity feedback gain is 1 (Gv = 1). However, delamination necessitates not only the consideration of boundary conditions and location, but also the analysis of the geometry of the structures.

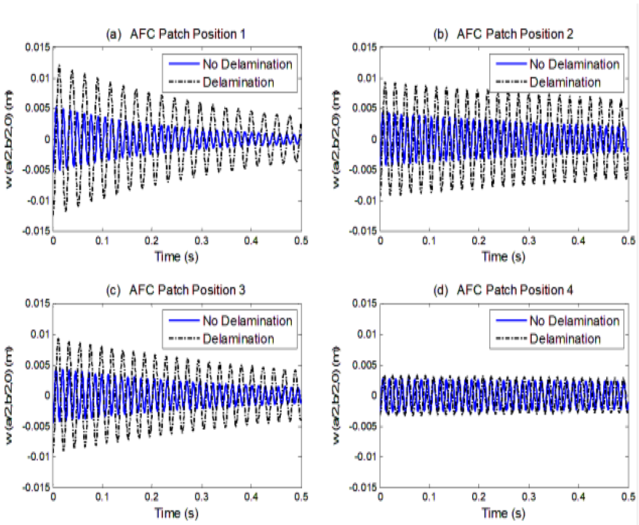

Fig. 9. Time response of delaminated and without delaminated plate at different locations of AFC patches (Gv = 1) in simply supported boundary condition [86]

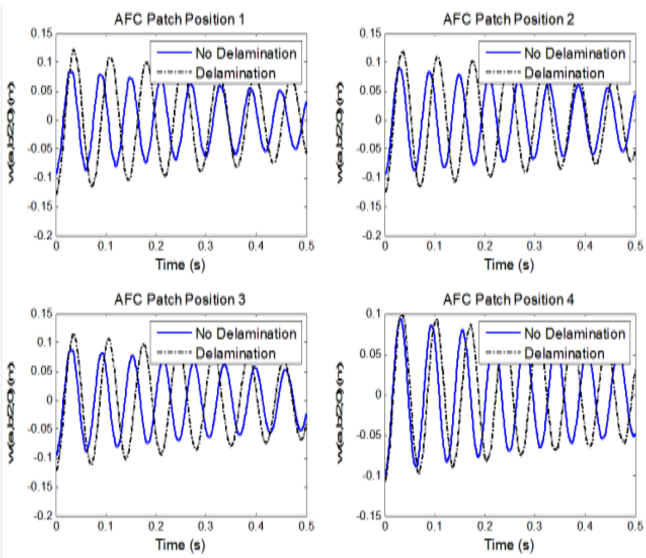

Fig. 10. Time response of delaminated and without delaminated plate at different locations of AFC patches (Gv = 1) in clamped–clamped boundary condition [86]

4.3 Forced vibration

Bovsunovskii et al. [50] used forced vibration of a simply supported laminated composite beam with 10 viscoelastic layers to determine delamination characteristics. At frequencies of 1 kHz and 2 kHz, respectively, 73 and 146 times higher than the beam's initial natural frequency, harmonic excitation was applied. The delamination produced additional multiples of the excitation frequencies as a result of the periodic opening and closing of the delamination, with their value varying depending on the size and location of the delamination, according to the nonlinear analysis. On the other hand MH Kargarnovin et al. [87] discussed the existence of the delamination changes the stiffness of the structure, and the effects of the size, depth, location of the delamination and the load velocity on the dynamic response.

4.4 Wave propagation

The most common damage in plate structures is delamination. The delamination effect in plate structure laminates has been widely modelled for free vibration, transient analysis, frequency analysis and damage detection [88-89]. Nonlinear analysis of regular composite plates and shells for free vibration, bending and buckling has been carried out in many studies based on classical laminate theory [90], first-order shear deformation theory [91], higher-order shear deformation theory [92] and other refined theories [93]. By studying the interaction between the structure and the foundation in the context of the used material, Yushu et al. [94] obtained a good understanding that would enable prediction of the structural response. The composite plate can be predicted using different delamination sites before delamination and modelling is to be performed using layer-by-layer theory and finite element method.

Many researchers have addressed the problem of nonlinearity [95, 96] and are studying the static analysis of laminated shells resting on a nonlinear elastic foundation. They use a method to subsequently determine stresses and strains and initially assume a perfect bond. Shen et al. [97] investigate the nonlinear behaviour of a composite panel resting on an elastic foundation with imperfect bonding using transverse normal stress, transverse shear stress and transverse nonlinear stress transferred from the foundation to the plate. The result was obtained, but they only consider the base of nonlinearity, and the material nonlinearity and geometric nonlinearity are not considered.

N. Houhat et al. [98] focus on the vibro-acoustic modulation (VAM) technique and optimise nonlinear damage detection in multiple scattering. They use automatic optimisation and gain time reduction. It is a good basis for detecting damage to scattering radiators, but information about the local minima of the correlation coefficient is important for locating the nonlinear scattering radiators. X. Zhang et al. [99] proposed a nonlinear Lamb wave imaging technology to detect barely visible impact damage (BVID) in carbon fibre reinforced composites (CFRP). They did this at three different impact energies with corresponding impact surfaces. In their result, the relative nonlinearity coefficients of the second harmonic Lamb wave packets were received at a distance of 103 mm. However, at a distance greater than 103 mm, the relative nonlinearity coefficients become weak. According to their results, the Lamb wave fluctuated after it exceeded a certain value. So it is important to consider the noise.

According to Castellano et al. [100], they used the sideband peak count index (SPC-I) in composite plates to improve the application of the nonlinear ultrasonic technique in the detection of impact-induced damage. They improved the SPC-I technique by introducing a narrow band SPC-I technique. Then, lead zirconate titanate (PZT) of non-permanent adhesive gel was used, and they

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reported that it showed reliable results in detecting damage in composite plates both at initiation and propagation. However, these techniques cannot determine the size of the defect and the type of defect that caused the damage. In the case of nonlinearity, it is important to understand the shape, dimensions and location of the cracks. Moreover, nonlinearity is caused not only by the crack, but also by other factors, such as material behaviour [101]. Therefore, such a consideration is essential to detect damage in the structure.

Q. Wei et al. [102] used the nonlinear acoustic resonance method to characterise fatigue damage in composite materials, especially CFRP. Based on their work, the progressive fatigue damage in composites was introduced by pendulum impact tests with different impact energies and investigated by infrared thermography and optical microscopy. They concluded that the damage index (DI) is a complex parameter influenced by the impact energy, impact angle, impact area and other factors; so, these factors should be considered. The boundary conditions and the effects of temperature should also be well thought-out.

S. Sikdar et al. [103] studied a geometric nonlinearity problem based on 3D numerical simulation and experimental analysis of guided wave propagation in smart composite structures based on finite element methods, and compared sources with and without breathing and bonding in the smart composite structures. Based on their results, the online SHM strategy has demonstrated its potential to locate sources with breathing and bonding using elastic wave signals, but operating conditions must be considered.

According to X. Wang et al. [104], the random pore model was introduced to perform nonlinear ultrasonic simulations to account for the randomness of pore size, morphology and distribution properties. They performed nonlinear ultrasonic simulations and experimental investigations for the real porous graphite composites. The results show that the relative nonlinear coefficient increases with increasing pore length and decreases with increasing pore width. However, the question arises as to what the result would be if they considered the effects of pore parameters and excitation parameters on the nonlinear coefficient. Also, a random pore model was used. Is there another way to get the same results?

The secondary or additional mass and spring is a primary structural device used to suppress vibrations [105]. However, such methods produce two resonant peaks with high amplitude. To minimise the vibration structure at both peaks and also at the resonant frequency, researchers have used optimisation techniques. Such optimisation techniques include linear quadratic controllers, genetic algorithms and so on [106, 107]. Both traditional and non-traditional models of vibration dampers are used to minimise the vibration of the structure, but there is a limit to suppressing the vibration of the main structure if the mass of the damper is not increased [108]. Therefore, researchers have used some modifications in absorber design, such as shape-memory alloys, magnetic suspensions and piezoelectric devices [109-110]. These modifications are called semi-active or active vibration absorbers.

According to Mohanty et al. [111], the effectiveness of active traditional and non-traditional vibration dampers in suppressing the vibrations of the main undamped system using PZT has been studied. Their work requires lower tension because the spring exerts a stronger blocking force on the PZT stack actuator, and thus lower tension is required.

Zhang and Zhao [112] studied the nonlinear vibrations of a rectangular cantilever composite plate excited in the plane and transverse directions. They found that the chaotic responses were sensitive to the change in forced excitation and damping coefficient.

L.S. Yousuf [113] studied the nonlinear dynamic behaviour (non-periodic motion and chaos) of a laminated composite plate due to temperature changes and combined loading using the concept of the largest Lyapunov exponent with the Wolf algorithm and fast Fourier transform (FFT). While their work provides a basis to proceed upon, the question arises as to what the result would be if the algorithm were to differ According to Dauson et al. [114], they used time reversal (TR) to focus acoustic energy and make acoustic measurements correlated with successive localised damage cycles in a laboratory-scale wellbore. They performed a damage detection study in wellbores by applying time reversal and nonlinear elastic wave spectroscopy, and the wellbore used in their study was a laboratory-scale wellbore. Based on their work, we might infer that this might be another way to locate damage in carbon fibre reinforced plastic (CFRP). J. Wang et al. [115] investigate a nonlinear Lamb wave TR technique for fatigue crack detection and quantification. Using the same technique, which is physical–virtual, TR can be extended to a network of active sensors for fatigue crack localisation and imaging.

According to Zhen et al. [116], a purely output-based approach using the Volterra series model to detect nonlinear structural damage is proposed, quantifying the nonlinear behaviour of structures without prior knowledge of the external excitations. Sometimes the structure responds either linearly or nonlinearly. When it responds linearly, the techniques may not be effective, and thus it is necessary to analyse the structural response. Many factors, such as geometric nonlinearity, nonlinear boundary conditions, connections of prefabricated structures and material nonlinearity, can cause the structural system to exhibit nonlinear behaviour; this needs to be fully considered.

In theory, V. Samaitis et al. [117] have analytically studied the mode conversion of Lamb wave directed from each other at the tip of delamination in a composite plate. According to Wang et al. [118], they have investigated reflection and transmission of wave mode in metal and composite beams containing delamination and inhomogeneity. The spectral element method (SEM) is used to simulate Lamb wave interaction with open and closed cracks. Additionally, cracked spectral element models are individually established for open and closed cracks. Nag et al. [119] used the SEM to model wave scattering at the location of embedded open delamination in composite beams.

5. FUTURE RESEARCH

The current challenges and future suggestions for the application of nonlinear vibration-based method in the detection of closed cracks and other defects in a beam and plate are discussed in this section.

In the beginning, a linear approach was used since the majority of researchers believed that cracks kept opening during vibration. Gudmunson [120] was the first to notice this phenomenon after conducting experiments to determine a correlation between the position and size of a crack and the variation in natural frequencies. N. Pugno et al. [51, 121] used the vibrational response to the harmonic force of a cantilever beam with cracks of different sizes and locations, and their analysis employs the harmonic balance approach. The method involves a significant reduction in comparison with direct numerical integration.

In order to determine the relative amplitude of the dominant harmonics of a rod-like structure with a closing crack at subharmonic and superharmonic resonances, Matveev et al. [122] presented an approximate analytical method. They demonstrated that the acceleration amplitude of the second harmonic is more sensitive than that of the fundamental harmonic. To accurately describe the stiffness change of the cracked beam, Long H. et al. [53] employed a new stiffness model that takes the influence of partial crack closure into consideration.

Based on the conclusions drawn by various researchers, it may be inferred that modelling of cracks is very important in the detection of the closed crack and in fully understanding its behaviour. The model does not fully consider the behaviour of the closed crack in beam.

Bispectrum analysis was used by Cui et al. [41] to examine the nonlinear dynamic properties of cracked beams in order to identify 'breathing' cracks. Their approach causes the first harmonics, which appear to be hidden, to dominate higher harmonics. M. Cao et al. [69] formulated a new phenomenon of energy modulation effect to enhance hidden higher harmonics, and their approach has shown considerable effectiveness in closed crack detection. However, their techniques caused fake peaks that can be mistaken for the crack-caused peaks. Consequently, subsequent researchers focused on excitation and measurement points, since these need to be properly selected to acquire large amplitudes of vibration responses.

In attempts directed at closed crack detection in both the beam and plate components of a structure, a high priority should be given to nonlinear behaviour in the absence of awareness of external excitations. The structure can sometimes respond linearly or nonlinearly. Analysis of the structural reaction is required because the procedures may not be effective if it responds linearly. The structural system may exhibit nonlinear behaviour due to a variety of causes, including geometric nonlinearity, nonlinear boundary conditions, connections between construction technique, and material nonlinearity; this needs to be fully considered.

6. CONCLUSION

The need for a safe structure has led to the development of diverse damage detection methods. These various damage detection methods have their own advantages and limitations, especially in the early stages and for small cracks. Damage diagnosis based on the use of nonlinear vibration effects is the first preference for use in a variety of structures. Most damage detection methods have been applied to beam structures, and there are few that are practical in laminated plate-like structures. The general advantages of nonlinearity damage detection methods include the high sensitivity in detecting cracks, particularly closed cracks. Under the nonlinearity effects, the closed crack in the structure is detected based on the crack size, and its location is investigated by changes in modal parameters such as natural frequency and damping factor. However, this approach suffers from two major limitations. First, the changes in natural frequencies have been shown to be significant only when the crack size is large, and second, the measured shift in natural frequency cannot be unambiguously attributed to the crack alone, as it may also be influenced by other factors such as wear, relaxation, etc. Modelling a crack is important to identify the closed cracks in a beam-like structure. It helps to fully understand the vibration characteristics generated by the closed crack. However, modelling is time consuming due to its complexity.

The subharmonic and superharmonic resonance methods have different nonlinear properties. On the other hand, the nonlinear vibration of a beam with a closed crack is poorly understood, the intrinsic relationship between the subharmonic and superharmonic resonance responses and the beam parameters is far from thoroughly understood and cracked beams require further investigation. Wave propagation methods are very sensitive in determining the size and location of cracks in both beams and plate-like structures. However, the various wave propagation methods have their own drawbacks, as there are various factors that affect the structure; and in assessing the effects of nonlinearity, noise and environmental conditions should be considered, even if such a consideration were to also take into account the possibility that different types of nonlinearity—namely nonlinear damping, geometric nonlinearity and nonlinearity of the material—would also affect the response of the methods. In the case of delamination, there are various methods to analyse the nonlinearity effects of the structure, and each method has its own advantages and disadvantages. Various researchers used nonlinear effects based on the subharmonic response of the structure. These methods are effective in detecting damage, but there were also some false indications, leading to an incorrect conclusion about defects.

VAM is a time reduction technique, but it requires additional information. Nonlinear Lamb wave imaging, another method for damage detection, can be used up to a certain distance, but after that distance it is difficult to detect the exact damage. One way to increase the sensitivity and reliability of nonlinear vibration diagnostics would be to develop special mathematical methods that can account for multiple conditions simultaneously, and implementing these methods would mean that even with the presence of closed cracks and delamination in the structure, the damage detection technique employed would prove effective.

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