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PERTURBATION ANALYSIS OF ELECTRIC FIELD AROUND OBJECTS IN SEA WATER

In marine environment there are different types of objects, such as ships or naval mines, which should be located and identified. One of the location methods is based on the electric potential distribution measurement [1, 2]. The object with an insulating or conductive casing, placed in an electric field, causes some field perturbations. For the purpose of the object location the dimensions of the area, where it is possible to determine electric field perturbations by measurement, are important. In this way one can pre-determine the extent of the system for locating and identifying objects placed in the seawater. This paper presents an analytical approach to the perturbation analysis of electric field distribution caused by a sphere and an ellipsoid placed in a uniform electric field. The distance, beyond which the uniform electric perturbations can be neglected, is determined.

1. INTRODUCTION

In marine environment there are different types of objects, such as naval mines, with different electrical properties, which should be located and identified [1]. One method of the non-ferromagnetic objects location is based on measurement of the electric potential distribution. In seawater, there is a uniform electric field induced by external sources. An object with an insulating or conductive casing, placed in this field, causes its perturbation. For the purpose of the object location, it is important to define the area dimension, in which the uniform electric field perturbation can be determined by a measurement method. In this way, one can define the range of the system to locate and identify objects placed in the seawater.

The paper presents an analytical approach to the perturbation analysis of the uniform electric field caused by a sphere and elongated ellipsoid of revolution placed in this field. One considered conductive and non-conductive sphere and non-conductive ellipsoid. It was assumed that the depth of the marine basin, in which there is an object, is much greater than the maximal object size, what allows to skip the non-conducting air – conducting water boundary conditions. Likewise, it was assumed that the seabed conductivity has the value equal to the seawater

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conductivity. On this basis, one assumed that the object is placed in an infinite homogeneous conductive environment. Taking into account the air- seawater conditions or the bottom conductivity different to the seawater, one requires a numerical analysis of the electric field distribution. Results, presented in this paper, were treated as a preliminary analysis of the range location by electric field perturbation measurement [3].

2. NON-CONDUCTIVE SPHERE IN UNIFORM ELECTRIC FIELD

A non-conductive sphere of the radius R, which can be realized as a metallic sphere coated with an insulating layer or made of a non-conducting material will be considered (Fig. 1).

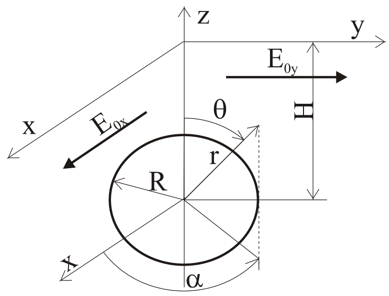

Fig. 1. Sphere in uniform electric field

Assuming that the depth H, on which the sphere is placed in seawater, is much larger than its radius R the influence of non-conducting region z>0 can be neglected and it can be assumed that the sphere is placed in a homogeneous medium with an electrical conductivity σ. Placing the non-conductive sphere in the uniform electric field **E** causes some perturbation of this field. This perturbation are described by an electrical potential φ(r,θ,α), where r,θ,α are the spherical coordinates (Fig. 1). The potential φ satisfies the Laplace's equation in the spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\varphi}{\partial\alpha^2}=0 \qquad (1)$$

and boundary conditions on the non-conductive sphere surface in the form:

$$\frac{\partial\varphi}{\partial r}=-\left(E_{0x}\cos\alpha+E_{0y}\sin\alpha\right)\sin\theta\Big|_{r=0} \qquad (2)$$

and the decay condition of the perturbation potential for the large distance from the sphere center, which can be written as follows:

$$\varphi \underset{r\to\infty}{\to} 0 \tag{3}$$

The solution of equation (1) with boundary conditions (2) and (3) is of the form:

$$\varphi(r,\theta,\alpha)=\frac{1}{2}\left(\frac{R}{r}\right)^2\left(RE_{0x}\cos\alpha+RE_{0y}\sin\alpha\right)\sin\theta \tag{4}$$

For the assessment of the area, in which the non-conductive sphere disturbs the uniform electric field distribution, one determines the ratio k(r,θ,α) as:

$$k(r,\theta,\alpha)=\frac{1}{2}\left(\frac{R}{r}\right)^3\left[1+3\left(\frac{E_{0x}\cos\alpha+E_{0y}\sin\alpha}{E_0}\sin\theta\right)^2\right]^{\frac{1}{2}} \tag{5}$$

where $E_0=\sqrt{E_{0x}^2+E_{0y}^2}$.

Assuming that it is possible to determine the perturbation of the electric field amplitude above 1% of the uniform electric module, the dependence (5) shows that at a distance 5R from the sphere center, the field perturbation is undetectable. An important conclusion follows from these calculations also: if a non-conductive object is located at the depth greater than 5R, the influence of the air- seawater boundary condition can be neglected.

3. CONDUCTIVE SPHERE IN THE UNIFORM ELECTRIC FIELD

Another extreme case is the conductive sphere with the electrical conductivity σ_k. In this case, the potential distribution Φ_1 inside the sphere as well as outside the sphere Φ_2 is considered Assuming, as before, that the sphere is placed in the uniform electric field $\mathbf{E_0}$ with the components E_{0x}, E_{0y} the perturbation potential fulfils Laplace's equation in both areas:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi_i}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi_i}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi_i}{\partial\alpha^2}=0 \tag{6}$$

where i=1,2 for r<R and r>R, respectively.

The electrical potential on the sphere surface is a continuous function, what results from the continuity of the tangential electric field component, and it leads to:

$$\Phi_1=\Phi_2\big|_{r=R} \tag{7}$$

and from the continuity of the normal current density component on the border between the conductors one obtains:

$$\sigma_k\frac{\partial\Phi_1}{\partial r}=\sigma\frac{\partial\Phi_2}{\partial r}\bigg|_{r=R} \tag{8}$$

In addition to the conditions expressing continuity of the electric field tangential components and the current density normal components on the sphere surface, the potential should be limited in the sphere center, namely:

$$\Phi_1 \underset{r\to 0}{<} \infty \tag{9}$$

and the disappearance of perturbations caused by the presence of the conducting sphere at large distance from it, which can be written in the form:

$$\frac{\partial \Phi_2}{\partial r} \underset{r\to\infty}{\to} \left(E_{0x}\cos\alpha + E_{0y}\sin\alpha\right)\sin\theta \tag{10}$$

The solution of equation (6) satisfying the conditions (7)÷(10) has the form:

$$\Phi(r,\theta,\alpha) = R\left(E_{0x}\cos\alpha + E_{0y}\sin\alpha\right)\begin{cases} \dfrac{3\sigma}{\sigma_k + 2\sigma}\dfrac{r}{R}\sin\theta & \text{for } r \le R \\ \left[\dfrac{r}{R} - \dfrac{\sigma_k - \sigma}{\sigma_k + 2\sigma}\left(\dfrac{R}{r}\right)^2\right]\sin\theta & \text{for } r \ge R \end{cases} \tag{11}$$

The ratio of the perturbation field module to the uniform field one specifies the relationship:

$$k(r,\theta,\alpha) = \left|\frac{\sigma_k - \sigma}{\sigma_k + 2\sigma}\right|\left(\frac{R}{r}\right)^3\left[1 + 3\left(\frac{E_{0x}\cos\alpha + E_{0y}\sin\alpha}{E_0}\sin\theta\right)^2\right]^{\frac{1}{2}} \tag{12}$$

Comparing equation (5) and (12), one can conclude that the metal sphere, for which there is $\sigma_k >> \sigma$, will be detectable at the distance of approximately 26% less then the non-conductive sphere with the same radius R.

Similarly, as for the non-conductive sphere, one can say that if the object is at the depth H > 5R, then the influence of the boundary condition between seawater and air can be neglected.

4. NON-CONDUCTIVE ELLIPSOID IN UNIFORM ELECTRIC FIELD

An ellipsoid of revolution can be a good approximation for a submarine or a mine. The model of the non-conductive ellipsoid of revolution has been adopted. The length of the longer ellipsoid axis (e.g. the hull of the ship) is 2L, and 2D is the length of the shorter ellipsoid axis (Fig. 2). In the ship's coordinates x,y,z the uniform external electric field $\mathbf{E_0}$ has three components E_{0x},E_{0y},E_{0z}. To determine the influence of the non-conductive ellipsoid on the electric field in its vicinity the elliptic coordinate system has been adopted.

$$x = a\sinh u \sin\theta\cos\alpha, \quad y = a\sinh u\sin\theta\sin\alpha, \quad z = a\cosh u\cos\theta \tag{13}$$

where $a = L\sqrt{1-\varepsilon^2}$ and $\varepsilon = \dfrac{D}{L}$.

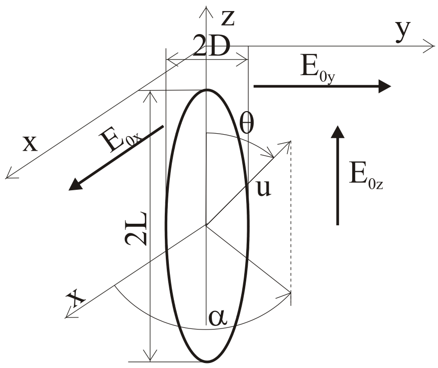

Fig. 2. Object model in the form of an ellipsoid of revolution

The perturbation potential φ satisfies the Laplace's equation in the elliptical coordinates u,θ,α:

$$\frac{1}{\sinh u}\frac{\partial}{\partial u}\left(\sinh u\frac{\partial\varphi}{\partial u}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right)+\left(\frac{1}{\sinh^2 u}+\frac{1}{\sin^2\theta}\right)\frac{\partial^2\varphi}{\partial\alpha^2}=0 \qquad (14)$$

In the case of the non-conductive ellipsoid placed in the uniform electric field, the current density normal component vanishes on the ellipsoid surface, what results in the boundary condition:

$$\left.\frac{\partial\varphi}{\partial u}\right|_{u=u_0}=-a\left(E_{0x}\cos\alpha+E_{0y}\sin\alpha\right)\cosh u_0\sin\theta-aE_{0z}\sinh u_0\cos\theta \qquad (15)$$

where: $u_0 = 0.5\ln\left(\dfrac{1+\varepsilon}{1-\varepsilon}\right)$.

In sufficiently large distance from the ellipsoid its influence on the electric field distribution vanishes, what is expressed by the condition:

$$\varphi \underset{u\to\infty}{<} \infty \qquad (16)$$

The perturbation potential satisfying equation (14) and conditions (15), (16) has the form:

$$\begin{aligned}\varphi(u,\theta,\alpha)=&-A\left(E_{0x}\cos\alpha+E_{0y}\sin\alpha\right)\left[\sinh u\ln\left(\frac{\cosh u+1}{\cosh u-1}\right)-2ctghu\right]\sin\theta+\\&-BE_{0z}\left[0.5\cosh u\ln\left(\frac{\cosh u+1}{\cosh u-1}\right)-1\right]\cos\theta\end{aligned} \qquad (17)$$

where $A = \dfrac{\varepsilon^2 L}{\dfrac{\varepsilon^2}{\sqrt{1-\varepsilon^2}} \ln\left(\dfrac{1+\sqrt{1-\varepsilon^2}}{1-\sqrt{1-\varepsilon^2}}\right) - 4\varepsilon^2 + 2}$, $B = \dfrac{\varepsilon D}{\dfrac{\varepsilon^2}{2\sqrt{1-\varepsilon^2}} \ln\left(\dfrac{1+\sqrt{1-\varepsilon^2}}{1-\sqrt{1-\varepsilon^2}}\right) - 1}$.

The ratio k(u,θ,α) of the perturbation field module to the uniform field module is defined by the relationship:

$$k(u,\theta,\alpha) = \frac{E_p(u,\theta,\alpha)}{E_0} \tag{18}$$

where: $E_p(u,\theta,\alpha) = \sqrt{[E_u(u,\theta,\alpha)]^2 + [E_\theta(u,\theta,\alpha)]^2 + [E_\alpha(u,\theta,\alpha)]^2}$,

$E_0 = \sqrt{E_{0x}^2 + E_{0y}^2 + E_{0z}^2}$.

$$E_u(u,\theta,\alpha) = \frac{-1}{a\sqrt{\sinh^2 u + \sin^2\theta}} \left\{ A\left(E_{0x}\cos\alpha + E_{0y}\sin\alpha\right)\left[\cosh u \ln\left(\frac{\cosh u + 1}{\cosh u - 1}\right) + \right.\right.$$

$$\left.\left. -2 + \frac{2}{\sinh^2 u}\right]\sin\theta + BE_{0z}\left[0.5\sinh u \ln\left(\frac{\cosh u + 1}{\cosh u - 1}\right) - ctghu\right]\cos\theta \right\}$$

$$E_\theta(u,\theta,\alpha) = \frac{-1}{a\sqrt{\sinh^2 u + \sin^2\theta}} \left\{ A\left(E_{0x}\cos\alpha + E_{0y}\sin\alpha\right)\left[\sinh u \ln\left(\frac{\cosh u + 1}{\cosh u - 1}\right) + \right.\right.$$

$$\left.\left. -2ctghu\right]\cos\theta - B\left[0.5\cosh u \ln\left(\frac{\cosh u + 1}{\cosh u - 1}\right) - 1\right]\sin\theta \right\}$$

$$E_\alpha(u,\theta,\alpha) = \frac{A}{a}\left(E_{0x}\sin\alpha - E_{0y}\cos\alpha\right)\left[\ln\left(\frac{\cosh u + 1}{\cosh u - 1}\right) - 2\frac{\cosh u}{\sinh^2 u}\right]$$

The calculation was performed for $E_{0x}=E_{0y}=E_{0z}=1$V/m, 2L=30m and 2D=6m. On the Figure 3 the curves, on which the ratio k is calculated, are given. Results of this calculation are given in Figures 4 and 5.

The perturbation ratio k curves, presented in Figure 4 and 5, show that perturbations in the uniform electric field, caused by the non-conducting ellipsoid, disappear (are less than 0.01 of the original field) at the distance equal to its longer half-axis starting from the ellipsoid center (Fig. 2). It appears from this calculation also that for objects submerged at a depth greater then their largest dimension one can, practically, neglect boundary conditions on the surface air-seawater, which means a substantial simplification in the calculations of the electric field distribution around such objects.

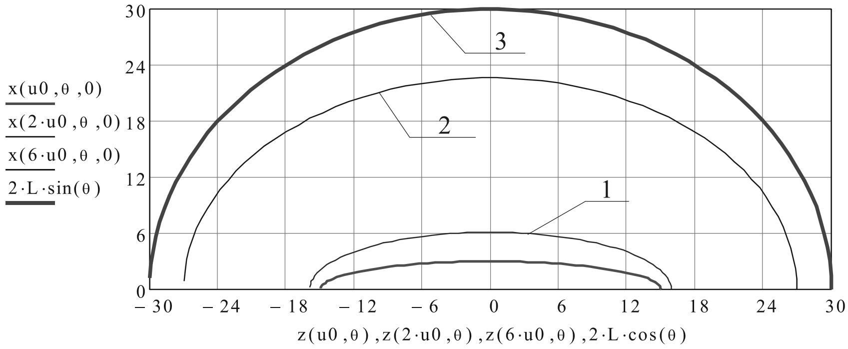

Fig. 3. The curves along which the ratio k was calculated. 0 – the ellipsoid surface in the plane x,y, 1 and 2 – curves, for which the perturbation ratio k has been calculated (Fig.4 and 5), 3 – the sphere of radius 2L

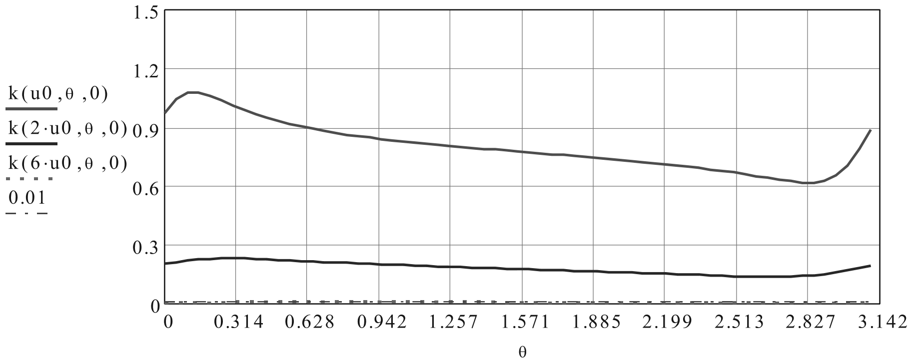

Fig. 4. The perturbation ratio k along curves 0,1 and 2 (Fig. 3)

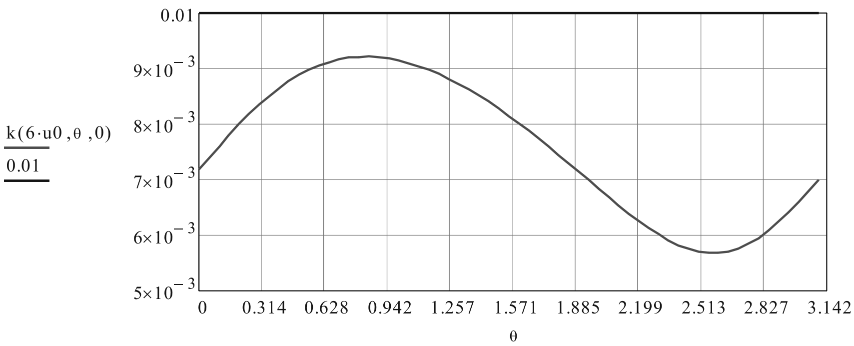

Fig. 5. Enlarged values of the perturbation ratio k on the curve 2 (Fig. 3)

5. SUMMARY

The following conclusions result from the performed analytical calculation of the electric field distribution around the sphere and ellipsoid:

- non-conductive sphere of radius R can be located at the distance less than 5R,
- conductive sphere of radius R can be located at the distance less then 3,8R,
- in the case of the non-conductive ellipsoid, it appears that the object of this shape can be located at the distance less then 2L from the ellipsoid center, where 2L is the long axis of the ellipsoid (Fig. 2).

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