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ON–LINE PARAMETER AND DELAY ESTIMATION OF CONTINUOUS–TIME DYNAMIC SYSTEMS

JANUSZ KOZŁOWSKI [a], ZDZISŁAW KOWALCZUK [a,*]

[a]Department of Robotics and Decision Systems
Gdańsk University of Technology, Narutowicza 11/12, 80-233 Gdańsk, Poland
e-mail: {jk23,kova}@eti.pg.gda.pl

The problem of on-line identification of non-stationary delay systems is considered. The dynamics of supervised industrial processes are usually modeled by ordinary differential equations. Discrete-time mechanizations of continuous-time process models are implemented with the use of dedicated finite-horizon integrating filters. Least-squares and instrumental variable procedures mechanized in recursive forms are applied for simultaneous identification of input delay and spectral parameters of the system models. The performance of the proposed estimation algorithms is verified in an illustrative numerical simulation study.

Keywords: delay systems, continuous-time models, discrete approximation, parameter estimation, least-squares estimator, instrumental variable estimator.

1. Introduction

In many practical realizations of diagnostic procedures the key information about the performance of a supervised system is gained from observations of the evolution of specific system parameters. Appropriate procedures of modeling the dynamics of the observed system and effective on-line identification of the model parameters are then necessary. Most commonly, the dynamics of monitored industrial processes are modeled by differential or difference equations, transfer functions or state space representations.

In the case of discrete-time descriptions, appropriate modeling is much easier because the necessary regression data simply have the form of shifted samples of the stored input and output signals. On the other hand, the parameters of such models have no physical interpretation and the estimated values heavily depend on the sampling frequency applied. When one uses the continuous-time representations, in turn, physically motivated parameters deliver additional diagnostic information. In addition, the numerical conditioning (sampling frequency) of the identification process does not affect the estimated values. Having in mind, however, that estimation of parameters is based on the system input-output data obtained in discrete instants, a numerical approximation of continuous-time regressors is necessary (Johansson, 1994; Middleton and Goodwin, 1990; Kowalczuk, 1995; Schoukens, 1990; Unbehauen and Rao, 1990; Young, 1981). Among various methods of such a discrete mechanization of differential equations (Kowalczuk, 1993; Sagara *et al.*, 1991), the idea of finite-horizon integrating filters is worth recommendation. In this approach, modeling accuracy is controlled by properly tuning an integration horizon, while numerically well-conditioned regressors result from the simple FIR processing of discrete-time measurements. Moreover, the omission of the free system response does not affect the results of identification.

Estimates of the parameters of the discretized description can easily be obtained using the popular least-squares (LS) routine, which is known to generate consistent estimates provided that additive measurement perturbation is zero-mean white noise. In realistic situations the corrupting noise is correlated, and this makes the estimates of parameters asymptotically biased. In order to cure this problem, the concept of instrumental variables (IVs) can be put into practice (Stoica and Söderström, 1983). Moreover, the deployment of special weighting mechanisms allows tracking the evolution of parameters of monitored non-stationary systems.

Both the LS and IV methods work perfectly for

*Corresponding author

non-delay systems or systems with a known input delay. There exist, however, complex industrial plants with input delays, which are undesirably changeable (time varying). Identification of such delays along with the parameters of the continuous models is definitely a challenging issue. Among the existing solutions, off-line estimation of input delay based on the gradient search can be applied (Ferretti *et al.*, 1991; Zhao and Sagara, 1991). A dedicated three-stage procedure for off-line identification of delay can be used in practical implementations (Kozłowski and Kowalczuk, 2009). The above-mentioned issues, discussed in this paper, lead us to propose an on-line procedure for simultaneous identification of the input delay and parameters of non-stationary continuous models. The performance of the proposed algorithms is verified by simulation, and the corresponding properties of these procedures are highlighted.

The paper is organized as follows. In Section 2, the well-known LS and IV estimators in algebraic and recursive forms are first recalled. With the aid of forgetting mechanisms, these algorithms are adjusted for the identification of non-stationary systems, i.e., for the tracking of time-variant parameters. An ordinary differential equation is introduced in Section 3 to represent the dynamics of the underlying continuous system, while the employment of properly tuned finite-horizon (FIR) integrating filters leads to an equivalent discrete model, which retains the original system parameterization. An additional stochastic part (disturbance) in the ultimate regression model follows from FIR filtering of measurement signals contaminated by noise. The original contribution can be attributed to Section 4, in which new methods of identification of differential equations with input delay are developed. Additionally, an outline of consistency analysis of the proposed estimation algorithms is presented. Moreover, in Section 5 the performance of the algorithms is verified via simulation. Finally, in Section 6 the fundamental properties of the new estimation procedures are highlighted, and some promising directions of further investigations in this area are indicated.

2. Estimation procedures

The dynamics of the monitored system are given by the following regression equation:

$$\gamma(k) = \phi^{\mathrm{T}}(k)\,\theta + e(k), \tag{1}$$

$$\phi(k) = [\ f_0(k)\ \ f_1(k)\ \ \ldots\ \ f_r(k)\ ]^{\mathrm{T}}, \tag{2}$$

$$\theta = [\ \vartheta_0\ \ \vartheta_1\ \ \ldots\ \ \vartheta_r\ ]^{\mathrm{T}}, \tag{3}$$

where $\gamma(k)$ is a reference signal, $e(k)$ stands for an equation error, while $\phi(k)$ and θ hold specific regression data and system parameters, respectively. The above model is assumed to be a discrete counterpart of an original standard differential equation (an example (21) is analyzed later) that describes the dynamics of the underlying continuous system. The components represented in (2) take the form of consecutive filtered derivatives of the input and output signals, while the quantities appearing in (3) are simply the evaluated coefficients of the differential equation and the estimated input-signal delay (see (24) and (25) discussed later on). Furthermore, the distinguished stochastic part $e(k)$ results from FIR filtering of measurement signals contaminated by noise (as demonstrated in (43) through (47) in the run of a consistency analysis).

The LS estimates of the parameters (3) can be directly obtained by minimizing the following performance index (Ljung, 1987):

$$\begin{aligned} V(\theta) &= \sum_{l=1}^{k} w_l\ [\ e(l)\ ]^2 \\ &= \sum_{l=1}^{k} w_l\ [\ \gamma(l) - \phi^{\mathrm{T}}(l)\ \theta\ ]^2, \end{aligned} \tag{4}$$

using a non-increasing ($w_l \geq w_{l+1}$) sequence of non-negative ($w_l \geq 0$) weights such that the so-called memory of the estimator (i.e., $\Gamma = \sum w_l$, calculated for the indices in the range of $l = -\infty, \ldots, k$) is finite (i.e., $\Gamma < \infty$).

By zeroing the gradient of (4), an algebraic form of the LS routine (Ljung, 1987) is obtained:

$$\begin{aligned} \hat{\theta}(k) = &\left[\ \sum_{l=1}^{k} w_l\ \phi(l)\,\phi^{\mathrm{T}}(l)\ \right]^{-1} \\ &\times \left[\ \sum_{l=1}^{k} w_l\ \phi(l)\,\gamma(l)\ \right]. \end{aligned} \tag{5}$$

The basic asymptotic properties of the non-weighted ($w_l = 1$) LS estimator employed for identification of systems with input delay are discussed in Section 4.

It is known that, in the case of measurement noises mutually correlated with the analyzed regression data, the LS estimates are usually asymptotically biased. In order to cure this problem, the instrumental variable method (Söderström and Stoica, 1981) can be put into practice.

The idea behind the IV solution lies in a simple modification of the algebraic LS procedure. In this solution an auxiliary (noise-free) vector $\xi(k)$ is used in place of genuine regressors $\phi(k)$, while the transpose regression vector remains unmodified. Thus the IV estimates of the system parameters result from (Ljung, 1987)

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$$\hat{\theta}(k) = \left[\sum_{l=1}^{k} w_l \, \xi(l) \, \phi^{\mathrm{T}}(l) \right]^{-1} \times \left[\sum_{l=1}^{k} w_l \, \xi(l) \, \gamma(l) \right]. \tag{6}$$

The respective consistency analysis (considering the non-weighted identification of delay systems) is submitted in Section 4.

Both the presented estimation procedures, LS and IV, suffer from numerically inconvenient matrix inversion. In order to overcome this issue, the respective formulas can be easily written in appropriate recursive forms.

Assuming that the utility weighting mechanism is represented by an exponential window ($w_l = \lambda^{k-l}$), the explicit algebraic formula is given by

$$\hat{\theta}(k) = \left[\sum_{l=1}^{k} \lambda^{k-l} \, \psi(l) \, \phi^{\mathrm{T}}(l) \right]^{-1} \times \left[\sum_{l=1}^{k} \lambda^{k-l} \, \psi(l) \, \gamma(l) \right]. \tag{7}$$

Employing the well-known matrix inversion lemma, we find an equivalent recursive procedure involving evaluation of the prediction error $\varepsilon(k)$, update of the covariance matrix $P(k)$ and correction of the estimates of the parameters,

$$\varepsilon(k) = \gamma(k) - \phi^{\mathrm{T}}(k) \, \hat{\theta}(k-1), \tag{8}$$

$$P(k) = \frac{1}{\lambda} \left[P(k-1) - \frac{P(k-1) \, \psi(k) \, \phi^{\mathrm{T}}(k) \, P(k-1)}{\lambda + \phi^{\mathrm{T}}(k) \, P(k-1) \, \psi(k)} \right], \tag{9}$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k) \, \psi(k) \, \varepsilon(k). \tag{10}$$

With $\psi(k) = \phi(k)$, Eqns. (7) and (8)–(10) yield the LS estimates, and by substituting $\psi(k) = \xi(k)$ the IV estimates are obtained. The bounded forgetting factor ($0 < \lambda < 1$) influences the length of the estimator's memory: $\Gamma = 1 / (1 - \lambda)$. Obviously, in the non-weighting case ($\lambda = 1$) this memory is infinite.

As an alternative, a rectangular window (referred to as a sliding window) of observations (when $w_l = \mathbf{1}(k - l) - \mathbf{1}(k - l - M)$, with $\mathbf{1}(k)$ standing for the unit-step function) can be used in both the estimation formulas LS and IV. With this, the resultant estimator assumes the following algebraic form:

$$\hat{\theta}(k) = \left[\sum_{l=k-M+1}^{k} \psi(l) \, \phi^{\mathrm{T}}(l) \right]^{-1} \times \left[\sum_{l=k-M+1}^{k} \psi(l) \, \gamma(l) \right]. \tag{11}$$

Again, with the aid of the matrix inversion lemma, a convenient recursive realization involving evaluation of prediction errors, update of covariance matrices and ultimate correction of estimates can be derived:

$$\varepsilon(k) = \gamma(k) - \phi^{\mathrm{T}}(k) \, \hat{\theta}(k-1), \tag{12}$$

$$\tilde{\varepsilon}(k) = \gamma(k-M) - \phi^{\mathrm{T}}(k-M) \, \hat{\theta}(k-1), \tag{13}$$

$$\tilde{P}(k) = P(k-1) - \frac{P(k-1) \, \psi(k) \, \phi^{\mathrm{T}}(k) \, P(k-1)}{1 + \phi^{\mathrm{T}}(k) \, P(k-1) \, \psi(k)}, \tag{14}$$

$$P(k) = \tilde{P}(k) + \frac{\tilde{P}(k) \, \psi(k-M) \, \phi^{\mathrm{T}}(k-M) \, \tilde{P}(k)}{1 - \phi^{\mathrm{T}}(k-M) \, \tilde{P}(k) \, \psi(k-M)}, \tag{15}$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k) \, \psi(k) \, \varepsilon(k) - P(k) \, \psi(k-M) \, \tilde{\varepsilon}(k). \tag{16}$$

As before, with $\psi(k) = \phi(k)$, Eqns. (11) and (12)–(16) describe the LS estimation, while by employing $\psi(k) = \xi(k)$ the IV estimates are obtained. This time the length of the sliding window ($M > 0$) simply equals the length of the estimator's memory: $\Gamma = M$.

It is of fundamental importance that the LS information matrix appearing in (5), and composed as the sum of the vector products $\phi(k) \, \phi^{\mathrm{T}}(k)$, is invertible. A rigorous proof of this property can be found in the work of Sagara and Zhao (1990), where multi-sine excitations are applied to the input of the identified continuous-time system.

In the case of the IV information matrix represented in (6), and gained by summing the products $\xi(k) \, \phi^{\mathrm{T}}(k)$, the corresponding argument and the proofs demonstrating the non-singularity of this matrix and the local convergence of parameter estimates can be found in the work of Söderström and Stoica (1981).

Commonly, the recursive LS estimators (8)–(10) and (12)–(16) are initiated using diagonal covariance matrices (e.g., $P(0) = \mathrm{diag}\,(10^5, \ldots, 10^5)$). In the case of the IV schemes, in turn, the respective initiation has to be based on the results gained from an LS algorithm running in parallel. This is so because generating instrumental variables usually requires a preliminary knowledge of the sought parameter estimates. Clearly, with a properly evaluated instrumental variable $\xi(k)$, the auxiliary LS routine can finally be switched off and the estimation can be continued using solely the IV procedure.

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3. Discrete-time approximation of continuous systems

The dynamics of a continuous system can be represented by an ordinary differential equation ($n \geq m \geq 0$):

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = b_m u^{(m)} + b_{m-1} u^{(m-1)} + \cdots + b_0 u, \quad (17)$$

where $u = u(t)$ and $y = y(t)$ stand for the input and output signals, respectively.

Since identification of unknown parameters a_i and b_i is based on records of discrete-time data, suitable discrete approximation of (17) retaining the original parameterization has to be performed. There exists a method in which consecutive derivatives of (17) are evaluated numerically, using the so-called 'delta' operator. In another approach, multiple integration of both the sides of (17) is performed to yield a discrete-time equivalent of the original model. Having in mind, however, that high-pass differentiation operators are certain to attenuate noise, heavily biased estimates can result from identification of the 'delta' models. In the case of integral modeling, in turn, the regression data given by multiple integrals diverge to infinity, while the non-zero initial conditions of (17) influence the results of parameter estimation. The method of finite-horizon integrating filters, in which both the sides of (17) are n times integrated over a fixed time interval ($h > 0$), overcomes the above-mentioned problems (Sagara *et al.*, 1991).

The operator of finite-horizon n-th order integration of an i-th derivative of a signal $x(t)$ can be shown as

$$J_i^n x(t) = \int_{t-h}^{t} \int_{t_1-h}^{t_1} \cdots \int_{t_{n-1}-h}^{t_{n-1}} x^{(i)}(t_n)\, \mathrm{d}t_n\, \mathrm{d}t_{n-1} \ldots \mathrm{d}t_1. \quad (18)$$

A numerical realization of integrals (18) leads to convenient FIR processing of sampled data ($x(kT) = x(t)\,|_{t\,=\,kT}$). With a trapezoidal rule involving the bilinear operator, the respective filtering can be expressed as follows (Sagara *et al.*, 1991):

$$J_i^n x(t)\,|_{t=kT} \approx I_i^n x(kT) = \left(\frac{T}{2}\right)^{n-i} (1+q^{-1})^{n-i}(1-q^{-1})^i \times \left(1 + \sum_{j=1}^{L-1} q^{-j}\right)^n x(kT), \quad (19)$$

where q^{-1} is the backward shift operator (i.e., $q^{-1}x(kT) = x(kT-T)$) and $L = h/T$ represents the number of samples falling within the integration horizon h. In order to suppress accumulation of possible numerical errors, spline methods can be used in discrete mechanizations (Kowalczuk, 1991). Ultimately, by performing numerical integration of both the sides of (17), the discrete counterpart preserving the parameters is obtained:

$$a_n\, I_n^n y(kT) + \ldots + a_0\, I_0^n y(kT) = b_m\, I_m^n u(kT) + \ldots + b_0\, I_0^n u(kT). \quad (20)$$

The resultant equation (20) can be written in a suitable regression form (1)–(3).

4. Parameter and delay estimation

Among existing methods of identification of delay systems, a solution employing the gradient search algorithm for minimization of an auxiliary quality function is worth noticing (Zhao and Sagara, 1991). Also a three-stage procedure involving a preliminary guess of the delay, estimation of parameters and final identification of the delay can be functional (Kozłowski and Kowalczuk, 2009). However, in both approaches the underlying processing is off-line, and the parameters are assumed to be time-invariant. In this study an on-line method based on Taylor's expansion of the delayed input is proposed.

For clarity of reasoning, the second order ($n = 2$) delay system is considered:

$$a_2 y''(t) + a_1 y'(t) + a_0 y(t) = b_0 u(t-\tau), \quad (21)$$

where τ represents an unknown system delay and, without loss of generality, it is assumed that $b_0 = 1$. The differentiable input signal $u(t-\tau)$ can be written as

$$u(t-\tau) = u(t) + \frac{(-\tau)}{1\,!} u'(t) + \frac{(-\tau)^2}{2\,!} u''(t) + \ldots + \frac{(-\tau)^r}{r\,!} u^{(r)}(t) + \ldots. \quad (22)$$

Assuming that the delay is relatively small, the higher order components multiplied by decaying weights ($\tau^r/r\,!$) can be disregarded. In the simplest approach the partial expansion (22) obtained for $r = 2$ can be considered in (21). Now, by employing the operator of finite-horizon integration (19), the original model (21) assumes a discrete-time regression form of $\gamma(k) = \phi^T(k)\theta + e(k)$:

$$\gamma(k) = I_0^2 u(k), \quad (23)$$

$$\phi(k) = [\, I_1^2 u(k) \quad I_2^2 u(k) \quad I_0^2 y(k) \quad I_1^2 y(k) \quad I_2^2 y(k)\,]^{\mathrm{T}}, \quad (24)$$

$$\theta = [\, \tau \quad -\tfrac{1}{2}\tau^2 \quad a_0 \quad a_1 \quad a_2\,]^{\mathrm{T}}, \quad (25)$$

where, for brevity, the index k stands for kT. Finally, by applying an estimation procedure (Section 2), both the model parameters and the delay can be estimated.

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As can be observed in the results of identification, by ignoring higher-order components of the expansion (22), the obtained estimates (25) are made asymptotically biased. In order to improve the consistency of estimation, more terms of the expansion (22) can be considered in the regression equation. Unfortunately, such an improvement encounters some practical limitations. First, the increased number of regressors in (24) requires the input $u(t)$ to be suitably modified, in such a way that the excitation is persistently exciting of a sufficient order. Second, in the case of second order systems (21), the finite-horizon integration (19) of order $n = 2$ cannot be applied for the derivatives exceeding the system order. Yet, additional regressors result in an increase in the variance of the estimation process.

Specific compensation of the rejected components of the expansion (22) can be proposed as an alternative approach, where the input signal is represented as a linear combination of sinusoidal components. Application of such excitations brings several advantages, to mention avoidance of non-zero offsets, facilitated generation and simplicity in selecting the proper order of persistency of excitation (each new sine wave increases this order by the value of 2). The resulting input dynamics

$$u(t) = \sum_{j=1}^{p} (A_j \sin \omega_j t + B_j \cos \omega_j t) \tag{26}$$

meet the requirements imposed upon the selection of persistently exciting signals in open-loop experiments (Schoukens *et al.*, 1994).

In the case of identification of differential equations (17), it usually suffices that the periodic input (26) is composed of $n + 1$ distinct sinusoids $\sin \omega_j t$ with the corresponding normalized frequencies $\Omega_j = \omega_j T$ (i.e., expressed in radians) falling below π (Sagara *et al.*, 1991). Usually, the periodic signal (26) is generated by a digital computer and forwarded to the plant via a D/A converter. It is therefore realistic that the applied frequencies are known and time-invariant (i.e., the period of a given sinusoid is simply a multiplicity of the 'stable' and known clocking period of the CPU in the PC). As a result, the values of magnitudes A_j and B_j can be evaluated on-line from the regression model $\gamma(k) = \phi^T(k)\theta + e(k)$:

$$\gamma(k) = u(k), \tag{27}$$

$$\begin{aligned}\phi(k) = [\, &\sin \Omega_1 k \quad \dots \quad \sin \Omega_p k \\ &\cos \Omega_1 k \quad \dots \quad \cos \Omega_p k \,]^{\mathrm{T}},\end{aligned} \tag{28}$$

$$\theta = [\, A_1 \quad \dots \quad A_p \quad B_1 \quad \dots \quad B_p \,]^{\mathrm{T}}, \tag{29}$$

where $\Omega_j = \omega_j T$ stand for the already mentioned normalized frequencies.

Considering that the input (26) is noise-free (by definition), the recursive LS estimator can be used to yield consistent estimates of the magnitudes (29). Based on the identified quantities (29), one can easily evaluate (in discrete-time instants) higher order derivatives of the input as follows:

$$\begin{aligned}\hat{u}^{(i)}(k) = &\sum_{j=1}^{p} \hat{A}_j \left(\frac{\Omega_j}{T}\right)^i \sin(\Omega_j k + i\frac{\pi}{2}) \\ &+ \sum_{j=1}^{p} \hat{B}_j \left(\frac{\Omega_j}{T}\right)^i \cos(\Omega_j k + i\frac{\pi}{2}).\end{aligned} \tag{30}$$

Ultimately, an improved regression model

$$\gamma(k) = \phi^T(k)\theta + e(k)$$

of the delay system (with the compensation of the additional terms in (22)) can be used

$$\begin{aligned}\gamma(k) = I_0^2 u(k) &- \frac{1}{3!} \left(\overline{\hat{\tau}}\right)^3 I_0^2 \hat{u}^{(3)}(k) \\ &+ \frac{1}{4!} \left(\overline{\hat{\tau}}\right)^4 I_0^2 \hat{u}^{(4)}(k),\end{aligned} \tag{31}$$

$$\begin{aligned}\phi(k) = [\, &I_1^2 u(k) \quad I_2^2 u(k) \quad I_0^2 y(k) \\ &I_1^2 y(k) \quad I_2^2 y(k) \,]^{\mathrm{T}},\end{aligned} \tag{32}$$

$$\theta = [\, \tau \quad -\tfrac{1}{2}\tau^2 \quad a_0 \quad a_1 \quad a_2 \,]^{\mathrm{T}}, \tag{33}$$

with an averaged estimate of the delay (i.e., an arithmetic mean based on L past values of the estimate of τ) used in (31).

A further improvement in consistency of estimation can result from the use of the IV method. The respective vector $\xi(k)$ can be determined as

$$\begin{aligned}\xi(k) = [\, &I_1^2 u(k) \quad I_2^2 u(k) \quad I_0^2 \hat{y}(k) \\ &I_1^2 \hat{y}(k) \quad I_2^2 \hat{y}(k) \,]^{\mathrm{T}},\end{aligned} \tag{34}$$

with an estimate of the noise-free output $y(k)$ found from an auxiliary filtering involving current estimates of the parameters (Sagara *et al.*, 1991). A deterministic filter can be obtained by substituting the bilinear operator into the Laplace transfer function of the system model (21). Based on current estimates of the parameters (33), the respective Laplace transfer function of the system (21) provides the following estimate of the system output:

$$\hat{Y}(s) = \frac{\hat{b}_0}{\hat{a}_2 s^2 + \hat{a}_1 s + \hat{a}_0} \mathrm{e}^{-s\hat{\tau}} U(s), \tag{35}$$

where $U(s)$ and $Y(s)$ stand for the Laplace transforms of signals $u(t)$ and $y(t)$, respectively, while $e^{-s\tau}$ yields the delay of the input signal. Now, application of the bilinear operator (i.e., Tustin's rule of trapezoidal integration)

$$\frac{T}{2} \frac{1 + q^{-1}}{1 - q^{-1}} \tag{36}$$

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in place of the Laplace operator s^{-1} of genuine integration immediately leads to the following 'discrete-time' counterpart of the 'continuous-time' transfer function represented in (35):

$$\frac{\hat{b}_0 \left(\frac{T}{2}\frac{1+q^{-1}}{1-q^{-1}}\right)^2}{\hat{a}_2 + \hat{a}_1 \left(\frac{T}{2}\frac{1+q^{-1}}{1-q^{-1}}\right) + \hat{a}_0 \left(\frac{T}{2}\frac{1+q^{-1}}{1-q^{-1}}\right)^2}. \quad (37)$$

The above formula allows a convenient on-line evaluation of the noise-free output $y(k)$ based on recorded sequences of the input $u(k)$

$$\begin{aligned} \hat{y}(k) &= \frac{C(q^{-1})}{D(q^{-1})}\, u(k), \\ C(q^{-1}) &= \hat{b}_0 \frac{T^2}{4}(1 + q^{-1})^2, \\ D(q^{-1}) &= \hat{a}_2(1-q^{-1})^2 \\ &\quad + \hat{a}_1 \frac{T}{2}(1-q^{-1})(1+q^{-1}) \\ &\quad + \hat{a}_0 \frac{T^2}{4}(1+q^{-1})^2. \end{aligned} \quad (38)$$

It is of fundamental importance that the operator (36) makes the auxiliary filter (37) stable, provided the underlying continuous system (21) is stable. One should also notice that the delay τ appearing in (35) is disregarded in the final formulas (37)–(38). This hint is by all means acceptable, because with the delayed input used in (38) the underlying correlation $E[\xi(k)e(k)]$ gets even more reduced (Söderström and Stoica, 1981), and therefore the asymptotic bias of estimates is also suppressed.

Based on (34) and (38), the recursive IV procedures (8)–(10) or (12)–(16) can be directly applied to identify the system with delay (21).

4.1. Outline of consistency analysis for higher-order systems. In the above discussion, for simplicity, we took into account a second-order delay system. It is, however, fairly straightforward to generalize the developed methodology of identification of systems with delay for higher-order differential equations.

Consider now a general case of differential equations of order n ($n > 2$):

$$a_n y^{(n)}(t) + \cdots + a_1 y^{(1)}(t) + a_0 y(t) = u(t-\tau). \quad (39)$$

The respective regression representation of the identified model takes the form $\gamma(k) = \phi^T(k)\theta + e(k)$, where

$$\begin{aligned} \gamma(k) &= I_0^n u(k) - \frac{1}{3!}\left(\bar{\hat{\tau}}\right)^3 I_0^n \hat{u}^{(3)}(k) \\ &\quad + \frac{1}{4!}\left(\bar{\hat{\tau}}\right)^4 I_0^n \hat{u}^{(4)}(k), \end{aligned} \quad (40)$$

$$\begin{aligned} \phi(k) = [\, & I_1^n u(k) \quad I_2^n u(k) \quad I_0^n y(k) \\ & I_1^n y(k) \ \ldots \ I_n^n y(k)\,]^{\mathrm{T}}, \end{aligned} \quad (41)$$

$$\theta = [\, \tau \quad -\tfrac{1}{2}\tau^2 \quad a_0 \quad a_1 \ \ldots \ a_n\,]^{\mathrm{T}}. \quad (42)$$

For known theoretical and practical reasons, it is important to verify the asymptotic behavior of the algorithms LS and IV used in identification of continuous models with input-signal delay. First, it is assumed that the discrete measurement of the output signal $y(k)$ is corrupted by an additive white noise process $v(k)$ (thus, effectively, $y(k) + v(k)$ is measured at the output), while the input signal $u(k)$ is assumed to be noise free. Note that part of the components of the regression vector (41) follow from the FIR filtering (19) of the measured output signal. Thus the resulting processing

$$I_i^n[\, y(k) + v(k)\,] = I_i^n\, y(k) + I_i^n\, v(k) \quad (43)$$

(in the presence of noise) generates a correlated disturbance component $e_v(k)$ of the type of moving average (MA):

$$\begin{aligned} e_v(k) \ = \ & a_0 I_0^n v(k) \ + a_1 I_1^n v(k) \\ & + \ldots + a_n I_n^n v(k). \end{aligned} \quad (44)$$

Second, by taking into account that higher-order constituents of Taylor's representation of $u(t-\tau)$ are excluded from the regression vector (41), a deterministic error $e_u(k)$ appears in the regression equation:

$$e_u(k) = \frac{1}{3!}\,\tau^3 I_3^n\, u(k) - \frac{1}{4!}\,\tau^4 I_4^n\, u(k) \ldots. \quad (45)$$

In summary, the total error $e(k)$ as shown in the regression model has the following form: $e(k) = e_v(k) + e_u(k)$.

Now, the asymptotic behavior of the (non-weighted) procedures LS and IV can be gained based on a simple rearrangement of the respective algebraic formulas of LS and IV. By substituting (1) into (5) and (6), one immediately obtains

$$\begin{aligned} \hat{\theta}(k) = \theta + & \left[\frac{1}{k}\sum_{l=1}^{k} \psi(l)\,\phi^{\mathrm{T}}(l)\right]^{-1} \\ & \times \left[\frac{1}{k}\sum_{l=1}^{k} \psi(l)\, e(l)\right], \end{aligned} \quad (46)$$

where $\psi(l)$ stands for $\phi(l)$ or $\xi(l)$ for the methods LS and IV, respectively. Assuming that the processes occurring in (46) are ergodic, the above averaging in the discrete-time domain represents simply correlation:

$$\hat{\theta}(k) = \theta + \left\{ E\left[\psi(k)\,\phi^{\mathrm{T}}(k)\right]\right\}^{-1} \left\{ E[\psi(k)\, e(k)]\right\}. \quad (47)$$

It is evident from (40) that the influence of the deterministic error (45) is to a large extent compensated;

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the effect of the other, stochastic part $e(k) = e_v(k)$ has to be considered in the analysis.

Naturally, with the FIR-filtered regressors $\psi(k) = \phi(k)$, inevitably correlated with the MA process (44), i.e., $E[\phi(k)\, e(k)] \neq 0$, the LS method suffers from asymptotically biased estimates. In the case of the IV method, in turn, the deterministic instrumental variables $\psi(k) = \xi(k)$, obtained from the auxiliary processing (38), are certainly uncorrelated with (44), i.e., $E[\xi(k)\, e(k)] = 0$. Therefore, the estimates (47) are asymptotically consistent. Hence the application of compensation techniques (40) and the utilization of instrumental variables are fully justified.

At this point of our presentation it can be instructive to recollect all stages which constitute the procedure of the so-called 'discrete-time identification of continuous-time systems'. First, the original differential equation (39) of the continuous-time model is integrated equally on both sides using the finite-horizon integrating operators (18). Next, the respective operators are discretized using suitable numerical techniques (in order to suppress accumulation of numerical errors, the normal integration methods can be put into practice; trapezoidal integration can be applied in (19) in the simplest case). As a result, the original continuous model is transformed into the discrete representation (20), which retains the original parameters. Ultimately, as the discrete-time measurement $y^*(kT)$ of the genuine/sampled output signal $y(kT)$ is assumed to be corrupted by an additive white noise sequence $v(kT)$, the employed FIR processing (19) subject to (43) is able to produce a correlated disturbance given by (44). Hence, the resultant discrete-time stochastic model assumes the fundamental regression form of (1)–(3).

5. Numerical example

In this study a second order $(n = 2)$ non-stationary continuous-time system (21) was simulated with the following scenario. A periodic input (26) composed of harmonic terms $\sin(\omega_j t)$ with equal magnitudes $(A_j = 1)$ and the respective frequencies ($\omega_1 = 0.31$ rad/s, $\omega_2 = 0.71$ rad/s, $\omega_3 = 1.39$ rad/s, $\omega_4 = 1.67$ rad/s, $\omega_5 = 2.23$ rad/s) satisfying the already-mentioned condition $\Omega_j = \omega_j T < \pi$ was applied. The model coefficients $a_2 = 0.9$ and $a_0 = 4.0$ were time invariant, while the other parameters $a_1 = 3.0, \ldots, 2.5$ and $\tau = 0, \ldots, 0.37$ were changing gradually within the interval of [200 s, 600 s]. The measurement $(y(k) = y(t)\,|_{t\,=\,kT})$ of the output signal was contaminated by a normally distributed white noise process $v(k)$, so that the resultant noise-to-signal ratio (i.e., the quotient of the respective standard deviations) was equal to $\sigma_v/\sigma_y = 5\%$.

The vital discrete-time representation of the continuous system (21) was obtained using the finite-horizon integrator (19) with $L = 30$. The exponentially weighted LS and IV algorithms ($\lambda = 0.995$) were used to track the evolution of the parameters of this system, while the non-weighted LS routine was applied to identify the magnitudes of the periodic input components of (26). The initial value of the covariance matrix in the LS algorithms was fixed to $P(0) = \text{diag}\,(10^5\,, \ldots\,, 10^5)$, while the IV routine was initiated based on results of a temporary (running in parallel) LS estimator. Input and output signals of the examined continuous system were sampled at the frequency of 20 Hz (i.e., the sampling interval was $T = 0.05$ s) and each simulation run was carried out for a period of 800 s (16000 sampling instants).

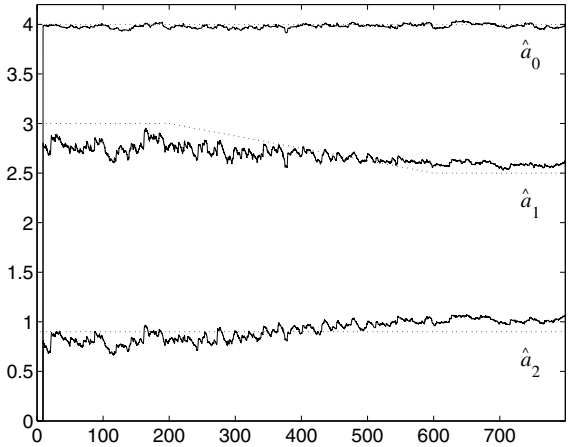

Fig. 1. LS estimates of parameters using the simple model.

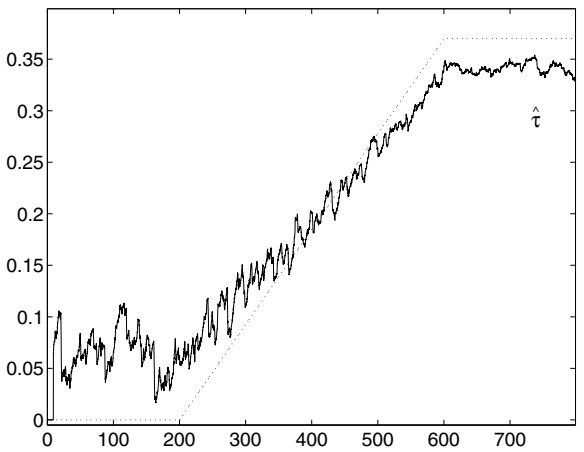

Fig. 2. LS estimate of delay using the simple model.

The results of the LS identification of the simple model (23)–(25) are illustrated in Figs. 1 and 2 . It is clear that the negligence of higher order terms (22) of the input $u(t - \tau)$ makes the estimation process asymptotically biased. The estimation consistency has been improved (as shown in Figs. 3 and 4) by using the IV algorithm working

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on the model (31)–(33) with compensated input terms.

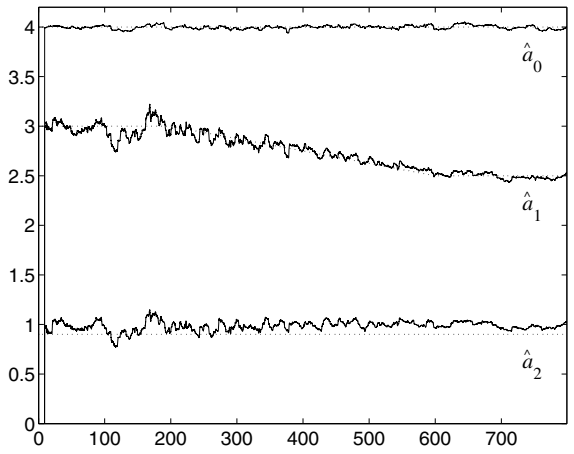

Fig. 3. IV estimates of parameters using the improved model.

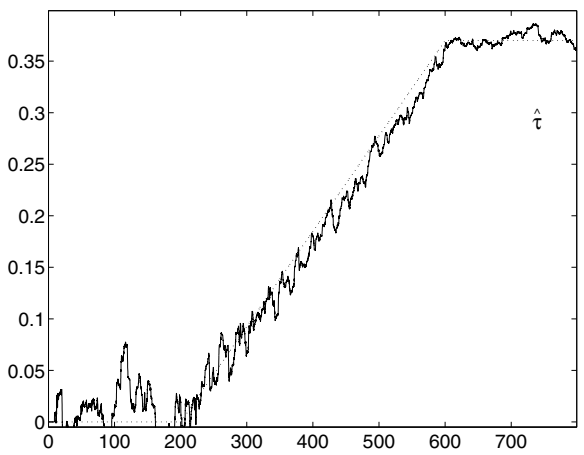

Fig. 4. IV estimate of delay using the improved model.

6. Conclusions and further investigations

The presented methods of on-line identification of continuous-time delay systems constitute the principal original contribution of this study. Employing Taylor's truncated series in the system's modeling, the coefficients of the underlying differential equation along with the unknown input delay can be easily estimated. Moreover, the adapted idea of instrumental variables, together with the proposed compensation of Taylor's disregarded constituents, allows an improved evaluation of system parameters. Finally, the outline of the consistency analysis and the straightforward enhancement of the presented method for identification of higher-order systems can be considered interesting achievements in the consequent course of the authors' investigations (Kowalczuk and Kozłowski, 2000; 2011; Kozłowski and Kowalczuk, 2009).

The reported simulation results show the applicability of the finite-horizon integration operators in the problem of discrete-time approximation of continuous-time dynamic systems. The subject filters of the FIR type are convenient for implementation, while low-pass filtering provides elimination of additive measurement noise. An additional improvement in the accuracy of numerical integration can be easily achieved by using the spline approach (Kowalczuk, 1991; Kowalczuk and Kozłowski, 2000) in the discrete mechanization of (18). Moreover, by selecting the integration horizon L such that the bandwidth of the filter (18) matches as close as possible the band of the identified system (21), the required tuning of integrating filters is considerably facilitated.

It is of practical importance that such finite-horizon integration operators can also be used for convenient numerical mechanization of partial differential equations (representing distributed parameter systems) and continuous models with certain non-linear components (Kozłowski and Kowalczuk, 2007).

The presented results of numerical tests confirm that the developed method of delay system identification allows on-line tracking of gradual changes in system parameters and input delay. Both of the described modeling techniques (23)–(25) and (31)–(33) are conceptually simple and convenient for numerical implementation.

It is also worth emphasizing that, in the proposed solution, the evaluated delay is not expressed as a multiplicity of the sampling period applied.

A suitably modified method can also be applied for identifying linear continuous-time models using differentiated inputs.

Since the FIR processing of measurements introduces correlation into the additive noise (of the moving average type), the LS estimates are certain to be asymptotically biased. Application of instrumental variables significantly improves the consistency of parameter and delay estimates. It is also worth noting that the proposed technique of delay-systems modeling has been developed irrespective of the routines used for the ultimate identification of the parameters. Therefore, by replacing the classical LS algorithm with an estimator in the sense of the least sum of absolute errors, the results of identification become insensitive to sporadic outliers in measurement data (Janiszowski, 1998; Kozłowski and Kowalczuk, 2007; Kowalczuk and Kozłowski, 2011).

At this stage, several promising directions of further investigation can be specified. In particular, the developed method of delay systems identification appears to be helpful in solving the following three problems.

1. *Identification of continuous-time delay MIMO systems*. In the underlying domain of continuous-time

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linear models, the generalizing transition from SISO systems to MIMO ones does not introduce any difficulty. Specifically, for the matrix transfer function models, the necessary modification is straightforward. Some results concerning the identification of continuous-time (non-delayed) MIMO systems are reported by Chao *et al*. (1987) as well as Sagara and Zhao (1989).

2. *Identification of delay SISO systems with non-linear dynamics*. The main problem of identification of non-linear systems is that the developed methods are usually dedicated for particular applications (e.g., Konen *et al.*, 1999; Inoue *et al.*, 1994). However, the method of Hartley modulating functions applied to non-linear Hammerstein models also gained some attention in literature (Mzyk, 2007). Enlightening numerical examples were supplied by Unbehauen and Rao (1997), for instance, in their instructive tutorial on system identification.

3. *Identification of distributed parameter systems with input delay*. The fundamental problem is attributed to the issue of handling non-trivial system descriptions, usually taking the form of partial differential equations. Based on specific approximations of such models, several classical estimation routines can be used for identification. Practical solutions concerning identification of non-delayed distributed systems are described, for instance, by Sagara and Zhao (1990).

Finally, it is worth noting that there exist modern, unconventional methods (such as genetic algorithms or neural networks, for instance), which can also be effectively applied for identification of continuous systems (e.g., Goldberg, 1989; Willis *et al*., 1992; Uciński and Patan, 2010).

There is a relevant question concerning the uniqueness of identification of the delay parameter τ. This issue is challenging, requires further investigation, and, at this stage of the research, only some heuristic arguments can be suggested. By employing the proposed idea of compensation (30)–(33) of higher order constituents of the (infinite) delay term (22), the ultimate model has a finite number of the identified parameters (33). With a persistently exciting input (26) the model is identifiable and the LS results of identification are definitely unique (the estimates though can be biased). The sketched way of reasoning, however, is only intuitive and requires precise analytical examination, which we leave for further research.

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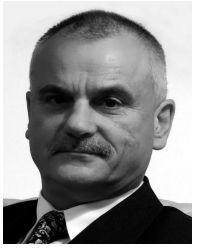
Janusz Kozłowski received his M.Sc. (1990) in automatic control and his Ph.D. (2000) in electronics, both from the Gdańsk University of Technology. He obtained industrial experience by working at ABB Strömberg (Finland), where his activity was focused on the design of communication software and implementation of communication protocols. At present he is with the Faculty of Electronics, Telecommunications and Informatics at the Gdańsk University of Technology (TUG), where his main scientific research is concentrated on the modeling and identification of continuous-time systems, signal processing and adaptive control.



Zdzisław Kowalczuk, Prof., D.Sc., Ph.D., M.Sc.E.E. (2003, 1993, 1986, 1978). Since 1978 he has been with the Faculty of Electronics, Telecommunications and Informatics at the Gdańsk University of Technology, where he is a full professor of automatic control and robotics, and the chair of the Department of Robotics and Decision Systems. He held visiting appointments at University of Oulu (1985), Australian National University (1987), Technische Hochschule Darmstadt (1989), and George Mason University (1990–1991). His main interests include robotics, adaptive and predictive control, system modeling and identification, failure detection, signal processing, artificial intelligence, control engineering and computer science. He has authored and co-authored about 15 books and 50 book chapters, over 90 journal papers and 200 conference publications. He is a recipient of the 1990 and 2003 Research Excellence Awards of the Polish National Education Ministry, and the 1999 Polish National Science Foundation Award in automatic control.

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