

# Novel Structure and EM-Driven Design of Miniaturized Microstrip Rat-Race Coupler

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**Abstract**—In this paper, a novel structure and design procedure of a miniaturized microstrip rat-race coupler (RRC) is described. Small size of the RRC is achieved by folding the transmission lines of the conventional circuit into its interior, as well as by implementation of the structure on three layers. The final size of the coupler realized for the operating frequency of 1 GHz is only 220 mm<sup>2</sup>, which gives over 95% footprint reduction w.r.t. a reference RRC. At the same time, rigorous optimization of coupler dimensions using variable-fidelity EM models permits rapid EM-driven design closure with the final design featuring 240 MHz bandwidth, equal power split, and obtained at the computational cost corresponding to only 12 simulations of the circuit at the high-fidelity level.

**Keywords**—compact couplers, design optimization, EM-driven design, rat-race couplers.

## I. INTRODUCTION

Rat-race couplers (RRCs) are important components of many microwave systems. Conventional RRCs, implemented in microstrip technology, are characterized by large footprints which limit their usefulness for modern applications such as portable or wearable devices. At the same time, size reduction of couplers is a challenging task, because small dimensions stay in conflict with maintaining required circuit performance [1]–[4]. Size reduction is often achieved through topological modifications of the coupler structure. Popular approaches include replacing transmission line sections by compact microstrip resonant cells or defected ground structures [4], [5]. Alternatively, the footprint can be reduced by folding transmission lines to the interior of the RRC [6]. Such geometrical changes allow for obtaining small dimensions along with acceptable performance. However, the resulting structures have complex geometries with multiple cross-couplings. Therefore, electromagnetic (EM) simulations are required for reliable evaluation of their performance.

In this work, a novel topology of a miniaturized RRC has been proposed. Small footprint of the circuit have been achieved by folding its transmission lines. Further size reduction has been obtained by implementing its 180 degree section on a separate layer connected to the remaining part of the circuit through viaholes. The final design is eleven times smaller compared to the conventional RRC structure and around 50% smaller compared to a single-layer compact design. At the same time, the structure features 24% bandwidth and equal power-split for the center frequency of 1 GHz. Low RRC design cost, corresponding to only to about

12 simulations of the high-fidelity model, has been achieved using variable-fidelity EM simulation models.

## II. PROPOSED RRC STRUCTURE

Consider a hybrid RRC shown in Fig. 1. The circuit is implemented on a Taconic RF-35 dielectric substrate ( $\epsilon_r = 3.5$ ,  $h = 0.762$  mm,  $\tan\delta = 0.018$ ). Compact structure dimensions are achieved by folding its 70.7-ohm sections to the interior (cf. Fig. 1 (a)). For further size reduction, its 180 degree section has been implemented on a separate layer and connected to the remaining part of the circuit through viaholes (cf. Fig. 1 (b)). This modification allows for reducing size by about 50% compared to the planar implementation of the RRC. The design parameters of the three-layer circuit are:  $\mathbf{x} = [l_1, l_2, l_3, d, w]^T$ , whereas dimensions  $r = 0.3$  (diameter of the viaholes) and  $w_0 = 1.7$  (width of the input ports) remain fixed. The unit for all parameters is mm. The operating frequency is set to 1 GHz. The lower and upper bounds are set to  $\mathbf{l} = [2, 2, 6, 0.2, 0.3]^T$  and  $\mathbf{u} = [10, 15, 20, 1, 1.2]^T$ .

The high-fidelity model  $\mathbf{R}_f$  of the three-layer coupler (cf. Fig. 1 (b)) consists of  $\sim 200,000$  tetrahedral mesh cells and its typical evaluation time on a dual Intel Xeon E5540 machine is 10 min. The coarsely-discretized planar realization of the structure (cf. Fig. 1 (a)) is used in the design process as the low-fidelity model  $\mathbf{R}_c$  ( $\sim 20,000$  tetrahedrons; simulation time: 130 s). Both models are implemented in CST Microwave Studio and evaluated using its FEM solver.

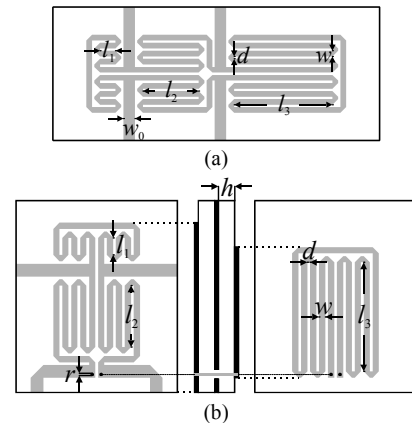


Fig. 1. Topology evolution of the proposed RRC: (a) folded structure and (b) three-layer implementation of the coupler. Viaholes are marked using black circles.

### III. DESIGN OPTIMIZATION METHODOLOGY

Design closure of the proposed RRC is conducted with the aim of obtaining the best possible electrical performance parameters, in particular (here,  $S_{11}$ ,  $S_{21}$ ,  $S_{31}$  and  $S_{41}$  stands for matching, transmission, coupling and isolation, respectively;  $f_0$  is the operating frequency;  $\mathbf{x}$  is a vector of geometry parameters of the RRC):

- Minimization of  $S_{\max} = \max(\min\{S_{11}(\mathbf{x}), S_{41}(\mathbf{x})\})$  at  $f_0$ ;
- Obtaining equal power split at  $f_0$ ,  $d_S = |S_{21}(\mathbf{x}) - S_{31}(\mathbf{x})| = 0$ ;
- Ensuring that  $f_{S_{11}}(\mathbf{x})$  and  $f_{S_{41}}(\mathbf{x})$ , i.e., the frequencies realizing minimum of  $S_{11}(\mathbf{x})$  and  $S_{41}(\mathbf{x})$ , respectively, are as close to  $f_0$  as possible.

In order to take into account all of these goals the following objective function is minimized:

$$U(\mathbf{x}) = \max(\min\{S_{11}(\mathbf{x}), S_{41}(\mathbf{x})\}) + \beta_1 (\max\{d_S - \varepsilon, 0\})^2 + \beta_{f_1} |(f_{S_{11}}(\mathbf{x}) - f_0) / f_0|^2 + \beta_{f_2} |(f_{S_{41}}(\mathbf{x}) - f_0) / f_0|^2 \quad (1)$$

Obtaining possibly equal power split, as well as allocating the minima of return loss and isolation characteristics at the operating frequencies are enforced by the appropriate penalty terms. The coefficients  $\beta_1$ ,  $\beta_{f_1}$ , and  $\beta_{f_2}$  are all set to  $10^3$ .

The objective function (1) is minimized using a trust-region gradient search [7] of the form:

$$\mathbf{x}^{(i+1)} = \arg \max_{\mathbf{x}, \|\mathbf{x} - \mathbf{x}^{(i)}\| \leq \delta^{(i)}} L^{(i)}(\mathbf{x}), \quad (2)$$

where  $\mathbf{x}^{(i)}$ ,  $i = 0, 1, \dots$ , is a sequence approximating the optimum design  $\mathbf{x}^*$ , whereas  $L^{(i)}$  is an auxiliary objective function at iteration  $i$ , with high-fidelity-simulated  $S$ -parameters replaced by their local linear approximation models (first-order Taylor expansions). In order to reduce the CPU cost of the optimization process,  $S$ -parameter Jacobians are estimated using low-fidelity EM coupler model, which is a circuit of Fig. 1 (a) with coarser discretization.

### IV. RESULTS

The initial design of the coupler of Fig. 1 (b) is set to  $\mathbf{x}_0 = [4.2, 4.5, 16.9, 0.2, 0.6]^T$ . The final design  $\mathbf{x}^* = [2.7, 8.75, 14.2, 0.45, 0.8]^T$  has been obtained after only a few iterations of (2). The size of the optimized structure is  $11.7 \times 18.8 = 220 \text{ mm}^2$  which is over 11-fold smaller compared to conventional RRC design implemented at RF-35 substrate ( $47.5 \times 95.5 = 4536 \text{ mm}^2$ ) [2]. Fig. 2 shows response characteristics of the RRC at the initial and final designs. The detailed performance figures of the proposed coupler are shown in Table I. For the optimized design, the bandwidth  $BW_1$ , defined as the frequency range where both  $|S_{11}|$  and  $|S_{41}|$  are below  $-20 \text{ dB}$ , is 24%. At the same time bandwidth  $BW_2$ , defined as a range where  $d_S \leq 0.2 \text{ dB}$ , is 20%.

The proposed structure has been compared with other state-of-the-art RRC structures in terms of the bandwidth  $BW_1$  and size reduction (expressed in terms of the guided wavelength  $\lambda_g$  defined for the center frequency and the given substrate properties). The results shown in Table II indicate that the proposed structure provides the highest miniaturization rate while ensuring competitive bandwidth compared to remaining RRCs.

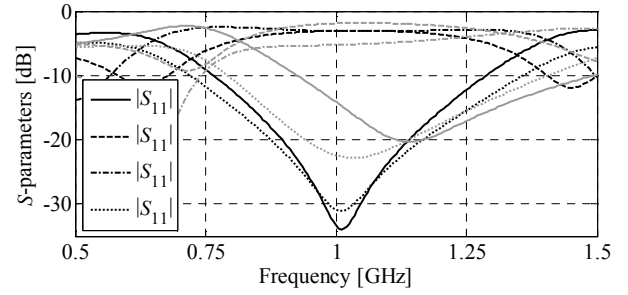


Fig. 2.  $S$ -parameter responses of the proposed RRC at the initial (gray) and final (black) designs.

TABLE I. PERFORMANCE FIGURES OF THE PROPOSED RRC

Obtained w.r.t. $f_0$							
$ S_{11} $ [dB]	$ S_{21} $ [dB]	$ S_{31} $ [dB]	$ S_{41} $ [dB]	$d_S$ [dB]	$BW_1$ [MHz]	$BW_2$ [MHz]	$\angle(S_{21} - S_{31})$ [°]
-33.4	-3.01	-3.07	-31.0	0.06	240	200	0

TABLE II. COMPARISON WITH STATE-OF-THE-ART RRCs

Coupler	Bandwidth [%]	Dimensions [mm × mm]	Effective $\lambda_g$	Size reduction [%]*
Design [1]	16.8	$22.4 \times 22.4$	$0.14 \times 0.14$	85.8
Design [2]	20.2	$22.8 \times 17.0$	$0.13 \times 0.09$	91.5
Design [3]	15.1	$6.67 \times 52.5$	$0.04 \times 0.28$	92.2
Design [4]	25.6	$14.9 \times 26.3$	$0.07 \times 0.13$	93.4
This work	24.0	$11.7 \times 18.8$	$0.06 \times 0.10$	95.2

\* w.r.t. conventional RRC (effective  $\lambda_g: 0.26 \times 0.53$ , size:  $4536 \text{ mm}^2$ ) [2].

The computational cost of the design optimization corresponds to 12.3 simulations of the high-fidelity model  $R_f$  (~2 hours) and includes 43 evaluations of the low-fidelity model  $R_c$  and 3  $R_f$  model simulations. For the sake of comparison, a high-fidelity model of the RRC of Fig. 1 (b) has been directly optimized using algorithm of [7]. The final design has been obtained after 26 model evaluations, over 50 percent more compared to optimization using procedure of Section III.

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