Reexamination of determinant-based separability test for two qubits

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It was shown in Augusiak *et al.* [Phys. Rev. A **77**, 030301(R) (2008)] that discrimination between entanglement and separability in a two-qubit state can be achieved by a measurement of a single observable on four copies of it. Moreover, a pseudoentanglement monotone π was proposed to quantify entanglement in such states. The main goal of this Brief Report is to show that the close relationship between π and concurrence reported there is a result of sharing the same underlying construction of a spin-flipped matrix. We also show that monogamy of entanglement can be rephrased in terms of π and prove the factorization law for π.

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Entanglement, first recognized by Schrödinger and Einstein *et al.* [1], lies at the heart of quantum-information theory. Without a doubt, it is the most important resource of this rapidly developing branch of science and serves as the building block for the huge number of information tasks, just to mention, for example, teleportation [2] and dense coding [3]. From this point of view, full recognition of this ‘spooky action at a distance’ [4] is fundamental for our understanding of quantum mechanics. Much effort has been put into recognizing its nature and, not surprisingly, major progress has been achieved in the case of the simplest bipartite quantum states—states of two qubits. One of the most important qualitative results concerning such systems is the necessary and sufficient condition for inseparability—the celebrated Peres-Horodeckis criterion of nonpositive partial transposition [5]. On the other hand, research toward quantitative description of entanglement of two-qubit states has culminated in the introduction of entanglement measures, among which the most notable are entanglement of formation [6] and concurrence for whom closed expressions have been found [7]. Unfortunately, both of them still have not been shown to be directly measurable, and it is reasonable to conjecture that they are not, in general. Very recently, however, it has been demonstrated that *single* collective measurement of the specially prepared observable on four copies of an unknown two-qubit state can *unambiguously* discriminate between entanglement and separability, additionally *quantifying*, to some extent, entanglement contained in the system by providing sharp lower and upper bounds on concurrence [8].

In this Brief Report, we continue research on the pseudoentanglement monotone π, which was introduced in Ref. [8] for entanglement quantification purposes.

Let us start with an introduction of necessary concepts. Consider a two-qubit mixed state ρ_{AB}. Define (conjugation in a standard basis) a spin-flipped state [7],

$$\tilde{\rho}_{AB} = \sigma_y \otimes \sigma_y \rho^*_{AB} \sigma_y \otimes \sigma_y. \tag{1}$$

Let $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \lambda_4$ be the square roots of the eigenvalues of $\rho_{AB}\tilde{\rho}_{AB} := M_{AB}$. Note that we can safely write inequalities since they are real (moreover, they are non-negative). We define concurrence to be

$$C(\rho_{AB}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} =: C_{AB}. \tag{2}$$

Eligibility of such a constructed quantity to be a good measure of entanglement is justified by the invariance of the eigenvalues λ_i under local unitary operations and by the fact that $0 \leqslant C_{AB} \leqslant 1$ with extreme values taken on separable and maximally entangled states, respectively. When ρ_{AB} is a partial trace over C from the tripartite pure qubit state $|\psi_{ABC}\rangle\langle\psi_{ABC}| =: \psi_{ABC}$, there are only two nonzero eigenvalues; thus, we just have $C_{AB} = \lambda_1 - \lambda_2$. Then, we also define tangle [9] to be

$$\tau_{ABC} = 4\lambda_1\lambda_2. \tag{3}$$

It was shown that eigenvalues of either of the matrices M_{AB}, M_{AC}, M_{BC} can be used in the above. These quantities can be combined to give the so-called monogamy relation [9,10]

$$C^2_{AB} + C^2_{BC} + \tau_{ABC} = C^2_{B(AC)} = 4 \det \rho_B, \rho_B = \mathrm{tr}_A\, \rho_{AB}. \tag{4}$$

Concurrence $C_{B(AC)}$ is the meaningful quantity since we consider the pure state of three qubits; thus, effectively $B(AC)$ is a two-qubit-like state. This relation provides an interpretation for tangle as a measure of tripartite correlations.

One also defines [11] concurrence of assistance C^a, which is the maximum over ensembles of average concurrence of pure states in the ensemble. In the case of two qubits, we just have $C^a = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$.

In Ref. [8], it was shown that the separability of an unknown two-qubit state ρ can unambiguously be settled in a single collective measurement on four copies of this state (i.e., one needs, at one time, $\rho^{\otimes 4}$). This was obtained on the basis of two facts: (i) Partially transposed density matrix ρ^Γ of an entangled two-qubit state ρ is full rank (has four nonzero eigenvalues), (ii) there can only be one negative eigenvalue of ρ^Γ. The preceding led to the conclusion that it is sufficient to measure $\det \rho^\Gamma$, and the strict negativity of the latter indicates entanglement. The authors of the mentioned paper showed that, indeed, such a measurement is possible using a noiseless circuit [12]. They also proposed a simple alternative scheme to measure this determinant. The question of the usage of $\det \rho^\Gamma$ for quantitative description was addressed further. It

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was shown that the quantity, which, in this Brief Report, we will call the *determinant–based measure*,

$$\pi(\rho) = \begin{cases} 0, & \text{for} \quad \det \rho^\Gamma \geqslant 0 \\ 2\sqrt[4]{|\det \rho^\Gamma|}, & \text{for} \quad \det \rho^\Gamma < 0 \end{cases}. \tag{5}$$

is a monotone under pure local operations preserving dimensions and classical communication and provides tight upper and lower bounds on concurrence as follows:

$$C(\rho) \leqslant \pi(\rho) \leqslant \sqrt[4]{C(\rho)\left(\frac{C(\rho)+2}{3}\right)^3}. \tag{6}$$

Normalization in Eq. (5) is chosen to impose agreement of determinant–based measure and concurrence on pure states. From the previous inequalities, we also have immediate bounds for entanglement of formation E_f [7] as follows: $E(r^{-1}(\pi(\rho))) \leqslant E_f(\rho) \leqslant E(\pi(\rho))$, where $E(x) = H(\frac{1+\sqrt{1-x^2}}{2})$ with $H(y)$ as the Shannon entropy of a probability distribution $(y, 1-y)$ and $r(x) = \sqrt[4]{x(\frac{x+2}{3})^3}$. One can also prove that π shares the nice property of being continuous in the input density operator [13].

For the purpose of this Brief Report, we propose the extension of our definition for entanglement between qubit A and qubits BC in a pure state ψ_{ABC} to $\pi_{A(BC)} \equiv 2\sqrt{\det \rho_A}$ (i.e., we define it to be equal to $C_{A(BC)}$ on such states). Such an extension is the most natural since we keep the two most important properties of π: mentioned equality with concurrence and the possibility of direct measurement.

Let us now turn to the main body of this Brief Report. We start with considerations analogous to the one from Ref. [14], where the local unitary interaction of one part of the maximally entangled state ψ^+_{AB} with two level environments E was considered [i.e., the global state after the evolution was $|\psi_{ABE}\rangle = \mathbb{I}_A \otimes U_{BE}(|\psi^+\rangle_{AB} \otimes |0\rangle_E)$].

Its bipartite reductions of interest will be denoted as ρ_{AB} and ρ_{AE}. In the mentioned paper, the author showed, by random sampling, that there is no correlation between singlet fraction [15] $F(\rho_{AB}) := F_{AB}$ after the action of the channel and concurrence C_{AB} of the decohered state and showed analytically, for the chosen class of channels, that $F_{AB} = \frac{1}{4}(1 + C_{AB})(1 + \sqrt{1 - C^2_{AE}})$. It turned out that this relation held true for all channels, which was shown by random generation of channels.

We pursue the same approach using the determinant–based measure instead of concurrence. First, let us consider the relation of F_{AB} and π_{AB} after the action of the random channel. The result is shown in Fig. 1, which was obtained by random generation of 10 000 U_{BE}'s [16].

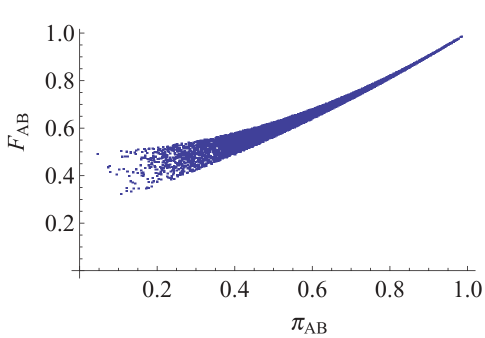

FIG. 1. (Color online) Singlet fraction F_{AB} vs determinant–based measure π_{AB} after action of the local channel.

In our case, one can see that there is some connection between these two quantities; however, there still is no analytical formula linking them. We will comment on this connection later when monogamy equality will be obtained.

Following Ref. [14], let us consider a class of local channels implemented by unitaries defined by

$$|00\rangle_{BE} \to \sqrt{1-q}|00\rangle_{BE} + \sqrt{q}|11\rangle_{BE}, \tag{7}$$

$$|10\rangle_{BE} \to \sqrt{1-p}|10\rangle_{BE} + \sqrt{p}|01\rangle_{BE}. \tag{8}$$

For such channels, we obtain the following (with previously established notation):

$$\pi_{AB} = \sqrt{|p+q-1|}, \tag{9}$$

$$\pi_{AE} = \sqrt{|p-q|}, \tag{10}$$

$$F_{AB} = \begin{cases} \frac{2-p-q+2\sqrt{(1-p)(1-q)}}{4} & \text{for} \quad p+q-1<0 \\ \frac{p+q+2\sqrt{pq}}{4} & \text{for} \quad p+q-1 \geqslant 0 \end{cases}. \tag{11}$$

Direct calculation reveals that

$$F_{AB} = \frac{1}{4}(1+\pi^2_{AB})\left(1 + \sqrt{1 - \frac{\pi^4_{AE}}{(\pi^2_{AB}+1)^2}}\right). \tag{12}$$

As in the case of concurrence, the relation we obtained can be shown to hold for all channels and is independent of whichever maximally entangled state we choose to be the input. Note the close resemblance of both forms.

The closed formula for the singlet fraction using determinant–based measure π opens up hope for a monogamy relation of entanglement in terms of it. In what follows, we prove the existence of such an equation.

Consider a pure state of three qubits ψ_{ABC}. As shown [17], as far as the entanglement properties are concerned, such a state can be parametrized by five real numbers as

$$|\psi_{ABC}\rangle = \gamma_0|000\rangle + \gamma_1 e^{i\varphi}|100\rangle + \gamma_2|101\rangle + \gamma_3|110\rangle + \gamma_4|111\rangle, \tag{13}$$

with $\gamma_i \geqslant 0$, $\sum_i \gamma_i^2 = 1$, and $\varphi \in \langle 0,\pi\rangle$. From this, we obtain eigenvalues of the matrix M_{AB} and the determinant of the partially transposed matrix of the reduced state of qubits A and B,

$$\lambda_1^2 = \gamma_0^2\left(2\gamma_3^2 + \gamma_4 + 2\gamma_3\sqrt{\gamma_3^2+\gamma_4^2}\right), \tag{14}$$

$$\lambda_2^2 = \gamma_0^2\left(2\gamma_3^2 + \gamma_4 - 2\gamma_3\sqrt{\gamma_3^2+\gamma_4^2}\right), \tag{15}$$

$$\det \rho^\Gamma_{AB} = -\gamma_0^4\gamma_3^2\left(\gamma_3^2+\gamma_4^2\right), \tag{16}$$

which immediately yields

$$\pi_{AB} = \sqrt{\lambda_1^2 - \lambda_2^2}. \tag{17}$$

Recalling Eqs. (2) and (3), we obtain an analytical relationship among π_{AB}, C_{AB}, and τ_{ABC} in a pure three-qubit state,

$$\pi_{AB} = \sqrt{C_{AB}\sqrt{C^2_{AB} + \tau_{ABC}}}. \tag{18}$$

This can be put into a nice compact form

$$\pi_{AB} = \sqrt{C_{AB} C^{a}_{AB}}, \tag{19}$$

which means that, in the case of rank 2 states, the determinant–based measure is the geometric mean value of concurrence and concurrence of assistance. We also conclude that the bound in Eq. (6) can be tightened for such states to obtain

$$\pi_{AB} \leqslant \sqrt{C_{AB}}. \tag{20}$$

Equation (18) leads us to a simple corollary stating that, for a given pure three-qubit state ψ_{ABC}, one has $\pi_{AB} = C_{AB}$ if and only if $C_{AB} = 0$ or $\tau_{ABC} = 0$. With the results of Ref. [18], this means that both measures agree when ψ_{ABC} is either of the following classes: GHZ with separable reduction AB, W, biseparable, or a product.

One can now argue that the pattern in the plot of singlet fraction (Fig. 1) is the result of quantifying, to some extent, tripartite correlations by the determinant–based measure.

Now, let us reverse Eq. (18) to get

$$C^2_{AB} = \frac{-\tau_{ABC} + \sqrt{\tau^2_{ABC} + 4\pi^4_{AB}}}{2}. \tag{21}$$

Inserting this into Eq. (4), one obtains the advertised elegant monogamy relation in terms of the determinant–based measure,

$$\sqrt{\left(\frac{\tau_{ABC}}{2}\right)^2 + \pi^4_{AB}} + \sqrt{\left(\frac{\tau_{ABC}}{2}\right)^2 + \pi^4_{BC}} = \pi^2_{B(AC)}. \tag{22}$$

This also gives the recipe to measure tangle on ten copies of the state relying directly on the measurements of the determinants of two partially transposed density matrices (four plus four copies) and the determinant of the reduced qubit density matrix (two copies). The question of the optimality of such measurements is beyond the scope of this Brief Report (see Ref. [19]).

Now, we will argue that the determinant–based measure π in a general case of mixed states of arbitrary rank is an analytical function of the eigenvalues of the matrix M. Consider states,

$$\rho_{B\,\mathrm{diag}} = p_1\psi_+ + p_2\psi_- + p_3\phi_+ + p_4\phi_-, \tag{23}$$

which are diagonal in the Bell basis $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, $|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$. Such states are entangled iff one of the probabilities is larger than $\frac{1}{2}$. Without loss of generality, we assume that it holds for p_1. One then has

$$\pi(\rho_{B\,\mathrm{diag}}) = \sqrt[4]{|-p_1 + p_2 + p_3 + p_4|(p_1 - p_2 + p_3 + p_4)(p_1 + p_2 - p_3 + p_4)(p_1 + p_2 + p_3 - p_4)}. \tag{24}$$

We also easily compute that $\lambda_i = p_i$. Motivated by the form of π for $\rho_{B\,\mathrm{diag}}$, we further define

$$C_1 = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \equiv C, \tag{25}$$

$$C_2 = \lambda_1 - \lambda_2 + \lambda_3 + \lambda_4, \tag{26}$$

$$C_3 = \lambda_1 + \lambda_2 - \lambda_3 + \lambda_4, \tag{27}$$

$$C_4 = \lambda_1 + \lambda_2 + \lambda_3 - \lambda_4, \tag{28}$$

and

$$\hat{\pi}(\rho) = \sqrt[4]{C_1 C_2 C_3 C_4}. \tag{29}$$

Notice that C_2, C_3, C_4 are always non negative.

The main result of this part of this Brief Report is the following.

Theorem 1. For any two-qubit states ρ, one has

$$\pi(\rho) = \hat{\pi}(\rho). \tag{30}$$

Proof. Let A and B be non singular local filters. The initial state ϱ_1 after the transformation under these filters is $\varrho_2 = (1/p) A \otimes B \varrho_1 A^\dagger \otimes B^\dagger$, where $p = tr A \otimes B \varrho_1 A^\dagger \otimes B^\dagger$. It follows from the results of Ref. [20] that $C_i(\varrho_2) = (|\det AB|/p) C_i(\varrho_1)$. Moreover, it holds true that $\pi(\varrho_2) = (|\det AB|/p)\pi(\varrho_1)$ [8]. Assume now that ϱ is a rank 4 state. It was shown [20] that such states can be reversibly obtained with A, B from a Bell diagonal state ϱ_{Bdiag}. As we already know, the assertion of the theorem is true on the latter. We thus have $\pi(\varrho) = (|\det AB|/p)\sqrt[4]{\Pi_i C_i(\varrho_{Bdiag})}$ and because of the transformation rule for C_i it follows that for full rank states it holds $\pi(\varrho) = \hat{\pi}(\varrho)$. For singular states, we can take their full rank perturbations $\sigma_\epsilon = \epsilon\varrho + (1-\epsilon)\mathbb{I}/2$. Then, $\pi(\sigma_\epsilon) = \hat{\pi}(\sigma_\epsilon)$ and we can apply the preceding argument and take the limit $\epsilon \to 0$. The result then follows from continuity of π and C_i. ■

We see that, π can be regarded as some kind of symmetrization of concurrence allowing for experimental direct accessibility. The natural question is to what extent does determinant–based measure also quantify tripartite correlations in the general case. Unfortunately, we have not been able to find a definite answer so far [21].

At the end, we prove the factorization law, which was originally stated for concurrence [22].

Theorem 2. Determinant–based measure π obeys the factorization law, that is, for an arbitrary channel Λ, a pure state ϕ, and a Bell state ψ_+, it holds

$$\pi(\mathcal{I} \otimes \Lambda(\phi)) = \pi(\mathcal{I} \otimes \Lambda(\psi_+))\pi(\phi). \tag{31}$$

Proof. The assertion is trivially true for separable $\mathcal{I} \otimes \Lambda(\psi_+)$ (i.e., when Λ is entanglement breaking) so we may assume entanglement of the latter. Any state $|\phi\rangle$ can be written as $A \otimes \mathbb{I}(|\psi_+\rangle)$ with $tr A^\dagger A = 2$. We then have $\pi(\mathcal{I} \otimes \Lambda(\phi)) = 2\sqrt[4]{|\det[\mathcal{I} \otimes \Lambda(A \otimes \mathbb{I}(\psi_+)A^\dagger \otimes \mathbb{I})]^{\Gamma_B}|} = 2\sqrt[4]{|\det[A \otimes \mathbb{I}(\varrho_\Lambda^{\Gamma_B})A^\dagger \otimes \mathbb{I}]|}$, where $\varrho_\Lambda = \mathcal{I} \otimes \Lambda(\psi_+)$ and we have used the fact that $(X \otimes \mathbb{I}\varrho Y \otimes \mathbb{I})^{\Gamma_B} = X \otimes \mathbb{I}\varrho^{\Gamma_B} Y \otimes \mathbb{I}$. Using now the following property of the determinant $\det XY = \det X \det Y$ and the fact that $\pi(\phi) = |\det A|$ [8] we arrive at $\pi(\mathcal{I} \otimes \Lambda(\phi)) = \pi(\phi)\pi(\mathcal{I} \otimes \Lambda(\psi_+))$ which is the desired. ■

Thus, the determinant–based measure provides a *factorizable* measurable bound on concurrence (see Ref. [23] for a recent attempt in this direction).

We have not been able to find an analytical proof of the extension of the factorization law to the mixed state domain, as it was in [22], nevertheless, by random sampling, we have verified that such an extension is indeed valid, that is $\pi(\mathcal{I}\otimes\Lambda(\varrho))\leqslant\pi(\varrho)\pi(\mathcal{I}\otimes\Lambda(\phi_+))$.

In conclusion, we have provided a monogamy relation for entanglement quantified by the determinant–based measure π. As a by-product, we obtained explicit formulas for the latter in terms of other entanglement quantities. We showed that a close relation with concurrence is the result of bearing the similar construction in its roots. We also provided evidence that the disagreement of π and C on general mixed states stems from the fact that π quantifies, to some extent, both bipartite and tripartite correlations. The natural question motivated by the result of this Brief Report is about the possibility of constructing other measurable quantifiers of entanglement, which are based on the analogous procedure and provide better bounds on the concurrence of an unknown state. We hope our results will stimulate research on this topic and will provide some tools for the improved understanding of two-qubit entanglement. The issue of using the determinant–based measure for detecting and quantifying entanglement in higher-dimensional systems is the subject of ongoing research [24].

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