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Scattering and Propagation Analysis for the Multilayered Structures Based on Field Matching Technique

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***Abstract*—A semi-analytical method is employed to the analysis of scattering and guiding problems in multilayer dielectric structures. The approach allows to investigate objects with arbitrary convex cross section and is based on the direct field matching technique involving the usage of the field projection at the boundary on a fixed set of orthogonal basis functions. For the scattering problems the scattered field in the far zone is calculated and for the guiding problems a complex root finding algorithm is utilized to determine the propagation coefficients. The results are compared with the alternative solutions in order to verify the validity of the proposed method.**

***Index Terms*—Cylindrical guides, Cylindrical structure, Field matching, Propagation, Root finding, Scattering.**

I. Introduction

Scattering and propagation problems are important issues in the analysis and design of microwave and optical devices. There are many methods and mathematical models which allow to analyze such structures with various efficiency and accuracy. For structures with simple geometries (e.g. cylindrical or elliptical) the analytical methods such as mode matching technique can be applied [1]–[5], which are characterized by high efficiency and low numerical costs. However, these methods are dedicated only to a specific structural shape and become difficult to use when the object shape is complex. The integral equation methods [6]–[8] can be utilized for the investigation of scattering from almost any obstacle shape. However, from a numerical point of view the efficiency of these methods depends on the choice of electric and magnetic current bases and accuracy of calculation of Green's function. The most popular methods nowadays, for the analysis of fully arbitrary structures, are space discretization techniques, such as finite element method (FEM) [9] or finite difference (FD) method [10]. However, their high numerical cost results in low efficiency of the solution. Recently, the field matching technique has been developed and utilized to the analysis of scattering [11] and propagation problems [12]. The methods is semi-analytical and the solution is based on the decomposition of the fields in the area of the structure into Fourier-Bessel series with unknown coefficients. An important advantage of this approach is also simple and intuitive implementation. Although it is restricted to the convex shapes of the structure's cross section (due to the assumed field representation), it allows analysis of a wide spectrum of electromagnetic problems. In this paper we utilize this method to the investigation of multilayer structures. For this purpose a T-matrix approach is used. Several examples of scattering and guiding structures are investigated and the results are verified with the finite element method calculations.

II. Formulation of the problem

The investigated structure is a multilayered cylindrical object of arbitrary, however convex, cross section as illustrated in Fig. 1, which is invariant along z axis. It is assumed that the structure is composed of L different dielectric layers and $L+1$ layer is the outer region (commonly air).

In order to investigate both scattering and propagation problem we utilize the semi-analytical field matching method, which requires to describe the electromagnetic fields in each layer of the structure by appropriate set of functions. As proposed in [11] we utilize a set of cylindrical functions and therefore the z components of the electric and magnetic fields in lth region have the following form (suppressing $e^{j\omega t}$ time dependence):

$$F_z^l = \sum_{m=-M}^{M} \left(A_m^{F,l} J_m(\kappa_l \rho) + B_m^{F,l} H_m^{(2)}(\kappa_l \rho) \right) e^{jm\phi} e^{-\gamma z} \quad (1)$$

where $F = \{E, H\}$, $\kappa_l^2 = \omega^2 \mu_l \varepsilon_l + \gamma^2$, ω is the angular frequency, $J_m(\cdot)$ and $H_m^{(2)}(\cdot)$ are Bessel and Hankel functions, respectively, of order m and A_m^F and B_m^F are unknown field coefficients. γ is the mode propagation coefficient in the case of guide problem or $\gamma = k_0 \cos(\theta_0)$ in the case of plane wave scattering problem, where k_0 is a wavenumber of free space and θ_0 is an angle of plane wave incidence, defined with respect to the z-axis. The other components of the electric and magnetic fields can be derived from Maxwell's equations [13]. As stated in [11] only convex shapes of the post or waveguide cross sections can be analyzed, due to the assumed field representation.

In order to determine scattered field or mode propagation coefficients we need to satisfy the continuity conditions for the tangential field components on the boundaries of the post or guide. Introducing functions $\rho^l = \varrho^l(s)$ and $\phi^l = \varphi^l(s)$, which describe the surface of the lth boundary (i.e. between lth and $(l+1)$th regions), where s is the curvilinear coordinate that

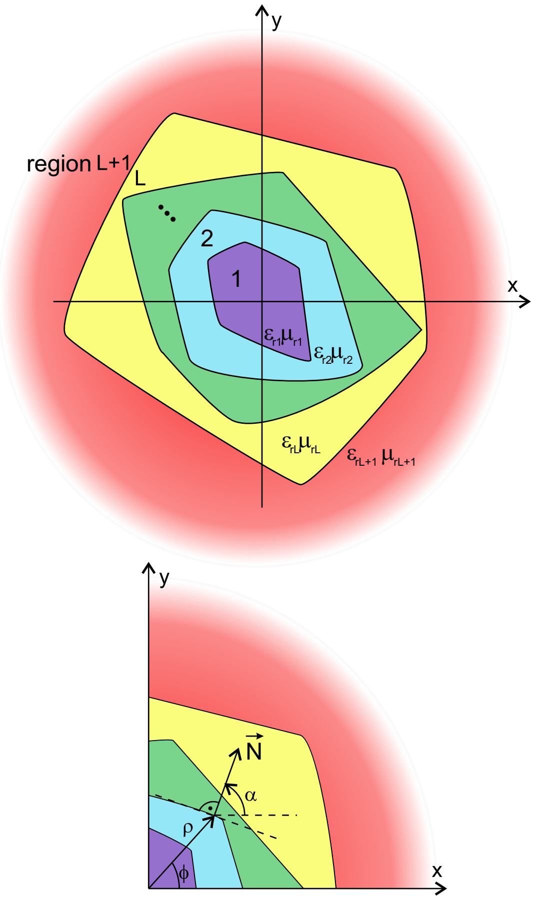

Fig. 1. The geometry of the investigated structure.

follows the surface, the continuity conditions for tangential components can be written as follows:

$$F_z^l(\varrho^l(s),\varphi^l(s),z) = F_z^{l+1}(\varrho^l(s),\varphi^l(s),z) \tag{2}$$

$$F_t^l(\varrho^l(s),\varphi^l(s),z) = F_t^{l+1}(\varrho^l(s),\varphi^l(s),z) \tag{3}$$

for $z \in \mathbb{R}$ and $s \in [0, S^l]$, where S^l is a total length of the lth boundary. Tangential field components $F_t^{(\cdot)}(\cdot) = (\sin\varphi\cos\alpha - \cos\varphi\sin\alpha)F_\rho^{(\cdot)}(\cdot) + (\cos\varphi\cos\alpha + \sin\varphi\sin\alpha)F_\phi^{(\cdot)}(\cdot)$ and $\alpha = \alpha(s)$ is an angle between the x-axis and the normal outgoing vector $\vec{N}$ to the cylinder surface.

Utilizing a projection on the orthogonal set of the functions $w_n^l(s) = \exp\left(j2\pi ns/S^l\right)/\sqrt{S^l}$ for $(n = -M \dots M)$ in the meaning of the inner product:

$$\langle g|w_n^l\rangle = \int_0^{S^l} g(s)w_n^l(s)^*\,ds \tag{4}$$

we can rewrite the continuity condition in the matrix form:

$$\mathbf{M}_A^{F,l,l}\mathbf{A}^l + \mathbf{M}_B^{F,l,l}\mathbf{B}^l = \mathbf{M}_A^{F,l+1,l}\mathbf{A}^{l+1} + \mathbf{M}_B^{F,l+1,l}\mathbf{B}^{l+1} \tag{5}$$

where the matrices $\mathbf{M}_{A(B)}^{F,\xi,\chi}$ and vectors $\mathbf{A}^\xi$ and $\mathbf{B}^\xi$ are defined in [11] for the scattering case and in [12] for the guiding problems; $\xi = \{1,\dots,L+1\}$ denotes the region and $\chi = \{1,\dots,L\}$ denotes the boundary between regions. It is worth noting that in the core region (region 1) only Bessel functions are involved, thus coefficients $B_m^{F,1} = 0$. In the outer region (region $L+1$) the coefficients $A_m^{F,L+1}$ are known for the case of scattering problem (as the incident wave is assumed e.g. plane wave), whereas these coefficients equal zero for the guiding problems (no sources).

Generalizing the procedure involving T-matrix formulation described in [11] we can calculate $\mathbf{T}^l$ matrix for each region, which represents the relation between coefficients (ingoing and outgoing waves) in $(l+1)$th region:

$$\mathbf{T}^l = \left(\mathbf{Z}^l\mathbf{M}_B^{H,l+1,l} - \mathbf{M}_B^{E,l+1,l}\right)^{-1}\left(\mathbf{M}_A^{E,l+1,l} - \mathbf{Z}^l\mathbf{M}_A^{H,l+1,l}\right) \tag{6}$$

where $\mathbf{Z}^l$ is an impedance matrix defined on lth boundary as follows:

$$\mathbf{Z}^l = \left(\mathbf{M}_A^{E,l,l} + \mathbf{M}_B^{E,l,l}\mathbf{T}^{l-1}\right)\left(\mathbf{M}_A^{H,l,l} + \mathbf{M}_B^{H,l,l}\mathbf{T}^{l-1}\right)^{-1} \tag{7}$$

The utilization of the T-matrix allows us to eliminate the field coefficients in each inner region (from 1 to L), and in result, for the scattering problem, we obtain the relation between coefficients in outer region as follows:

$$\mathbf{B}^{L+1} = \mathbf{T}^L\mathbf{A}^{L+1} \tag{8}$$

from which the scattered field in near or far zone can be easily obtained. For the guiding problem we utilize impedance relation and write:

$$\left(\mathbf{M}_B^{E,L+1,L} - \mathbf{Z}^L\mathbf{M}_B^{H,L+1,L}\right)\mathbf{B}^{L+1} = \mathbf{N}\mathbf{B}^{L+1} = 0 \tag{9}$$

Nontrivial solutions of the homogenous system (9) exist if the determinant of matrix $\mathbf{N}$ vanishes. The roots of this determinant represent complex propagation coefficients for particular modes and in order to find the solution the complex root finding algorithms [15]–[17] are employed.

An important advantage of the T-matrix approach utilization is the possibility of displacement of the chosen region (centers of the layers can be shifted) with the use of additional theorem of Bessel functions [14] by simple multiplication of T-matrix by translation matrix (from local coordinate system of the region to global coordinate system of the entire structure).

III. Results

In order to verify the validity of the presented approach, a few examples of electromagnetic field scattering and guiding structures have been analyzed. In the first and second examples, a plane wave scattering from a dielectric triangular-rectangular and rectangular-triangular posts is considered. The plane wave incidence angles $\theta_0 = 30°$, $\phi_0 = 30°$, and polarization rotation angle $\psi = 30°$ are assumed (see Fig. 2). For the assumed angle of incident the wave has both TE and TM components. The scattered fields in the far zone (at distance $100\lambda_0$) were calculated for two location of the inner post within the outer structure and the results are depicted in Figs. 3 and 4. In both cases the results were compared with the finite element method calculation and a good agreement was achieved.

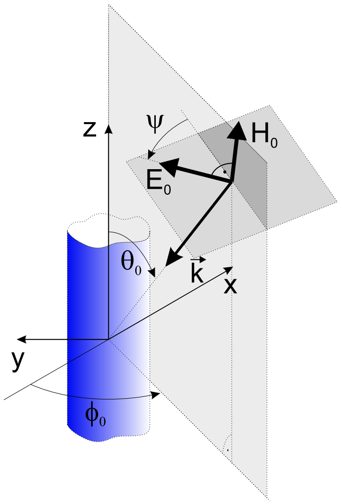

Fig. 2. Plane wave illumination at an arbitrary angle.

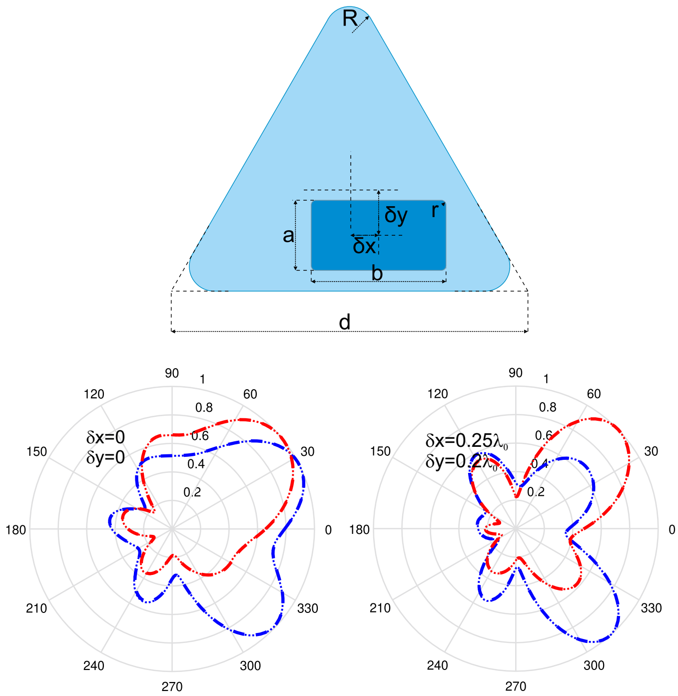

Fig. 3. Normalized amplitude of scattered electric (blue) and magnetic (red) fields from the dielectric ($\varepsilon_{r1} = 5$, $\varepsilon_{r2} = 2$, $\varepsilon_{r3} = 1$) triangular-rectangular post, for plane wave incidence angle $\theta_0 = 30°$, $\phi_0 = 30°$, and polarization rotation angle $\psi = 30°$ with the following dimensions: $a = \lambda_0/2$, $b = \lambda_0/4$, $r = \lambda_0/40$, $d = 2\lambda_0$, $R = \lambda_0/20$. Dashed line - this method, dotted line - finite element method.

The calculations were performed in a Matlab environment on an Intel Core i7-5930 3.5 GHz. In order to obtain convergent results it was sufficient to select $M = 25$ mode expansion functions and c.a. 700 integration points evenly covering the boundary to evaluate integrals (4) (trapezoidal rule). The calculation of a single frequency point took approximately

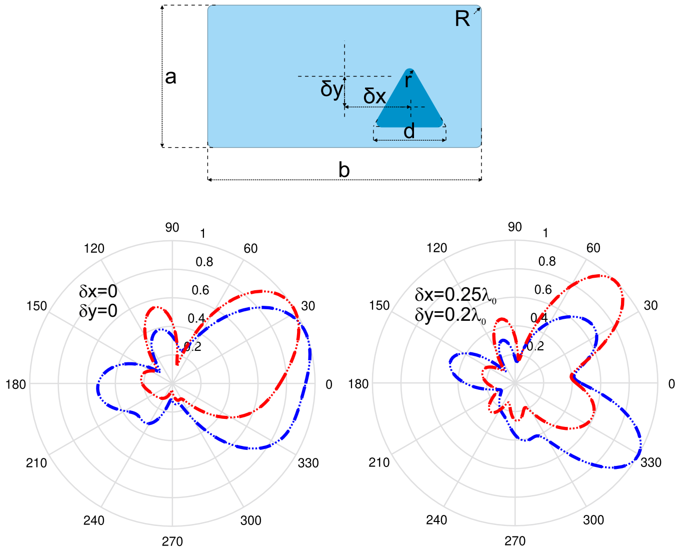

Fig. 4. Normalized amplitude of scattered electric (blue) and magnetic (red) fields from the dielectric ($\varepsilon_{r1} = 5$, $\varepsilon_{r2} = 2$, $\varepsilon_{r3} = 1$) rectangular-triangular post, for plane wave incidence angle $\theta_0 = 30°$, $\phi_0 = 30°$, and polarization rotation angle $\psi = 30°$ with the following dimensions: $a = \lambda_0$, $b = 2\lambda_0$, $r = \lambda_0/40$, $d = \lambda_0/2$, $R = \lambda_0/20$. Dashed line - this method, dotted line - finite element method.

1.1 s for both examples. The calculation time of a single frequency point with the use of finite element method (2.5D problem) was at least 10 times longer than with the use of the proposed method. Although the method is dedicated only to the restricted class of structures, the results clearly show that such a model is suitable for utilization in the optimization process.

The last example considers the triangular open waveguide with the cross section presented in Fig. 3 and dimensions $a = 0.5$ mm, $b = 15$ mm, $d = 30$ mm with displacement $\delta x = 7.5$ mm and $\delta y = 6$ mm and dielectric parameters $\varepsilon_{r1} = 2$, $\varepsilon_{r2} = 4$ and $\varepsilon_{r3} = 1$. The calculations were performed for the frequency $f_0 = 3$ GHz and the propagation coefficients of four first modes were calculated (two guided and two leaky modes). The results are presented in Table I. The calculations were performed using $M = 7$ mode expansion functions, and the integrals (4) were evaluated with the use of c.a. 700 points evenly covering the boundary contours. These values were sufficient to obtain convergent results. A satisfactory agreement between the results obtained from proposed method and finite element method used for comparison was achieved. Again in this case the calculation time utilizing finite element method (2.5D problem) was at least 10 times longer then with the use of proposed approach.

In the presented examples the structure cross-sections are assumed to have rounder corners, however the sharp corners can be analyzed as well. It requires denser discretization of the boundaries, which result in increase of calculation time (see [11], [12]). Also the dimensions of the presented structures are assumed to be comparable withe the wavelength, but this is

TABLE I
NORMALIZED PROPAGATION COEFFICIENTS OF THE INVESTIGATED WAVES IN TRIANGULAR-RECTANGULAR DIELECTRIC FIBER.

mode	this method	finite element method
guided mode 1	$j1.3569$	$j1.3579$
guided mode 2	$j1.3884$	$j1.3897$
leaky mode 1	$0.3176 + j0.7572$	$0.3122 + j0.7633$
leaky mode 2	$0.3438 + j0.74964$	$0.3392 + j0.7553$

not the limitation of the method. Electrically bigger structure can be also analyzed, however it requires the utilization of greater number of basis functions M.

IV. CONCLUSION

A simple semi-analitical method was proposed for the analysis both scattering and guiding problems of structures with arbitrary convex cross sections. Although only convex shapes can be analyzed, the results are obtained at least ten times faster than with the use of space discretization techniques, thus the method can be utilized in the design and optimization of the electromagnetic structures. The validity of the proposed approach has been verified for the case of scattering and guiding by comparing the results with other numerical techniques.

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