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Selected problems of decision making modelling in power engineering

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ABSTRACT

The paper presents the selected problems of decision making modelling in power engineering specially investment risk evaluation methods. The proposed model can be used in the range programming the development and investing process in power engineering. Decision making problems in power engineering and the evaluation of investment effectiveness in particular are closely related to modelling which relatively accurately reflects the complexity of market economy mechanisms. In order to analyze the phenomena mentioned above, models of existing processes, which are later used to achieve a set goal during decision making in a real situation, are applied. Its mathematical representation is a formalization of a model of a decision making task. A significant task in the whole process is to design a mathematical model, optimized by means of a goal function, whose arguments are decision variables meeting defined boundary conditions. Descriptions of decision making processes show that variables usually have non-negative values. Also the economic environment influences the quality of modelling. The dynamics of events causes that models of decision making processes can be analyzed. An attempt to take into account the influence of the factors mentioned above on the modelling of decision making in power engineering is presented in this paper.

Introduction

Decision making problems in electrical power engineering and the evaluation of investment effectiveness in particular are closely related to modelling which relatively accurately reflects the complexity of market economy mechanisms.

In order to analyze the phenomena mentioned above, models of existing processes, which are later used to achieve a set goal during decision making in a real situation, are applied.

The description of a decision making situation is generally speaking a decision making problem. Its mathematical representation is a formalization of a model of a decision making task.

A significant task in the whole process is to design a mathematical model, optimized by means of a goal function, whose arguments are decision variables meeting defined boundary conditions. Descriptions of decision making processes show that variables usually have non-negative values.

Also the economic environment influences the quality of modelling. The dynamics of events causes that models of decision making processes can be analysed in terms of definiteness, uncertainty, risk and fuzziness.

An attempt to take into account the influence of the factors mentioned above on the modelling of decision making in electrical power engineering is presented in the paper.

Formalization of decision-making

Decision making and its modelling result basically from the nature of a problem and a decision making situation. When there are many solution variants and only one must be chosen, a decision making problem appears. In the modelling and decision-making analysis of investment processes, the decision-maker should apply not one but many criteria simultaneously. For example, planned investments should incur minimal costs and maximize profits from every unit of shareholders' equity as well as income.

Generally speaking, a particular situation and knowledge about possible states, options, structures of sets of choices and results, factors, relations between choice criteria and others influence the modelling of decision-making problems.

In general, decision- making situations can be classified as follows:

- decision-making under conditions of certainty,
- decision-making under conditions of uncertainty,
- decision-making under conditions of risk,
- decision-making under conditions of vagueness (indefiniteness, ambiguity, fuzziness etc.),
- decision-making under conditions of ignorance.

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From the point of view of rational management, the decision-maker must choose between contradictory goals: maximisation of results – determined resources or minimisation of the use of resources – determined final results. Decision-makers often consciously or subconsciously formulate such goals.

The decision-maker often must make a choice from a defined set of solutions, which can be characterised by a finite set of features, by taking into account many criteria. The issues mentioned above concern multi-criteria decision-making tasks, in which many criteria are considered simultaneously using the Pareto approach, a *meta*-criterion based on the sum of values of partial criteria or the weighted sum of partial criteria or threshold techniques [1]. Another significant problem which often appears in the evaluation process is the decision-maker's sensitivity to differences in criteria values and their weights, and the decision-maker's personal attitude to the problem of choice as well as other non-criteria factors.

In the author's opinion, considering applications used to solve decision making problems, the methods of digital modelling and mixed models and methods are particularly interesting. Digital modelling (also called digital simulation) and mixed experimental methods connecting digital models, people, and possibly physical models may be particularly useful when considering decision making problems under conditions of uncertainty and a significant investment risk in the power sector. Problems related to ordering and classification of investment options, and decision-making in the case of many criteria can be resolved using multi-criteria programming and other techniques of solving decision-making problems. It is particularly important when assessing the risk of investing in the power industry. It is this issue that the next part of the paper is devoted to, where in subsection 4 the author's proposition of solving this problem is presented, extending the earlier concept which was presented in scientific journal energy conversion and Management, vol. 43, nr 4, 2002 [2]). Accounting for uncertainty in decision making processes

General comments

In the decision making practice one often has to deal with situations in which one does not know what the state of the environment will be, or what decisions will be taken on the market or what the results of the decisions will depend on.

Situations presented in the rest of the paper in a sense classify the problems of decision-making under uncertainty, in particular the problems of game theory. These situations are a starting point for a more detailed description of the problem issue. However, the analysis is limited to two basic cases, namely sum zero games (for example, in the case of a smart opponent) and game with nature (in the case of adverse effects minimization), always with "two participants", no coalition allowed. It is worth noticing that in relation to power engineering, "states of nature" are specific situations, decision-making in the energy market, and the "participant", "competitor", "opponent" are respectively a manufacturer, distributor or other participant in the energy market.

Referring to mentioned issues the structure of this section is arranged, as the following: general principles of the formulation of the rules of the game are presented, and the decision choice rules are described shortly.

General principles of the formulation of the rules of a game

It is assumed that in a game involving two players: (*A*) cautious (*B*) intelligent, each of them has the opportunity and make their own decisions independently. Assuming that participant *A* has *n* possibilities and participant *B* - *m* possibilities. For each pair (*i*, *j*) of participant *A*'s and *B*'s decisions, there is a known number a_{ij} denoting a win for participant *A* in the case when this player takes a decision *i*, and participant *B* - decision *j*. Assuming that the pay-off matrix of player A is a matrix of elements $[a_{ij}]$, and if a_{ij} is the number of participant *A*'s gains and participant *B*'s losses, we have a sum zero game.

It is easy to see that the sum of participant A's gains and participant B's losses (and this is equal to the negative value of the winning participant A) is equal to zero. It seems clear that the "cautious" participant A will endeavor to maximize their gains, and "intelligent" participant B - to minimize their loss. Thus, the interests of both participants are contradictory, because by choosing a certain decision making system, one participant can gain more and the other one can lose more. In general, two types of situations can be observed:

- There is an equilibrium point in the game, that is participant A gets the largest possible number of wins by decision $\boldsymbol{d}_i$, when participant B gets the least possible loss by decision dj, and both are interested in not changing their decisions. Both players also realize how much they will gain (or lose), the size of a win or loss is called the game value, and the point of equilibrium is sometimes called the saddle point (maxmin-minmax)
- An equilibrium point does not exist and the participants cannot make a decision in advance which best suits them

Analyzing a decision process for the aforementioned conditions, one can distinguish two strategy concepts:

- Clear strategy – taken by the participant only once
- Mixed strategy – a linear convex combination of clear strategies

It should be discerned that mixed strategies are generally used in two types of situations, i.e. in the case of playing the same game several times (each time is independent of the others) – the strategies show the frequency and not the order of various decisions made; or in the case when the area of decision application can be divided into sufficient partial areas – then they determine the ratio between partial areas.

In the game with two participants, " the cautious" and "the intelligent", making a choice decision about a clear strategy according to the von Neumann-Wald criterion, none of the participants runs a risk, they strive to maximize their gain (minimize their maximum loss) instead.

If participants' decisions made according to von Neuman's rules lead to the same value of the win, the value is called the game value, and the game has the saddle point. If the value is 0, the game is said to be fair. Assuming that decision d_{i0} is called dominated by decision $d_{i1,}$ if the inequality described below occurs:

$$a_{i_0 j} \leqslant a_{i_1 \mathrm{j}}, \underset{\mathrm{j}}{\forall} : \mathrm{j} = 1, 2, \ldots \mathrm{k}.$$

In the decision analysis, one decision can be eliminated as ineffective. If one does it, and if both participants are cautious, it will appear that both will stick to their decisions; adding any value to the elements of the payoff matrix does not change the optimal game strategy, only the value of the game changes.

Decision choice rules

It is assumed that a subjective amount of uncertainty is defined in a space state, reflecting information data of nature states. An introduction of a general pay-off function *f* (a composition of function Φ and function $f(d, s) = u(\Phi(d, s))$, where $d \in \boldsymbol{D}$, $s \in \boldsymbol{S}$) for determined decision *d* results in: $f_d(s) = f(d, s)$, $\Phi_d(s) = \Phi(d, s)$. In case of uncertain situation (as opposed to a zero-sum game) is obtained the game, played with the assumption of a passive attitude of the other player, who does not intend to win at all.

The assumption of the other participant's passiveness exerts a change in decision choice rules with the rules applied below [2]:

- Wald test (pessimistic)
- Laplace –Bayes' theorem (optimistic)

- Hurwicz rule (pessimistic - optimistic)
- Hewitt-Savage zero-one law

In the first case, the decision is based on a rule analogous to the von Neumann-Wald rule. One takes a decision in which the minimum gain (because of the state of nature) for a decision made takes the highest value, i.e. one strives for such a i_0, for which the formula below is satisfied:

$$a_{i_0} = \max_i \min_j \{a_{ij}\} \tag{1}$$

The expected decision value $\boldsymbol{d}$ is then equal to:

$$E_d^p = \min_{s \in S} f_d(s) \tag{2}$$

One could claim that the rule is a safeguard (the choice of a lesser value guarantees minimum gain results), i.e. the rule of a cautious participant, yet of an intelligent one (hence the search for maximum).

Another rule, the Laplace-Bayes theorem, is based on the assumption that all future states of nature are equally likely. Therefore, it is possible to calculate the expected value of the outcomes of each decision, the best is such for which the expected gain is maximum. The decision problem can be written down as follows:

Find such i_0, for which:

$$E_{i_0} = \max_i \left\{ \sum_j a_{ij} \right\} \tag{3}$$

The expected decision value d (the highest value of pay-off function) is defined as:

$$E_d^p = \max_{s \in S} f_d(s) \tag{4}$$

Of course in this case such decisions for which the sum of all gains is the highest will be preferred, and therefore they may not necessarily be the best decisions in terms of each criterion.

Furthermore, the third quoted criterion is Hurwicz's. It combines the will to gain as much as possible with risk inclinations.

Hence:

$$a_i = \min_j \{a_{ij}\} \tag{5}$$

and

$$A_i = \max_j \{a_{ij}\} \tag{6}$$

d_i is calculated as follows [3]:

$$d_i = \gamma A_i + (1-\gamma) a_i, \tag{7}$$

where $\gamma$is the risk inclination coefficient, taking values from the range $\langle 0, 1 \rangle$.

The best decision in view of the Hurwicz criterion is defined by the maximum value of $\boldsymbol{d_i}$, this approach takes into account pessimism - optimism (when $y = 1$, the result is a pessimistic rule, and when $y = 0$ an optimistic one) [2].

Finally, according to the Hewitt-Savage rule, each nature state is equally probable i.e. a certain type of event will either almost surely happen or almost surely not happen. The best decision is calculated in two steps. The first step involves creating a "matrix of sorrow" defined as:

$$\mathrm{R} = \{r_{ij}\}, \text{where } \mathrm{r}_{ij} = \max_i \{c_{ij}\} - c_{ij}. \tag{8}$$

Next, in the second step for each matrix row, the highest value is sought, and then i^{th} row is sought in which this value is the lowest:

$$i \rightarrow i_0 : R_0 = \min_i \max_j \{r_{ij}\} \tag{9}$$

The presented rule can be explained as follows. Namely, for each/every of the nature state individually, and then for each/every possible decision, the amount of lost benefits that could be achieved is calculated (in relation to the best decision at a given state of nature). Next, one proceeds in accordance with the von Neumann-Wald rule, given that one has to deal with undesired values, i.e. the highest value from differences in row i^{th} is selected, and then the smallest one. The selected value is the optimal decision taken in view of the Hewitt-Savage criterion [2].

In the last matrix row, one selects the highest values in a column which give answer to a question: Which decisions are best under the states of nature analysed nature states.

In the second stage, it is necessary to calculate values of elements from "matrix of sorrow", which were defined as differences between the greatest values (from the column) and the corresponding elements of the matrix.

To conclude, this section presents only the basic elements of game theory. It must be stressed again that synthetically described decision making ways relate to decisions made under conditions of the complete ignorance of future nature states states of nature and the behavior of adversaries, in this case of two participants, when the sum of gains is equal to zero. Therefore, one does not take into account situations in which there can be more than two participants, and in which they can build strategic alliances and coalitions, based on common interest.

In many conflict situations under real economy conditions, the game is not with the zero sum. This may happen, for example, in the case of markets being shared by different companies. Strategic alliances can work for common good, they can attract new customers through advertising campaigns, but they can also win clients from others.

During games, the participants can become acquainted with each other one another and learn to anticipate opponent's behaviour on the basis of previous observations. It can happen in the case of situations in which one attempts to deduce the future strategy of the main competitor on the market on the basis of their past decisions and actions taken.

Topics related to uncertainties in modelling decision making problems are covered by the previously described issues of decision making in terms of risk, where probability distributions characterizing future nature states, stages of nature or competitor decisions are known. In such a case, in general, it is important to make a decision by which the expected value of gain (or loss) is the maximum (minimum).

Investment risk evaluation methods

General comments

In terms of market conditions, the risk associated with investments in energy is one of the most important issues of programming investments. The size of the risk affects both the market and technical factors, as well as the policy related to state power economy. There are several methods allowing taking into account the risk in calculating the efficiency of planned investments, such as a method to revise the effectiveness of the investment project, the account of sensitivity, probabilistic-statistical simulation methods, operating methods, taxometric methods [4]. Referring to mentioned issues the structure of this section is arranged, as the following: classic methods i.e. correcting the effectiveness of an investment project and sensitivity account are presented. Then probabilistic-statistical methods, Monte Carlo method and operative methods are described.

Correcting the effectiveness of an investment project

The method of correcting the parameters of the project concerns primarily the adjustment rate and rates of return, as these parameters are particularly vulnerable to major changes in the long run. With regard to power engineering, the above method is used to assess the risks of investing in distribution companies [5,6].

In the simplest analysis, the cost of risk is taken into account by increasing discount rate. Keeping financial accounts of the project is to develop different scenarios of investment for a period of N years, with different percentage rates. Values adopted for calculation result from the expected impact exerted by the state on the level of interest rates. These values should take into account both the rate of inflation as well as the risk premium paid. Using the revised interest rate, the rate of security investments r_s is fixed. The following formula is aimed to assess the profitability of investment:

$$NPV = \sum_{t=1}^{N} \frac{S_t}{(1 + r + r_k)^t} = \sum_{t=1}^{N} \frac{S_t}{(1 + r_s)^t} \quad (10)$$

where:
NPV – net present value,
S_t – balance of cash flow in year t,
r – coefficient of annual capital costs,
r_k – corrective value,
r_s – rate of security value, taking into account changing money value over time, and the risk connected with a project analysis.

Sensitivity accounting

Sensitivity account is a method of searching for critical values at which investments have economic benefits. The impact of many factors on the effectiveness of the draft is considered. This method is usually used in the effectiveness assessment of investment projects in power engineering and heat power.

The method of risk assessment using the account of sensitivity requires the following steps (Fig. 1):

- Determining the material scope of real investment
- Choice of uncertainty values being the account parameters whose influence on project effectiveness will be subject for analysis)
- Defining the variability range of uncertain values
- Constructing an evaluation process model
- Defining fluctuation range at an assumed variability of uncertain values (based on the existing model)

The purpose of sensitivity account is to determine how the selected input variables of the effectiveness of investment account affect the net discounted value of the NPV or internal rate of return *IRR* [7].

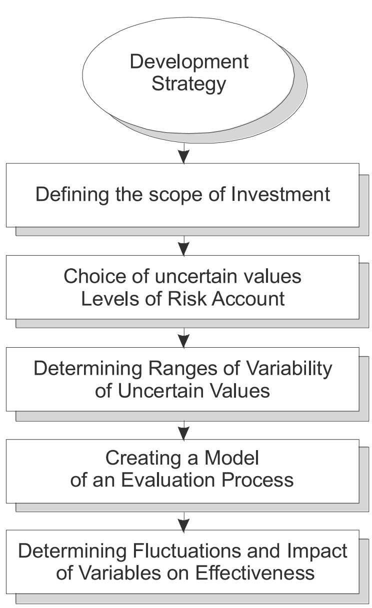

Fig. 1. Using sensitivity accounting for investment risk assessment (adopted from Kamrat W.: Methodology of investment evaluation in the local energy market. Energy Department Report, EN-D-65, Lappeenranta University of Technology, Finland, 1999 [3]).

Probabilistic-statistical methods

Probabilistic-statistical methods taking into account investment risks are expected to determine the efficiency parameters value of planned investments and the statistical measurement of risk involved in it. It is assumed that there are probability distributions of variables for the calculation of the efficiency of investments at a given level. These methods exploit the distribution of random variables, differentiation, standard deviation and variability coefficient. These methods of analysis of an investment risk include a method of decision tree. A decision tree is a method mainly used for the analysis of complex investment projects. Their use is particularly suited to sequential investment projects, where the decision depends on the results obtained in the successive stages. The results of the analysis are presented graphically in a tree of chronological events that includes the most important events from the first to the last, while the analysis of the tree takes the opposite direction (from the last to the first event), in this way enabling comparing the results after every/each decision. This methodology can be useful for evaluating network investments in the power system. A decision tree method is a dynamic method. Its important advantage is that it takes account of money volatility over time, economic impact for the decision-maker, both in the case of the implementation of specific projects and in the case of withdrawal from the implementation of a project. It also takes into account the conditions of competition in the market. Monte Carlo is a method quite often used for a risk analysis of investment projects related to power engineering systems. It allows more accurately than a scenario analysis or a decision tree analysis estimating the expected value of net present value E(NPV) and the standard deviation, which measures the risk of a draft. Before the beginning of the project analysis, an appropriate mathematical model for password-topic, which reflects the economic dependence between the variables and the model probability distributions of these variables must be built. For each/ every variable appearing in the model, a random number setting this variable is generated, which should be used to calculate the NPV in the first draw, namely during the first execution of the project. The draw of these values is repeated, for example, 500 or 1000 times, resulting in a probability distribution of possible net present values for the NPV of the project.

Monte Carlo simulation method

Monte Carlo is a method quite often used for a risk analysis of investment projects related to power engineering systems. It allows more accurately than a scenario analysis or a decision tree analysis estimating the expected value of net present value E(NPV) and the standard deviation, which measures the risk of a draft.

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Operative methods

Operating methods (used for example in the case of system expansion when it is necessary to coordinate a number of factors strongly affecting the achievement of the goal) use elements of a game theory to assess the risk. Using a strategy game in the assessment of an investment risk, performance indicators should be calculated for all the options in investment decisions, assuming that they can be attributed to different decision-making scenarios differing in all the possible combinations of uncertain factors. Different variants of investment decisions contain two or more scenarios, each with a different combination of uncertain factors and characterized by good economic efficiency.

Taxonometric methods

General comments

Referring to the author's earlier works, suggestion is that the investment risk inherent in a given venture can be gauged with the so–called risk rate identified based on a taxonomic variable, the construction of which can be found in earlier works [2,3].

A synthetic scale of the investment risk is constructed by employing the comparative cluster analysis method that enables ranking individual entities along the synthetic scale under the so–called model method.

Selecting the diagnostic variables is an important step in the suggested risk assessment method. Described using a set of diagnostic variables, the analyzed energy market investment strategies can be treated as real, multi–feature objects. Such objects can be analyzed for the risk involved under appropriate methods of comparative cluster analysis. The methods put numerical representations of the input variables in the centre of studies, i.e. treat them as the objects of study. This makes it possible to obtain information on uniformity within the set of the objects considered, i.e. uniformity within the considered set of numerical data. When numerical representations of the input variables are used to study different strategies, the set of the data analyzed is found non–homogenous, as it groups entities that differ in size, technology, and/or technical equipment. Therefore, to study regularities occurring in investment strategies the non–homogeneous data set should be split into relatively uniform subsets which can then be studied using methodologies and techniques of the comparative cluster analysis. Referring to mentioned issues the structure of this section is arranged, as the following: essential problems encountered when estimating the investment risk and way of construction of the measure of the risk in investing in the energy market are presented.

Essential problems encountered when estimating the investment risk in the energy market

In general are two types of basic data should be adopted for the purposes of synthetic studies of the risk inherent in investing on the local energy market, i.e.:

- A set of possible objects $\Omega = \{\omega_i\}$, $i = 1, 2, \ldots\ldots, m$
- A set of the diagnostic data $\Phi = \{\Phi_j\}$, $j = 1, 2, \ldots\ldots, n$,

where the diagnostic data can represent both physical, technical, and/or economic variables.

The notions of the object, ω_i, and the features, Φ_j, are considered to be prime in nature, hence need not be defined.

The features, Φ_j, can be interpreted to be the result of transformation of the object set Ω into a set of real numbers, i.e.:

$\Phi_j : \Omega \rightarrow \xi_j \subset \mathbf{R}^1$

Hence, each $\omega_i \in \Omega$ element has an $x_{ij} \in \xi_j$ value assigned. In this natural way one arrives at the following data matrix:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (11)$$

where: x_{ij} – representation of feature j^{th} in object i^{th}.

The rows, X_i, of the matrix, or the so–called vectors, $(x_{i1}, x_{i2}, \ldots, x_{in})$, can be considered representations or visual pictures of the ω_i objects. This should be understood as the following mapping: $\Omega \rightarrow \mathbf{G}^n{}_1 \subset \mathbf{R}^n$, where each $\omega_i \in \Omega$ element has an $X_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \mathbf{G}^n{}_1$ vector, i.e. a row in the **X** matrix, assigned.

The above notions should thus be understood as a process of transformation: $\Phi \rightarrow \mathbf{G}^m{}_2 \subset \mathbf{R}^m$, where each $\phi_j \in \Phi$ element has an $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})^T \in \mathbf{G}^m{}_2$ vector assigned. The $\mathbf{G}^n{}_1$ and $\mathbf{G}^m{}_2$ sets are thus called: the space of object graphic representations, and the space of feature graphic representations, respectively.

Studies do not deal directly with objects or features, but with numerical representations of features. In other words objects are compared with one another via their assigned numerical representations. Analysis of the kind very much relies on securing the base for comparisons. This is achieved by differentiating between the notions of a stimulant and destimulant . For example, feature ϕ_j will function as a stimulant for objects ω_1 and ω_2 when the numerical representation of feature j^{th} of the object ω_2, i.e. x_{2j}, is greater than the respective x_{1j} value of object ω_1.

To arrive at a general definition for objects ω_r and ω_s, feature ϕ_j is said to be a stimulant when:

$$\widehat{x_{rj}, x_{sj}} \left(X_{sj} \geqslant x_{rj} \right) \Rightarrow \omega_s \} \omega_r \quad (12)$$

The same, ϕ_j feature is said to be a destimulant when:

$$\widehat{x_{rj}, x_{sj}} \left(X_{sj} \geqslant x_{rj} \right) \Rightarrow \omega_s \{ \omega_r \quad (13)$$

where:

x_{rj}, x_{sj} – denote numerical relations of feature j^{th},

} – denotes "domination" of the object over another object.

To continue into further considerations, we need to adopt the following two assumptions:

- set $\Phi = \Phi_1 \cup \Phi_2$; $\Phi_1 \cap \Phi_2 = \varnothing$; where: Φ_1 – set of stimulants, Φ_2 – set of destimulants,
- For each destimulant there is a transformation that changes it into a stimulant. This indicates that upon transformation sets Φ and Φ_1 can be considered identical, i.e. $\Phi = \Phi_1$ or $\Phi_2 = \varnothing$.

The simplest way of transforming a destimulant into a stimulant is to reverse its numerical representation.

The procedure of assessing the investment risk is based on the assumption that the representations of the features included in the observation matrix constitute representations of one (and the same) chance variable, all of which refer to individual elements of the object set. A comparative cluster analysis of the risk inherent in investment strategies will be carried out using diagnostic variables of the characteristic features of a specific project planned. One of the key issues is to compile a correct set of diagnostic variables. In practice such sets are compiled based on familiarity with the objects studied. Cluster analysis proves useful also when the studied objects are not very well known, however then it is bound to be based on a large set of features. These are initially hard to rank in terms of their priority for the purposes of the conducted comparative studies.

All things considered, using a cluster and spatial measure to assess risk has the advantage of enabling the positioning of relatively uniform features along individual axes of the spatial model. In addition, this kind of measure can be easily interpreted in its simple graphic representation. Its disadvantage, compared to the synthetic (unilateral assessment measure), is the unavailability of a simple way to rank objects against a single, synthetic ratio. Furthermore, it is easy to end up with a set including non–diagnostic features which will hinder identification of characteristic types and increase the amount of work needed to complete the calculations. The latter argument is used to justify the use of the so–called reduction of the description of the studied space. The purpose here is to eliminate doubled information (e.g. closely correlated space constituents), data of low informative value (low information capacity), and little differentiation between the features of the objects grouped in the studied set. Comparative cluster analysis serves as a tool for comparing variables (reflecting the features and specificity of processes) that can be expressed in identical or different measurement units; it also allows for implementing the following analytical procedure:

- Stage I preliminary variable selection,
- Stage II identifying the co–relations between the variables,

- Stage III identifying the factors that underlie the co–relations,
- Stage IV forming synthetic variables to reflect the factors,
- Stage V arriving at aggregate variables that will allow objects to be ranked along the scale of investment risk.

The presented comparative procedure involves employing a set of statistical methods sequenced so that the results obtained under one method used at one stage of the analysis serve as the starting point for the subsequent stage that uses a different method. The stages meet the requirements of the above mentioned tasks comparative cluster analysis is expected to achieve. The process of identifying the risk usually involves three basic methods:

- Co–relation analysis – stages I, II
- Main constituent method – stages III, IV
- Taxonomic methods – stage V

Variable selection is carried out in several steps. Step one consists in analyzing the contents of the variables on the preliminary list which gathers all the available variables related to the aim and scope of the study. The variables should be precisely and unambiguously defined.

A formal analysis performed at step two aims at excluding all the variables whose informative significance is minor (i.e. are irrelevant) or which double the information contained in other variables from the preliminary list. The starting point for the analysis proper is the $\mathbf{B}_o = [x_{ij}]$ matrix of $t \times n$ size, where t stands for the number of observations, and n for the number of variables. The $\mathbf{B}_o$ matrix must be complete, i.e. must include information on the variables of each investment strategy. The described procedure eliminates those variables whose variation ratio (relation of the standard variation to the mean) is smaller than the adopted threshold value ε. Such data is insignificant, or quasi–stable. To reduce the number of variables, the present work seeks to incorporate a method once developed by Z. Hellwig [1]. Literature mentions is under the concept of the "developmental model method" used to rank energy market investment strategies in terms of risk. The description of the above can be found further in the paper.

Construction of the measure of the risk in investing in the energy market

For the present purposes and investment process is understood only to denote construction, development, or modernization of the sources of energy produced. Further on this is called as an investment strategy. Observations are positioned along a synthetic scale of the investment risk under the previously mentioned model method. Here, the Euclid distance (in multi–dimensional space, the borders of which are determined by the number of variables) is calculated for all values of the factor objects from the factor values of the hypothetical "model" object defined based on "the most desirable" values of the variables found in the entire set of the investment strategies. The objects "closest" to the model have the most preferable parameters in terms of the adopted criterion, i.e. represent the lowest investment risk. For formal reasons it is more convenient to use a standardized scale (between 0 and 1) to depict the positioning of the objects. The "best" object is represented by the highest value, the "worst" by the lowest. Thus, in the present study strategies involving the lowest risk are found at the beginning of the synthetic investment risk scale, while those burdened with the highest risk come at the end. This concept involves measuring an investment risk using rate risk measurement techniques. Rate risk should be assessed by taking into account the specific taxonomic variables. In order to build a synthetic scale of an investment risk, multivariate analysis of comparison was used, which hierarchically arranges units on a synthetic scale by applying the so-called standard method. At the end the investment risk is measured using the following *ep* synthetic rate risk formula:

$$e_p = 1 - \frac{d_p}{||D||} \tag{14}$$

where distance d_p:

$$d_p = \left[\sum_{j=1}^{k} \left(x_{pj} - q_j \right)^2 \right]^{\frac{1}{2}} \tag{15}$$

x_{pj} – normalized value of j^{th} variable in p^{th} investment strategy,

$|| D || = d(P, Q)$ - distance between "poles" (max., min.) characteristic of investment strategy.

Given the risk rate and multiplying it by the flow of investment costs K_{nd}, annual risk costs are obtained K_{ryz} according to the following formula [3]:

$$K_{ryz} = e_p \cdot K_{nd} \tag{16}$$

where:

e_p - risk rate,

K_{nd} - discounted flow of investment costs.

The annual risk costs defined above allow taking into account the risk impact on total annual costs, according to the formula:

$$K_r = K_s + K_z + K_{ryz} \tag{17}$$

where:

K_s - annual fixed costs,

K_z - annual variable costs.

Block diagram of the algorithm of identifying the investment risk is presented on Fig. 2.

An important fact is that the risk can be subdivided into several categories along the synthetic scale. Such subdivision is not only possible, but also recommendable. It seems purposeful to identify the following five investment risk categories:

- Low risk – for strategies with a ratio above < 0, 0.1)
- Medium risk – for strategies with a ratio between < 0.1, 0.2)
- Increased risk – for strategies with a ratio between < 0.2, 0.3)
- High risk – for strategies with a ratio between < 0.3, 0.4)
- Extreme risk – for strategies with a ratio below < 0.4, 1.0>

To summarize, it must be noted that the usefulness of methods taking into account risks depend on situational determinants and the specificity of decision-making processes.

An additional benefit gained here is the possibility to compare the cost of risk to the installed power. In this way we arrive at the unit risk cost for a specific investment strategy. The values of the risk rates range between <0, 1>. The closer the $\boldsymbol{e_p}$ value is to ***1***, the higher risk is involved in a given investment strategy.

The presented approach to estimating the cost of risk allows for identifying such cost in changing market conditions, with technical, economic, and location parameters characteristic for a given investment in the power industry recognized. This is particularly crucial in planning the processes of investing in regional power industry and local energy markets [8–10].

Conclusions

The presented investment risk evaluation methods in this paper can be a rational package of tools for assessing the risk of investing in the energy sector. The method presented by the author is fast, gives good reliable results and allows for efficient decision-making analyzes in the energy sector. The general principles of modelling and solving decision making problems in investment processes, specially investment risk evaluation methods outlined in this paper do not cover all the issues, yet they can provide a stimulus for further discussion aimed to develop more precise methods for evaluating investment projects. This will require, inter alia, providing reliable source materials concerning investment, cost, applied energy technologies, effects and opportunities for investment in electric power engineering.

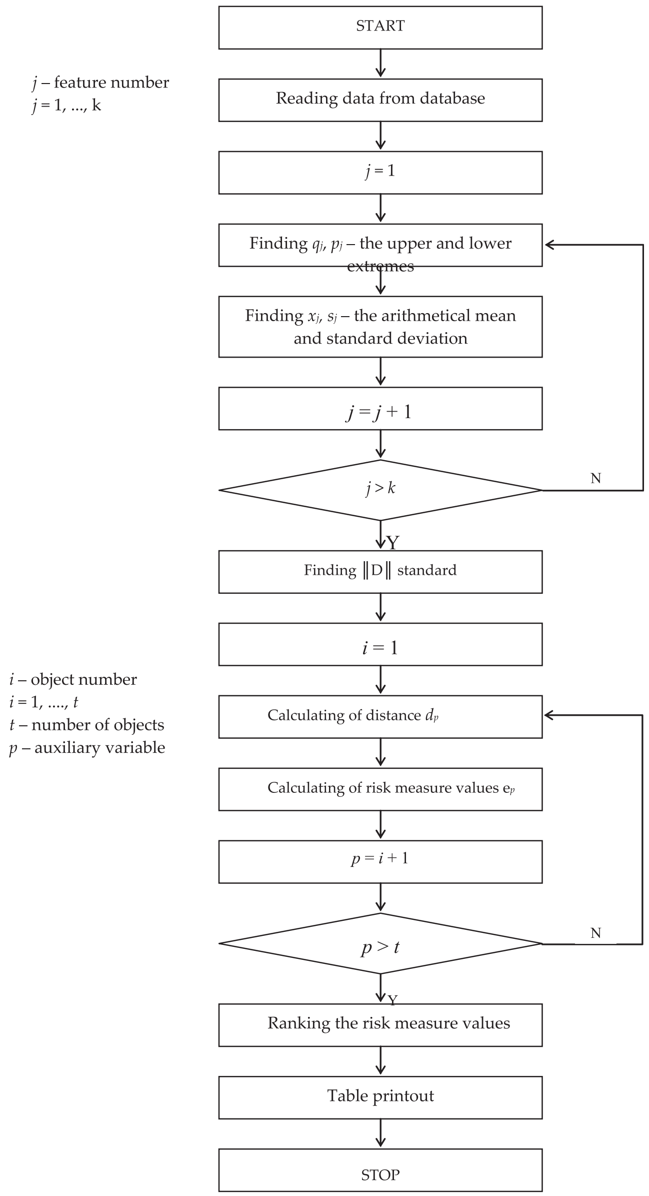

Fig. 2. Block diagram of the algorithm of identifying the investment risk (adopted from Kamrat W.: Methodology of investment evaluation in the local energy market. Energy Department Report, EN-D-65, Lappeenranta University of Technology, Finland, 1999 [3].

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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