Data Descriptor

On Computing Curlicues Generated by Circle Homeomorphisms

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Abstract

The dataset entitled Computing dynamical curlicues contains values of consecutive points on a curlicue generated, respectively, by rotation on the circle by different angles, the Arnold circle map (with various parameter values) and an exemplary sequence as well as corresponding diameters and Birkhoff averages of these curves. We additionally provide source codes of the Matlab programs which can be used to generate and plot the first N points of curlicues of these types and calculating related quantities. Illustrative figures are included as well.

Keywords: dynamical systems; curlicues; circle homeomorphisms; rotation on the circle; Arnold circle map

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Specification table (data records)

Subject area	Dynamical systems
More specific subject area	Low dimensional dynamics, Rotation theory
Type of data	Text; Figure; Source code
How the data was acquired	The data was generated using the Matlab program and information contained in (Signerska-Rynkowska, 2020)
Data format	.txt, .m, .eps
Experimental factors	The data contained in the dataset was not processed
Experimental features	Values of the geometric and dynamical quantities

Data source location	MOST Wiedzy Open Research Catalog, Gdańsk University of Technology, Gdańsk, Poland
Data accessibility	The dataset is accessible and is publicly and freely available for any research or educational purposes

Background

The notion of a curlicue originates mainly from the visual arts where it stands for a fancy curl that can serve as a decorative motif in architecture or calligraphy. However, in mathematics by a curlicue we shall mean a piece-wise linear curve on the complex plane, where, passing consecutively through the points , and , ,..., where:

A curlicue can be obtained from an arbitrary sequence of real numbers and for some sequences they can indeed form beautiful shapes (see e.g. Berry, Goldberg, 1988; Dekking, Mendès-France, 1981; Sinai, 2008). On the other hand, if the sequence is obtained from an orbit of a given map i.e. , then we can speak about dynamically generated curlicues. For mathematicians it is especially interesting to examine how the dynamical properties of are reflected in the geometric structure of . In particular, the recent work (Signerska-Rynkowska, 2020) studies curlicues generated by lifts of circle homeomorphisms and determines how the properties of the continued fraction expansion of its rotation number are connected with boundedness, the rate of diameter increase and the superficiality of such a curve (for definitions, see the cited paper and references therein).

The three functions Rotation.m, Arnold.m and Sequence.m can be used to plot the first points of a curlicue generated, respectively, by rotation on the circle by the angle of 2πρ, the Arnold circle map (with different parameter values of and) and the sequence . Additionally, the functions calculate corresponding Birkhoff averages and a diameter of a curlicue depending on number of iterates . A description of the functions and variables involved is provided as comments in the m-files. It is worth pointing out that the function Sequence.m can be easily modified to compute and draw a curlicue generated by an arbitrary sequence given by an explicit formula. Attached txt-files provide exemplary data obtained by these functions. Four figures (eps-files) were generated by the function Sequence.m for, respectively, (Fig. 38.1.eps,), (Fig. 38.2.eps,), (Fig. 38.3.eps,) and (Fig. 38.4.eps,).

Concerning curlicues generated by circle homeomorphisms with an irrational rotation number, it is known that an unbounded Birkhoff average implies that the diameter of a curlicue is unbounded as well. On the other hand, a bounded Birkhoff average can result both in the bounded and unbounded case. The bounded case (for transitive homeomorphisms) is equivalent to the fact that the corresponding cohomological equation has a continuous solution (see: Signerska-Rynkowska, 2020). The intriguing open question is to characterise the unbounded curlicues with a bounded Birkhoff average.

The dataset provided can be used for numerical experiments, perhaps assisting in arriving at new hypotheses concerning this problem. It can also be used for educational purposes (including laboratory classes) for students at the undergraduate level.

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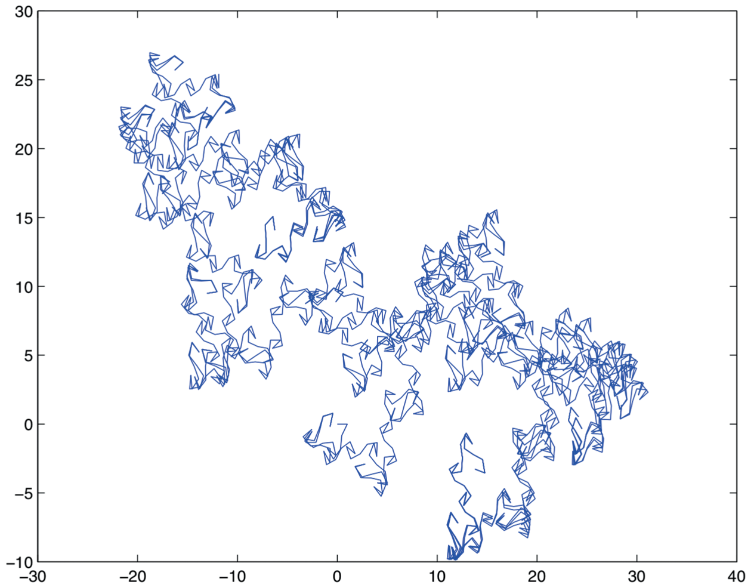

Fig. 38.1. Graph of the curlicue (first 2000 points) on the complex plane

Methods

The figures and datasets were obtained by programs in Matlab based on the definitions and information provided in (Signerska-Rynkowska, 2020). The source-codes for the programs, also attached, can be reused and easily modified to compute other quantities (e.g. the curlicue's diameter).

Data quality and availability

Dataset DOI
10.34808/anjg-q802

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References

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