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ABSTRACT

The characteristics of propagation of sawtooth periodic and impulsive signals at a transducer are analytically studied in this work. A plasma under consideration is motionless and uniform at equilibrium, and its perturbations are described by a system of ideal magnetohydrodynamic equations. Some generic heating/cooling function, which in turn depends on equilibrium thermodynamic parameters, may destroy adiabaticity of a flow and make the flow acoustically active. Planar waves with the wave vector forming a constant angle θ with the equilibrium straight magnetic strength are considered. This model has been proposed in previous publications listed in the Introduction. Conclusions are drawn for fast and slow magnetoacoustic waves of sawtooth shape and various cases of a nonlinear flow. These nonlinearities occur in accordance with a type of heating/cooling function under consideration. Amplitude and duration of signals are evaluated as functions of a distance from a transducer, θ, plasma-β, and a type of heating-cooling function. In particular, it is observed that the duration of an impulse enlarges infinitely in acoustically stable flows, while on the other hand, it tends to some limiting value in acoustically active flows of plasma.

I. INTRODUCTION

A reliable detection and interpretation of hydrodynamic disturbances in a plasma are of key importance in the astrophysical and technical applications, especially as these are related to the dynamics of the solar atmosphere. The wave perturbations not only transfer energy and momentum, but may be a reason for heating of a plasma and its bulk flows.[1,2] The difficulty is in a variety of magnetohydrodynamic (MHD) modes. These include waves, namely Alfvén, fast and slow magnetoacoustic modes, and nonwave modes, i.e., the entropy and the vortex modes. The important issue is the correct definition of these modes. The excitation of the nonwave modes is associated with magnetoacoustic heating and streaming, and occurs due to nonlinearity and nonadiabaticity of a flow in the field of intense magnetosound perturbations. Plasma is an open system, since there is an influx of energy to plasma, and radiative losses. This may be a reason for deviation of adiabaticity of a flow along with mechanical and thermal losses. Acoustics of open systems is well understood. Depending on the balance between attenuation and inflow of energy, the wave perturbations may increase in the course of propagation.[3–5] This happens due to a generic heating-cooling function which provides isentropical instability of a flow.[6,7]

Nonlinear distortions may drastically influence the wave process, even if perturbations are of moderate magnitude. Stationary waveforms exist due to nonlinearity in flows with dispersion and attenuation. The balance of the nonlinearity of a flow and dissipation may lead to formation of a shock wave, which is a solution to the Burgers equation. The solution for flows with dispersion were proposed, e.g., by Rudenko and Soluyan.[8] Nonlinearity, if taken alone without attenuation, results in waves with discontinuities of decreasing amplitude, which at the large distances from a transducer take the sawtooth shape. The stationary and self-similar waveforms are of special importance for fluid dynamics for the following reasons: They are common in many wave processes, including optic waves; they are usually the simplest mathematical case; the total perturbation often develops in a set of self-similar or stationary waveforms. The joint action of nonlinearity and nonadiabaticity arising from a heating-cooling function, may lead to appearance of self-similar waveforms with variable amplitude but stable shape. The periodic waves in plasma were discussed by Chin *et al.* in Ref. 9, and asymmetric pulses by Zavershinsky *et al.*[5] In open systems, the heating/cooling function may prevent formation of shock fronts, or, on the contrary, may enlarge the magnitude of perturbations accelerating their formation. This enhances the nonlinear effects. Thus, under some conditions, the waves with discontinuities propagate in a plasma with magnitudes determined not only by nonlinear attenuation, but also by the kind of heating-cooling function.[5,9] They have been predicted in open systems described by a similar dynamic equation though of different physical meanings.[6,7]

A flow of magnetic gas depends strongly on the geometry of a flow, strength of the magnetic field, and energy balance. The first studied cases consider planar waves along and across the straight magnetic field.[10–12] We concentrate on the planar, fast and slow magnetoacoustic longitudinal velocity of a plasma governed by equation derived by Nakariakov *et al.* in Ref. 13. This equation applies to the case of the constant straight magnetic field forming a constant angle with the wave vector. This dynamic equation may be readily derived by the projecting technique which has been exploited by the author in many problems of fluid dynamics.[14–17] It was reproduced by the author in Ref. 18. The equation is valid if one selected wave mode is dominant, that is, its specific perturbations are much larger than those of other modes. The authors of Ref. 9 have concluded that there is possibility of self-organization of initially sinusoidal MHD waves into shock waves and solitary pulses in a thermoconducting plasma. The subject of this study is the analysis of nonlinear evolution of the periodic and impulsive signals which take the sawtooth shape at a transducer.

The conclusions concern the dynamics of magnitude and duration of individual slow and fast perturbations in dependence on plasma-β and the degree of nonadiabaticity of a flow due to heating/cooling function. Nonlinear attenuation at the shock front occurs unusually in the adiabatically unstable flows of a plasma. We do not consider the electrical resistivity of plasma. Also, we do not consider attenuation due to mechanical viscosity and thermal conduction of a plasma. These factors are well understood separately in the context of nonlinear acoustics.[8,19] In the equilibrium flow, they enhance attenuation including nonlinear one, and counteract enlargement of magnitudes of perturbations in acoustically active flow. They may prevent formation of discontinuities or lead to smoothing of shock fronts.

II. THE EQUATIONS OF MHD FLOW

Following Nakariakov and co-authors, we consider the dynamics of totally ionized gas which is governed by MHD equations. The full set of MHD equations for perfectly conducting fluid includes the continuity equation, momentum equation, energy balance equation, and electrodynamic equations

$$\begin{aligned}
&\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{v})=0,\\
&\rho\frac{D\vec{v}}{Dt}=-\vec{\nabla}p+\mu_0(\vec{\nabla}\times\vec{B})\times\vec{B},\\
&\frac{Dp}{Dt}-\gamma\frac{p}{\rho}\frac{D\rho}{Dt}=(\gamma-1)L(p,\rho),\\
&\frac{\partial\vec{B}}{\partial t}=\vec{\nabla}\times(\vec{v}\times\vec{B}),\\
&\vec{\nabla}\cdot\vec{B}=0,
\end{aligned}\tag{2.1}$$

where p, ρ, $\vec{v}$, $\vec{B}$, are the pressure and density of a plasma, its velocity, the magnetic field strength, and μ_0 is the permeability of free space. The two last equations from the system are: The ideal induction equation, and the Maxwell's equation which ensures the solenoidal character of $\vec{B}$. Some generic heating-cooling function $L(p,\rho)$ accounts for the optically thin radiative cooling and unspecified inflow of energy.[13] The third equation in the set (2.1) concerns an ideal gas with the adiabatic index γ. An ideal gas consists of molecules of negligible size with an average molar kinetic energy dependent exclusively on temperature.

It is useful to remind the conditions and the geometry of a planar flow used in Ref. 13 along with conclusions from this model. The equilibrium magnetic strength $\vec{B}_0$ forms constant angle θ $(0\le\theta\le\pi/2)$ with the wave vector directed along axis z, and its y-component equals zero, so as

$$B_{0,x}=B_0\sin(\theta),\quad B_{0,y}=0,\quad B_{0,z}=B_0\cos(\theta).$$

Any thermodynamic quantity $f(z,t)$ is a sum of equilibrium value f_0 and perturbation $f'(z,t)$. We consider an initially homogeneous static plasma with $\vec{v}_0=\vec{0}$. The system (2.1) is nonlinear. The leading-order equations including linear and leading-order nonlinear, that is, quadratically nonlinear terms, follow from Eq. (2.1):

$$\begin{aligned}
&\frac{\partial\rho'}{\partial t}+\rho_0\frac{\partial v_z}{\partial z}=-\rho'\frac{\partial v_z}{\partial z}-v\frac{\partial\rho'}{\partial z},\\
&\frac{\partial v_x}{\partial t}-\frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z}=-v_z\frac{\partial v_x}{\partial z}-\frac{B_{0,z}}{\rho_0\mu_0}\rho'\frac{\partial B_x}{\partial z},\\
&\frac{\partial v_y}{\partial t}-\frac{B_{0,z}}{\rho_0\mu_0}\frac{\partial B_y}{\partial z}=-v_z\frac{\partial v_y}{\partial z}-\frac{B_{0,z}}{\rho_0\mu_0}\rho'\frac{\partial B_y}{\partial z},\\
&\frac{\partial v_z}{\partial t}+\frac{1}{\rho_0}\frac{\partial p'}{\partial z}-\frac{B_{0,x}}{\rho_0\mu_0}\frac{\partial B_x}{\partial z}=-\frac{\rho'}{\rho_0}\frac{\partial p'}{\partial z}-\frac{B_{0,z}}{\rho_0\mu_0}\rho'\frac{\partial B_x}{\partial z}\\
&\qquad-\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{B_x^2+B_y^2}{2\mu_0}\right)-v_z\frac{\partial v_z}{\partial z},\\
&\frac{\partial p'}{\partial t}+c^2\rho_0\frac{\partial v_z}{\partial x}-(\gamma-1)(L_pp'+L_\rho\rho')\\
&\quad=(\gamma-1)(0.5L_{pp}p'^2+0.5L_{\rho\rho}\rho'^2+L_{p\rho}p'\rho')-\gamma p'\frac{\partial v_z}{\partial z}-v_z\frac{\partial p'}{\partial z},\\
&\frac{\partial B_x}{\partial t}+\frac{\partial}{\partial z}(B_{0,x}v_z-B_{0,z}v_x)=-B_x\frac{\partial v_z}{\partial z}-v_z\frac{\partial B_x}{\partial z},\\
&\frac{\partial B_y}{\partial t}-\frac{\partial}{\partial z}(B_{0,z}v_y)=-B_y\frac{\partial v_z}{\partial z}-v_z\frac{\partial B_y}{\partial z},
\end{aligned}\tag{2.2}$$

where primes by components of velocity are dropped, and partial derivatives of the heating-cooling function $L(p,\rho)$ with respect to its variables are designated as

$$L_p=\frac{\partial L}{\partial p},\quad L_\rho=\frac{\partial L}{\partial\rho},\quad L_{pp}=\frac{\partial^2L}{\partial p^2},\quad L_{\rho\rho}=\frac{\partial^2L}{\partial\rho^2},\quad L_{p\rho}=\frac{\partial^2L}{\partial p\partial\rho}$$

and refer to the equilibrium state (p_0,ρ_0). L equals zero in the unperturbed state. Equation (2.2) is an initial point for further evaluations. The dispersion relations follow from the linearized version of Eq. (2.2), if one assumes the harmonic dependence of the perturbations is proportional to $\exp(i\omega(k_z)t-ik_zz)$, where k_z designates the wave number

$$f'(z,t)=\int_{-\infty}^{\infty}\tilde{f}(k_z)\exp(i\omega(k_z)t-ik_zz)dk_z.$$

They reflect the solvability of linearized version of Eq. (2.2). The first four roots relate to the slow and fast magnetosound modes of different directions of propagation

$$\omega_j = C_j k_z - iC_j D_j, \tag{2.3}$$

where $j = 1, \ldots 4$, C_j is one from the four roots of the equation

$$C_j^4 - C_j^2(c_0^2 + C_A^2) + c_0^2 C_{A,z}^2 = 0,$$
$$c_0 = \sqrt{\frac{\gamma p_0}{\rho_0}} \tag{2.4}$$

indicates the acoustic speed in unmagnetized gas in equilibrium, and

$$C_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$$

is the Alfvén speed, $C_{A,z} = C_A \cos(\theta)$. D_j is determined in the following way:

$$D_j = \frac{C_j(C_j^2 - C_A^2)(\gamma - 1)}{2c_0^2(C_j^4 - c_0^2 C_{A,z}^2)}(c_0^2 L_p + L_\rho).$$

For any nonzero magnetosound speed C_j, the denominator in the expression for D_j differs from zero: Zero $C_j^4 - c_0^2 C_{A,z}^2$ leads to equalities $|C_{A,z}| = \frac{c_0^2 + C_A^2}{2c_0}$ and $C_{A,x}^2 = -\frac{(c_0^2 - C_A^2)^2}{4c_0^2}$, with the exception of the case $|C_j| = c_0 = C_A$ which corresponds to the zero nominator. The next two roots ω_5, ω_6 specify the Alfvén modes

$$\omega_{5,6} = \pm C_{A,z} k_z,$$

and the last root ω_7 corresponds to the nonwave entropy mode

$$\omega_7 = \frac{i(\gamma - 1)L_\rho}{c_0^2}.$$

The dispersion relations Eq. (2.3) along with Eq. (2.4) were established by Chin *et al.*[9,13] and have been used in studies of various wave processes in a plasma. The condition of acoustic (adiabatic, isentropic) instability depends on the kind of heating-cooling function. In all flows in open systems, it sounds as[3,4]

$$c_0^2 L_p + L_\rho > 0. \tag{2.5}$$

This inequality means $C_j D_j > 0$ and enlargement of magnetosound perturbations of infinitely small magnitude in the course of propagation, accordingly to Eq. (2.3). Hence, this is the case of acoustically active flow. Nonlinear attenuation of finite-magnitude sound at the front of a shock wave along with mechanical and thermal damping and elecrical resistivity may prevent growth of wave perturbations.

The magnetosound modes determined by Eq. (2.3) may not be the wave processes if strongly attenuated. Wave motion occurs in the case of weak attenuation (or enhancement) during the characteristic duration of perturbations

$$\omega_j \gg |C_j D_j|, \quad j = 1, \ldots 4.$$

For the impulsive disturbances, ω_j should be replaced by an inverse characteristic duration of a signal, T_j^{-1}. This condition determines actually the domain of magnetosound frequencies to be considered in the case of slow and fast magnetosound perturbations. This point requires explanation. In fact, it demands smallness of impact of the heating-cooling function, so as

$$\frac{|c_0^2 L_p + L_\rho| T_j}{c_0^2} \ll \frac{2(C_j^4 - c_0^2 C_{A,z}^2)}{(\gamma - 1)C_j^2(C_j^2 - C_A^2)} \quad j = 1, \ldots 4. \tag{2.6}$$

Equation (2.6) may be valid for fast magnetosound modes and broken for slow ones or, on the contrary, may be valid for slow modes and broken for fast ones. The expression on the right-hand side of the inequality is always positive. Figure 1 shows variations of $\frac{C_j^2(C_j^2 - C_A^2)}{2(C_j^4 - c_0^2 C_{A,z}^2)}$ with $\beta\gamma$ and θ, where plasma-β is determined as as the ratio of thermodynamic and magnetic pressures

$$\beta = \frac{2}{\gamma}\frac{c_0^2}{C_A^2}.$$

The question at issue is that at $\theta = 0$ and $\theta = \pi$ there are two solutions to Eq. (2.4), $|C_j| = c_0$ and $|C_j| = C_A$. If $\beta < \frac{2}{\gamma}$, the fast mode has the speed C_A, and if $\beta > \frac{2}{\gamma}$, the fast mode is the sound wave with the speed c_0. That is the reason for the ratio $\frac{C_j^2(C_j^2 - C_A^2)}{2(C_j^4 - c_0^2 C_{A,z}^2)}$ to vary abruptly at $\beta\gamma = 2$ if $\theta = 0$ or $\theta = \pi$. The main conclusion is that for any $\beta\gamma$ and θ, and for any kind of magnetosound wave, fast or slow, $\frac{C_j^2(C_j^2 - C_A^2)}{(C_j^4 - c_0^2 C_{A,z}^2)} \leq 1$. This can be seen in Fig. 1 that refers to the fast modes. Hence, if

$$\frac{|c_0^2 L_p + L_\rho| T_j}{c_0^2} \ll \frac{2}{\gamma - 1}, \tag{2.7}$$

Equation (2.6) is satisfied for arbitrary $\beta\gamma$ and for both fast and slow modes. For a small $\beta\gamma$, it is valid with a large margin for the fast waves, and for large $\beta\gamma$, it is valid with a large margin for the slow waves. Summarizing, Eq. (2.6) determines the characteristic duration of perturbations and properties of a heating-cooling function supporting the wave processes.

III. NONLINEAR DYNAMICS OF A MAGNETOACOUSTIC WAVE WITH DISCONTINUITY

Nonlinearity of wave processes plays a crucial role even in the case of small magnitudes of perturbations. That is all the more true in acoustically active media. The dynamic equation for an individual

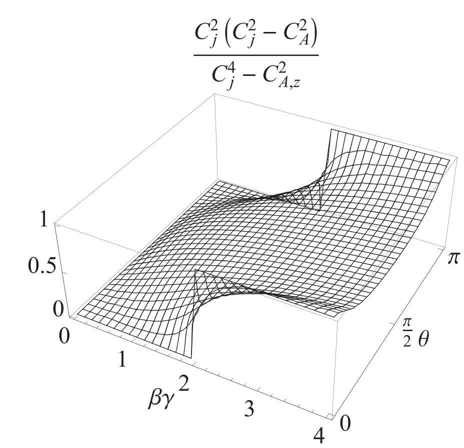


FIG. 1. Variation of the ratio $\frac{C_j^2(C_j^2 - C_A^2)}{(C_j^4 - c_0^2 C_{A,z}^2)}$ in the case of fast magnetoacoustic perturbations.

magnetosound wave follows from the system (2.2). It has been derived in the case of one dominant wave mode by Nakariakov *et al.* in Ref. 13. The dominance means that the absolute values of magnetosound perturbations are much bigger than those of other modes, at least in some spatial and temporal domains. The dynamic equation describing the nonlinear distortion of the longitudinal component of velocity $v_{z,j}$ in the dominant magnetoacoustic wave ordered as j, takes the form

$$\frac{\partial v_{z,j}}{\partial t} + C_j \frac{\partial v_{z,j}}{\partial z} - D_j C_j v_{z,j} + \varepsilon_j v_{z,j} \frac{\partial v_{z,j}}{\partial z} = 0, \tag{3.1}$$

with

$$\varepsilon_j = \frac{3c_0^2 + (\gamma+1)C_A^2 - (\gamma+4)C_j^2}{2(c_0^2 - 2C_j^2 + C_A^2)}.$$

Equation (3.1) may be readily derived by means of the projecting row which projects the total vector of perturbations into the specific vector of perturbations of the mode ordered as j. It projects the governing system of Eq. (2.1) into dynamic equations for dominant perturbations as well. The projectors are established by the author in Ref. 18. In the absence of the magnetic field and deviation from adiabaticity, Eq. (3.1) coincides with the well-known equation for velocity in the planar Riemann's wave propagating in the positive direction of axis z in an ideal gas with $D_j = 0$, $C_j = c_0$, $\varepsilon_j = \frac{\gamma+1}{2}$.[8]

Without loss of generality, we will consider magnetosound waves propagating in the positive direction of axis z, that is, positive C_j. Hence, the sign of D_j is determined exclusively by the sign of $c_0^2 L_p + L_\rho$. The understanding that the magnetic flows may and almost always will include discontinuities, comes from the early theoretical studies.[20] Equation (3.1) for the isentropic flow of an ideal fluid (the case $D_j = 0$) and $\theta = \pi/2$, for planar or cylindrically symmetric motion, was derived and analyzed in the context of propagation of a sawtooth impulse in Ref. 12. Equation (3.1) may be readily rearranged into the leading-order nonlinear equation, if $D_j \neq 0$, $C_j \neq 0$ (the lower index j will be omitted in all fore-coming formulas)

$$\frac{\partial V}{\partial Z} - \frac{\varepsilon}{C^2} V \frac{\partial V}{\partial \tau} = 0, \tag{3.2}$$

by means of new variables

$$V = v_z \exp(-Dz), \quad Z = \frac{e^{Dz} - 1}{D}, \quad \tau = t - z/C.$$

Note that Z is always positive for nonzero D. Equation (3.2) recalls the dynamic equations for perturbations in other media which may be acoustically active due to different reasons.[6,7] The important property of Eq. (3.2), which in fact is analogous to the equation for the simple wave, is the independence of $\int V d\tau$ on a distance from a transducer at any kind of excitation, since

$$\frac{\partial}{\partial Z} \int V d\tau = \int \frac{\varepsilon}{C^2} V \frac{\partial V}{\partial \tau} d\tau = 0. \tag{3.3}$$

The limits of integration may be set as $-T/2$, $T/2$ for periodic with the period T signals, or $-\infty, \infty$ for the impulsive ones. Equation (3.2) may be solved by the method of characteristics.[21] It has been established that discontinuity in the waveform always forms in acoustically active media (that is the case of $D > 0$) and may not arise otherwise due to attenuation.[6] The peculiarity of the wave motion in the magnetic gases is strong dependence of its parameters on θ, plasma-β and magnetic strength. By the way, they determine the rate of enlargement or decay of the signals. That is the subject of Secs. III A and III B. The particular case $D = 0$ corresponds to $c_0^2 L_p + L_\rho = 0$ or $C = C_A$. It is of minor importance. Equation (3.2) is still valid with $Z = z$ and $V = v_z$.

A. Periodic magnetoacoustic wave with discontinuities

We focus on the periodic velocity in the magnetoacoustic wave that is sawtooth initially. Its initial variations over one period T_0 is determined by the formula

$$\frac{V}{V_0} = \begin{cases} 1 - 2\dfrac{\tau}{T_0}, & 0 \le \tau \le \dfrac{T_0}{2} \\ -1 - 2\dfrac{\tau}{T_0}, & -\dfrac{T_0}{2} \le \tau < 0, \end{cases} \tag{3.4}$$

where V_0 denotes an amplitude of the periodic perturbation and T_0 is its period at a transducer which is situated at $z = 0$. Dynamics of the periodic sawtooth velocity of unmagnitized gas in the Cauchy problem was considered by Landau and Lifshitz.[21] At any distance from a transducer, the shape of a signal remains saw-edged. Figure 2 exhibits the distortions of one period of velocity in the course of propagation. The current amplitude v_A depends on the distance from a transducer in the following manner:

$$\begin{aligned} v_A &= V_A e^{Dz} = \frac{V_0 e^{Dz}}{1 + \dfrac{2\varepsilon V_0 (e^{Dz} - 1)}{DC^2 T_0}}, & D \neq 0, \\ v_A &= \frac{V_0}{1 + \dfrac{2\varepsilon V_0 z}{C^2 T_0}}, & D = 0. \end{aligned} \tag{3.5}$$

The conclusion is that v_A tends to zero if $D < 0$ and tends to

$$v_{A,\infty} = \frac{DC^2 T_0}{2\varepsilon} \tag{3.6}$$

if $D > 0$ when the distance from a transducer z enlarges infinitely. The limit does not depend on the initial magnitude of a triangular signal, V_0 and is proportional to its period T_0 which does not vary with the distance from a transducer. Quantities D, C, and ε depend on the

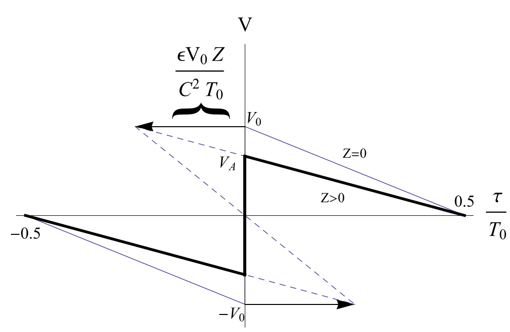


FIG. 2. Variation of velocity in the sawtooth periodic perturbation in the course of propagation, if $D \neq 0$. If $D = 0$, $Z = z$, and $V = v_z$.

magnetic strength, plasma-β, and the angle between the magnetic field and wave vector. Figure 3 shows the limiting magnitude of a periodic signal $v_{A,\infty}$ when z tends to infinity and D is positive (that is, $c_0^2 L_p + L_\rho > 0$). The first row in plots in Fig. 3 concerns fast wave perturbations, and the second row relates to the slow ones. The limiting values at $\theta = 0$ and $\theta = \pi$ depend on the domain of plasma-β. If $\theta = 0$ or $\theta = \pi$, the limiting magnitude changes abruptly at $\beta = \frac{2}{\gamma}$. In the case of the fast magnetosound perturbations and $\beta < \frac{2}{\gamma}$, $v_{A,\infty} = 0$, and in the case $\beta > \frac{2}{\gamma}$, $v_{A,\infty} = \frac{(c_0^2 L_p + L_\rho) T_0}{c_0^2} \frac{\gamma - 1}{\gamma + 1}$. In the case of slow perturbations, $v_{A,\infty} = 0$ jumps at $\beta = \frac{2}{\gamma}$ from $\frac{(c_0^2 L_p + L_\rho) T_0}{c_0^2} \frac{\gamma - 1}{\gamma + 1}$ until 0. In all evaluations, $\gamma = 5/3$. As for $\theta = \pi/2$ and case of slow modes, $C = 0$ and hence the limiting value of magnitude of velocity formally equals zero for any β. The theory does not consider the case $C = 0$ that does not reflect a wave process. The averaged over the period energy of a unit mass of a plasma equals

$$E = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v^2(\tau, z) d\tau.$$

It varies with a distance from a transducer and depends on θ, plasma-β, and degree of nonequilibrium, and it is proportional to v_A^2. When $|D|z$ tends to infinity and $D < 0$, it behaves as e^{2Dz}, and it tends to $\frac{D^2 C^4 T_0^2}{4\varepsilon^2} E_0$ in the case of positive D, where E_0 is the averaged over period kinetic energy of a unit mass at a transducer.

B. The sawtooth impulse

Evolution of triangular at a transducer positive impulse of velocity with an initial duration T_0 is shown in Fig. 4. Excess density and pressure in this impulse are also positive. We make use of Eq. (3.3) and Fig. 4 in evaluations of the magnitude at the front of the triangular impulse

$$v_A = V_A e^{Dz} = \frac{V_0 e^{Dz}}{\sqrt{1 + \dfrac{\varepsilon V_0 (e^{Dz} - 1)}{D C^2 T_0}}}, \quad D \neq 0,$$
$$v_A = \frac{V_0}{\sqrt{1 + \dfrac{\varepsilon V_0 z}{C^2 T_0}}}, \quad D = 0. \qquad (3.7)$$

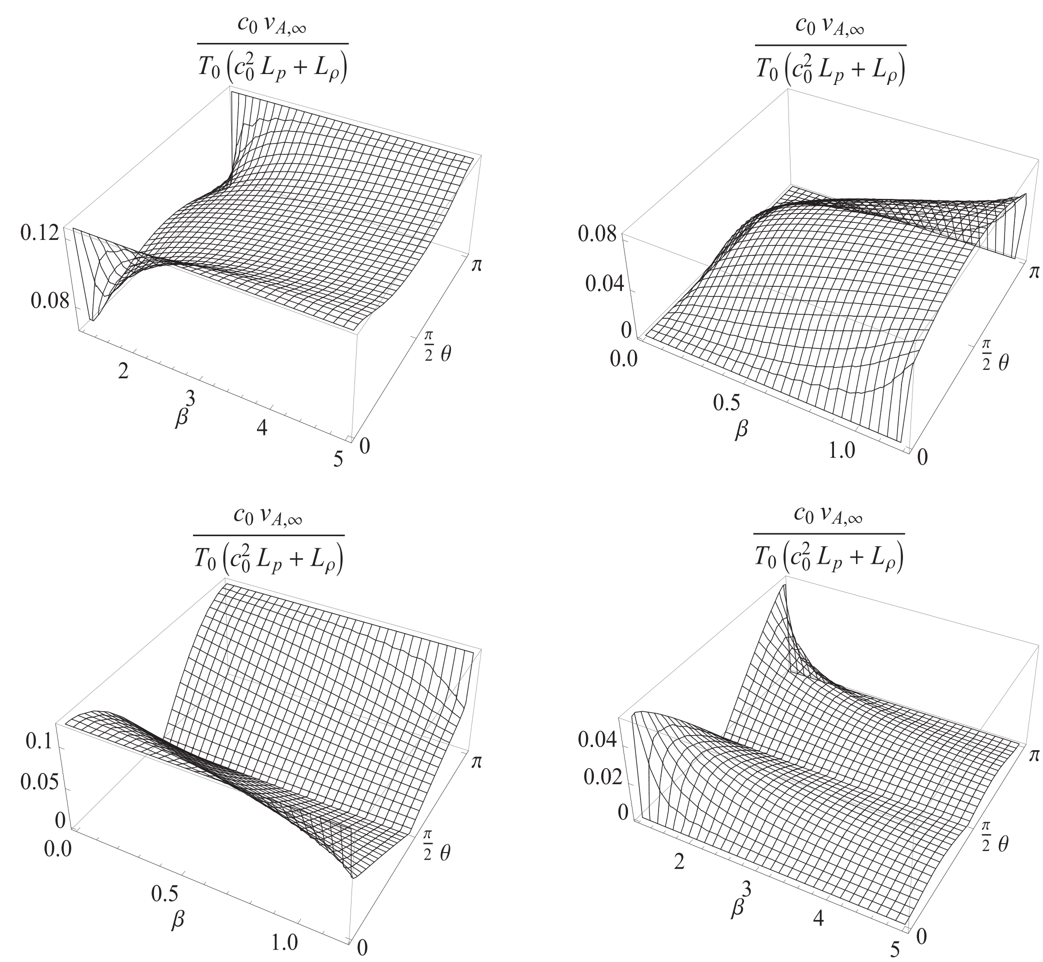


FIG. 3. Magnitude of velocity in the periodic sawtooth magnetosound waves with discontinuities in the acoustically active plasma with $D > 0$ for different domains of β: $0 \le \beta \le 2/\gamma$ (left panel) and $2/\gamma \le \beta$ (right panel). The top row corresponds to fast perturbations and the bottom row corresponds to slow perturbations.

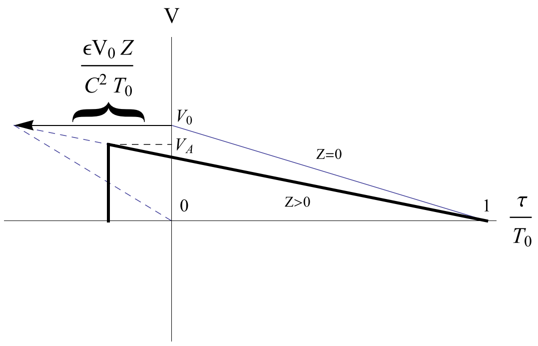


FIG. 4. Variation of the magnitude and duration of the triangular impulse in the course of propagation, if $D \neq 0$. If $D = 0$, $Z = z$, and $V = v_z$.

Its duration T varies with a distance from a transducer and equals

$$T = T_0\sqrt{1 + \frac{\varepsilon V_0(e^{Dz} - 1)}{DC^2 T_0}}, \quad D \neq 0,$$
$$T = T_0\sqrt{1 + \frac{\varepsilon V_0 z}{C^2 T_0}}, \quad D = 0.$$

If $D < 0$, v_A tends to zero at $|D|z \to \infty$, and T tends to

$$T_\infty = T_0\sqrt{1 - \frac{\varepsilon V_0}{DC^2 T_0}}.$$

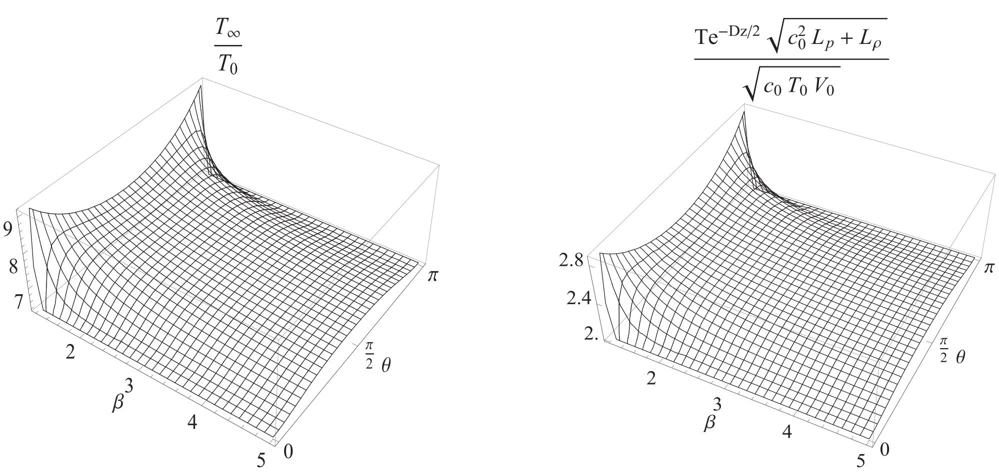
If $D > 0$, v_A enlarges at large distances $zD \gg 1$ as $Ce^{Dz/2}\sqrt{\frac{V_0 D T_0}{\varepsilon}}$, and T enlarges as $\frac{e^{Dz/2}}{C}\sqrt{\frac{\varepsilon V_0 T_0}{D}}$. Hence, while T tends to the limiting value T_∞ for $D < 0$, it increases infinitely for $D > 0$. For magnetosound perturbations such as a wave process, condition (2.6) must be satisfied with variable characteristic duration of an impulse, T. It might be broken with increase of the period, especially probably for positive D and large β, $\beta > \frac{2}{\gamma}$. Figure 5(a) shows the limiting duration of the triangular impulse in the case $D < 0$. The surface is determined by the value $\frac{V_0 c_0}{T_0(c_0^2 L_p + L_\rho)}$ which in fact is the ratio of the Mach number of a signal at a transducer in the unmagnetized plasma, $M = \frac{V_0}{c_0}$, and a parameter responsible for deviation from the adiabaticity of a flow, $\frac{(c_0^2 L_p + L_\rho)T_0}{c_0^2}$. The ratio may vary in dependence on a balance between nonlinearity and nonisentropicity of a flow. Inequality Eq. (2.7) is valid for a wave processes with any β and θ, and M is a small parameter, at least for the fast magnetosound perturbations. Figure 5(b) concerns the case $D > 0$. Evolution of v_A at large distances from a transducer, $Dz \gg 1$, is shown in Fig. 6. The kinetic energy of the unit mass of a plasma averaged over the duration of an impulse, equals

$$E = \frac{1}{T}\int_0^T v^2(\tau, z)d\tau = E_0\frac{e^{2Dz}}{1 + \frac{\varepsilon V_0(e^{Dz} - 1)}{DC^2 T_0}}, \quad D \neq 0,$$
$$E = \frac{E_0}{1 + \frac{\varepsilon V_0 z}{C^2 T_0}}, \quad D = 0.$$

It tends to zero proportionally to e^{2Dz} when $|D|z$ tends to infinity for negative D and increases as $\frac{e^{Dz} DC^2 T_0}{\varepsilon V_0}E_0$ for positive D. An impulse is more effective in transferring of energy than the periodic sawtooth signal.

IV. CONCLUDING REMARKS

The underlying assumption in this study is that the system of ideal MHD equations is applicable. Hence, spatial and temporal scales of perturbations in a flow must be much larger than gyrokinetic scales. The model imposes equal temperature of ions and electrons, the Maxwellian distribution function for particle momenta, and deals with plasma's equilibrium quantities. The MHD system does not consider relativistic and quantum effects.[22,23] The ideal gas equation of state completes the system of MHD equations. Most astrophysical plasmas



(a) (b)

FIG. 5. (a) The limiting duration of the triangular impulse T_∞ in the case $D < 0$ and fast magnetosound perturbations. The plot corresponds to $\frac{V_0 c_0}{(c_0^2 L_p + L_\rho)T_0} = -10$. (b) The case $D > 0$, fast magnetosound perturbations, and duration of the triangular impulse at large distances from a transducer, $Dz \gg 1$.

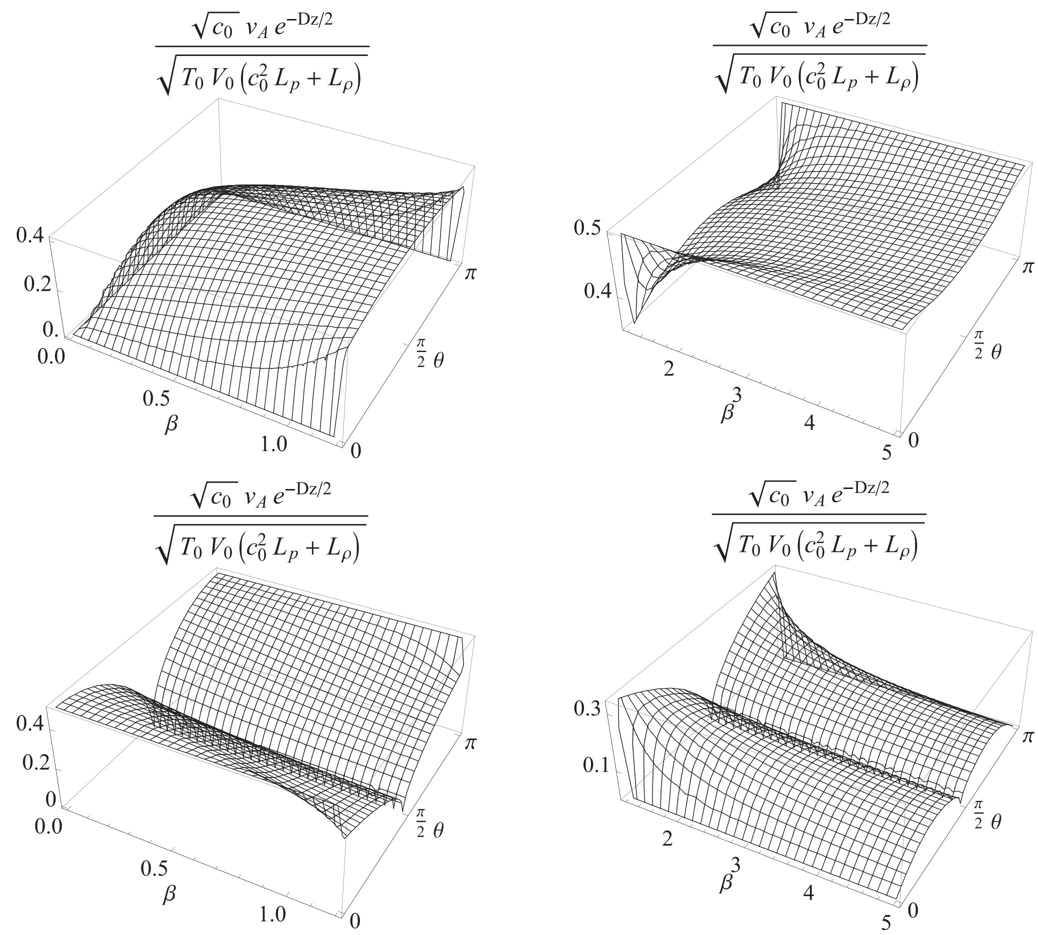


FIG. 6. The case $D > 0$ and fast magnetosound perturbations (top row) and slow magnetosound perturbations (bottom row). The amplitude of velocity in the triangular impulse at large distances from a transducer, $Dz \gg 1$. Left panels: $0 \le \beta < 2/\gamma$; right panels: $2/\gamma < \beta$.

are well described by the equation of state for an ideal gas. The contribution of the interparticle potential energies is less than ten percent of the kinetic energies in a fully ionized gas. The recent studies demonstrate that even coupled system of particles can be described by the equation of state for an ideal gas.[24] Ideal gas law is no longer valid in very cool and dense plasmas, or in the case of quantum degeneracy.

The exact solution of Eq. (3.2), which is sinusoidal at $z = 0$ with period T_0, reads

$$v_z = V_0 \exp(Dz) \sum_{n=1}^{\infty} \frac{2J_n(nK^{-1}(\exp(Dz)-1))\sin(2\pi n(t - z/C)/T_0)}{nK^{-1}(\exp(Dz)-1)}, \quad D \neq 0,$$

$$v_z = V_0 \sum_{n=1}^{\infty} \frac{2J_n\left(\frac{2\pi\varepsilon V_0 z}{C^2 T_0} n\right)\sin(2\pi n(t - z/C)/T_0)}{\frac{2\pi\varepsilon V_0 z}{C^2 T_0} n}, \quad D = 0, \qquad (4.1)$$

where $K = \frac{DC^2 T_0}{2\pi\varepsilon V_0}$. It is valid before formation of a discontinuity,[6] that is, if

$$0 < z < z_{sh} = ln(1+K)D^{-1}, \quad D \neq 0,$$
$$0 < z < z_{sh} = \frac{C^2 T_0}{2\pi\varepsilon V_0}, \quad D = 0.$$

A discontinuity always forms in acoustically active flows with $D > 0$ at the distance z_{sh}, and does not form at all if $K \le -1$.

In this study, the sawtooth like signals are considered. Once a discontinuity has formed at a transducer, it spreads without destruction independently of acoustical activity of a flow. This is important difference with respect to the case of initially harmonic excitation that may lead to formation of a discontinuity at some distance from a transducer. The signal remains triangular with variable amplitude and duration (in the case of an impulse). The conclusions relate to a flow without mechanical and thermal losses, and electrical resistivity. Equation (3.2) supplemented by the terms responsible for viscous and thermal damping, takes the leading-order form

$$\frac{\partial V}{\partial Z} - \frac{\varepsilon}{C^2} V \frac{\partial V}{\partial \tau} - \frac{\alpha}{2C^3(ZD+1)\rho_0} \frac{\partial^2 V}{\partial \tau^2} = 0, \qquad (4.2)$$

where α is responsible for attenuation of sound due to the bulk and shear viscosity, thermal conduction, and finite electrical conductivity

$$\alpha = \frac{C^4 + C^2(6c_0^2 - C_A^2) - 3c_0^2(c_0^2 + C_A^2)}{3c_0^2(2C^2 - c_0^2 - C_A^2)}\eta + \frac{C^2(C^2 - C_A^2)}{c_0^2(2C^2 - c_0^2 - C_A^2)}\xi + \frac{C^2 - C_A^2}{2C^2 - c_0^2 - C_A^2}\left(\frac{1}{C_V} - \frac{1}{C_p}\right)\kappa + \frac{C^2 - c_0^2}{2C^2 - c_0^2 - C_A^2}\frac{\rho_0}{\mu_0\sigma}.$$

The quantities η, ξ, and κ designate shear viscosity, bulk viscosity, and thermal conductivity, respectively, and C_V, C_p is the heat capacities under constant volume and pressure; σ is the electrical conductivity of a fluid (the reciprocal of electrical resistivity). If $D=0$, Eq. (4.2) rearranges to the Burgers equation which may be solved exactly.[8,19,21] Equation (4.2) contains a variable with the distance from a transducer damping factor by $\frac{\partial^2 V}{\partial \tau^2}$ tending to zero if $D>0$. Hence, the conclusions of this study are longer valid in acoustically active flows since the nonlinear effects increasingly prevail over attenuation. If the effective Reynolds number of a flow is initially high, $Re = \frac{V_0 C T_0}{\alpha} \gg 1$, it holds high at any distance from a transducer and a flow is mostly governed by nonlinearity. With a high degree of accuracy, the conclusions are valid in the equilibrium flow with $D<0$ when nonlinear effects remain stronger than attenuation, that is, at distances from a transducer satisfying inequality $z \ll \frac{1}{D}\ln\left(\frac{Re^{-1}}{\varepsilon}\right)$, which in turn demands the large initial Reynolds number.

This study considers ideal MHD flow of the planar geometry with constant equilibrium magnetic strength forming a constant angle θ with the wave vector, and weak distortions associated with nonisentropicity of wave perturbations over their characteristic duration. There is no restriction concerning strength of the magnetic field in this study, and hence, on the plasma-β. Features of waves with discontinuities are studied as functions of θ and plasma-β. The results may be addressed to different kinds of the function $L(p, \rho)$, and to low plasma parameter β (for example, a cold plasma in the inner magnetosphere), or finite plasma-β (for example, a rarefied plasma of the outer magnetosphere affected by a weak magnetic field). It is assumed that $L(p_0, \rho_0)=0$. This is justified by the weakness of deviation from adiabaticity. Otherwise, the background thermodynamic parameters vary, and the mathematical description becomes fairly difficult.

This paper brings out some new interesting features of dynamics of initially sawtooth periodic and impulsive signals in a plasma. Particularly, in equilibrium flows, the magnitude of an impulse tends to zero, but its duration tends to some limit at large distances from a transducer. The amplitude of periodic velocity in acoustically active flow tends to a limit that does not depend on initial magnitude, but on the equilibrium magnetic strength, angle between the magnetic strength, and the wave vector, β-plasma, and a kind of heating-cooling function. So there is an autowave form, with amplitude that is fully prescribed by the plasma and is independent from the initial conditions. Chin *et al.* have analyzed the evolution of the sinusoidal signal numerically for some parameters of a flow and established the stationary asymptotic solution in thermoconducting active flow analytically (approximately, in the form of the shock wave, despite thermal conductivity preventing formation of the exact sawtooth). Its amplitude is given by Eq. (3.2) from Ref. 9 and coincides with Eq. (3.6) for weak thermal conduction, having in mind that $\mu_1 = -CD$, and $\lambda = CT_0$. Equation (3.5) describes not only the asymptotic solution at large distances, but at any distance, and refers to the periodic sawtooth signal at a transducer. The particular case

$$\frac{2\varepsilon V_0}{DC^2 T_0} = 1$$

leads to a periodic waveform with the amplitude $v_A = V_0$ that is independent of z.

The character of propagation of periodic or impulsive perturbations may be useful in the analysis of plasma parameters and processes within, which are difficult or even impossible for direct measurement. This is so-called seismological techniques that could be applied in remote prediction of plasma's features, in particular, in coronal plasma. The idea of making use of wave parameters in the detection of properties of the medium is rapidly developing and promising.[25]

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