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SELECTING THE OPTIMUM LOCATION FOR LOGISTICS FACILITIES USING SOLVER – CASE STUDY

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ABSTRACT

Siting logistics facilities strategically in the most cost-effective geographic location is one of the key decisions a contemporary company will make.

The aim of the paper is to present a solution to this problem using the Solver add-on. In the case study discussed in the paper, the company's central warehouse location was selected based on the classic location theory, which addresses the need to minimize the cost of transport. The mathematical model of the exercise is based on the Euclidean coordinates metrics.

The method used in the study is summarised in the context of its strengths and weaknesses.

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INTRODUCTION

Siting of logistics facilities is an important issue pertinent to the field of logistics. Choosing the location is almost always a strategic decision with long-term consequences. Wrong location decisions will affect the volume and the cost of transportation, as well as the level of inventories kept in various locations, since the shape of the logistics network determines the flow of products (cargo) between logistics facilities. The volume and the frequency of these flows are a function of the distance between the points within the network.

Taking interest in the problem of logistics facilities location yields measurable benefits to businesses. Studies in this field contribute to the reduction of transportation cost and the duration of the delivery cycle, while enhancing the customer satisfaction and, eventually, increasing the competitive advantage of companies.

Many procedures have been developed to support the location decision-making. The most popular methods of siting a single logistics unit include: the centre of gravity method, the transportation method, the multi-stage transportation problem method (Hodczak-Sekulska & Redmer 2010), the network method and the method of finding graph centre[1]. Yet, the computational complexity of all these methods is often a barrier to practical application.

The purpose of this paper is to present a method for selecting the optimum location of a single logistics unit (e.g. a warehouse), using the MS Solver add-on. The spreadsheet used in the method automates the whole computation process, and thereby the drawbacks of other methods are overcome both in terms of labour-intensity and computation precision. The procedure described in the paper enables a very efficient selection of an optimum location for a facility by means of computing its geographic coordinates. By entering these geographic coordinates in digital maps (e.g. Google) it is possible to immediately visualise the location. This provides a wide range of additional information about the infrastructure existing around the potential location and significantly facilitates the decision-making process. The method presented in the study is a base point for adopting the final solution to the location problem seen in its qualitative and financial aspects.

SINGLE FACILITY LOCATION PROBLEM – A MATHEMATICAL APPROACH

Finding an optimum location for a logistics facility (e.g. a distribution centre, a central warehouse) is a strategic task. The first attempt to describe this problem in the form of a mathematical model was made by W. Launhardt in *Prachtische Standortbestimmung* (Budner 2004, p. 57). The studies on the selection of an optimum location for a single facility were continued by Alfred Weber (Krawczyk 2001, p. 97). In his proposal, the best location is ensured through identifying an accessible place at the lowest cost of transportation. This approach links the location to the specific benefit – cost savings. The procedure described by A. Weber remains relevant and valid.

Hence, a variant which minimizes the cost of transport should be looked for, based on the classic location theory. It is difficult to calculate the future cost of transport for a non-existent transportation network. Yet, if we assume that this cost is strictly related to distances to be covered to reach the new facility (e.g. a central warehouse), then they can be easily estimated. The mathematical model being designed here is based on a thesis that in the event the original objective variables cannot be measured, they should be replaced with the substitution variables. With this assumption, it is possible to create a mathematical model which contains good parameters and, consequently, to obtain the result sought.

To compute distances in models, two analytical approaches are used: the Euclidean distance and the Orthogonal (or Perpendicular) distance. For two points: $A_i\ (x_i, y_i)$ and $A_j\ (x_j, y_j)$ the Euclidean distance is expressed by the formula (Dattorro 2005, p. 346):

[1] With the variant decomposition of the graph mapping of the transportation network, this method can also be used for solving multi-site location problems.

$$d_{ij}^{e} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \qquad (1)$$

while the Orthogonal distance – by the formula:

$$d_{ij}^{o} = |x_i - x_j| + |x_i - x_j|. \qquad (2)$$

According to the Euclidean metric, based on the Pythagorean theorem (Wallace & West 2015, p. 399), the distance between two points on a plane equals the length of the line segment connecting them. On the other hand, according to the Orthogonal metric, the distance is measured as a total of line segments received after projecting the points onto coordinates axes. Relations between the distances are visualised in Figure 1.

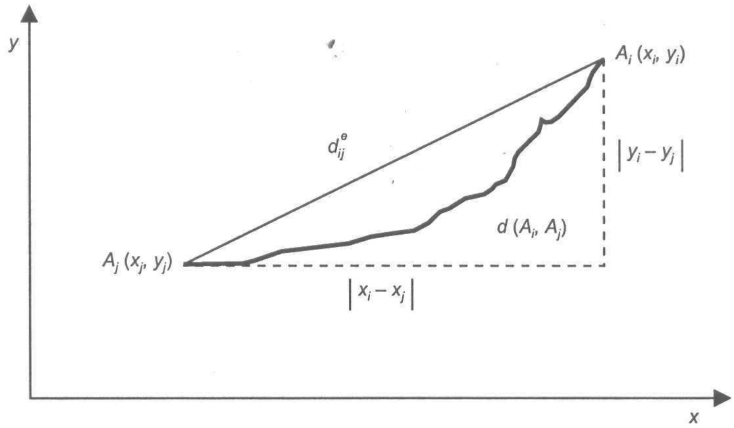

Figure 1. Relations between the Euclidean, orthogonal and real distance between two points
Source: S. Krawczyk, (2001). *Zarządzanie procesami logistycznymi*, Warszawa: PWE, p. 99.

Both these methods are subject to the measurement error bias in relation to the real distance $d(A_i, A_j)$. In practice, the distance to be covered in reality will be longer than that shown by the Euclidean metric and, as a rule, shorter than the Orthogonal one. This dependence is shown by the equation:

$$d_{ij}^{e} \leq d(A_i, A_j) \leq d_{ij}^{o} \qquad (3)$$

In a situation, where the set of recipients has already been identified (products will be sent from the logistics facility to various destinations) and the demand to be expected over, for example, one year has been estimated – the task of finding an optimum location is reduced to determining point $M(x_o, y_o)$ with the lowest cost of transportation.

The procedure of determining coordinates x_o, y_o of an object (irrespective of which metric has been used: the Euclidean or the Orthogonal one) involves laborious mathematical computations that have been discussed in detail by St. Krawczyk (2001, pp. 184-194). The results obtained in each of the metrics are slightly different. Yet, both indications are so close to each other that it is difficult to determine undeniably, which of the metrics better reflects the real distance between the two points in space. The computations will always retain the relation described by equation (3). The fact of obtaining similar computational results regardless of the metric used is also confirmed by Kuczyńska & Ziółkowski (2012, pp. 399-350).

The focus of this study is on solving the problem of selecting an optimum location for the company's facilities using the MS Solver add-on. The mathematical model of the exercise is based on the Euclidean coordinates metrics. The section below presents Solver and the way it works.

SOLVER ADD-ON

Solver[2] is one of the most advanced MS Excel-based analytical tools designed for solving single-criterion optimization problems. In order to use Solver, the mathematical model has to be saved in the spreadsheet work area. The optimization model consists of three key elements (Ragsdale 2015, pp. 52-53):

- the objective cell (the objective function),
- the variable cells (decision variables),
- the constraint cells (these may be used in computing the values in both the objective cell and the variable cells).

The objective function is the cell of the worksheet intended to take the minimum, maximum or pre-determined value in the form of a real number as a result of using Solver. Decision variables are represented by cells with the values to be found and are changed iteratively throughout the optimization process, i.e. they are inserted in the objective function by Solver, until the optimum solution is found. When building the optimization model, constraints need to be entered (in most cases). They are represented by the worksheet formulas, the value of which has to fit within the pre-defined limits or achieve some specific target values.

Solver is upgraded and improved on a regular basis. The effects of these modifications can be followed when comparing the consecutive versions of MS Excel. Today, the add-on can be used for solving problems, where the number of decision variables does not exceed 200 and contains up to 100 constraints (Gross, Akaiwa & Nordquist 2014, p. 598). The main change in MS Excel 2010 (as compared with the previous versions) is that it offers a choice of methods to be used for solving the optimization task. There are three methods available to the user (Baj-Rogowska 2013, p. 172):

- the LP Simplex method – used for solving linear optimization problems;
- the non-linear GRG[3] method – the objective cell and/or some constraints cells contain non-linear functions;
- the evolutionary method – the objective cell and/or some of the constraints cells contain non-smooth functions[4].

[2] Solver solutions are also available in open source software, e.g. OpenSolver (in OpenOffice). The Solver add-on provides the same computational results, regardless of the software version.

[3] One of the options for GRG (Generalized Reduced Gradient) method is to use multiple starting points. Entering a variety of starting points ensures that the answer reported by Solver is actually the optimal solution.

[4] A non-smooth function – its derivative is a discontinuous function.

When solving the optimization problem using Solver, one needs to enter the required characteristics referred to above in the spreadsheet. Then, after activating Solver, all acceptable solutions are identified and the one ensuring "the best" value of the objective function is chosen. This solution is referred to as the optimum one.

In the next section of the paper, a case study of selecting an optimum location for a company's central warehouse will be demonstrated as a practical application of the procedures described above.

PROBLEM OF WAREHOUSE LOCATION CHARACTERISED AND SOLVED USING SOLVER

A manufacturing Company delivers its products to regular customers in south-east Poland on a cyclical basis. Figure 2 shows locations of 15 recipients of the Company's products (1 – Łódź, 2 – Radom, 3 – Kielce, ..., 15 – Gliwice). Currently, the Company is planning to build a central warehouse in the area covered by its services. Therefore, a decision has to be made on the facility's location. The cost of transportation is the key and most often the only determinant of the company's facilities siting (Gołębska 2006, p. 140). Hence, a conclusion was reached that the central warehouse location – $M(x_o, y_o)$ – should ensure that distances to be covered by delivery trucks generate as low a cost as possible, which means that they should be as short as possible. In order to examine the chosen criterion it is worth to analyse historical data of the company's transportation activities, for example, from last year. According to the assumption referred to before, we may expect that this will be a basis for minimizing the total distance of all deliveries. Such a condition (minimizing) will set the direction of optimization in the model being designed.

Figure 2. Company's branches shown on the map
Source: Author's own analyses, using Google maps.

According to what has been discussed above, data presented in Figure 3 were entered into the Excel spreadsheet. Each of the cities (column C) has its determined longitude and latitude (columns D and E, respectively). Column F shows the number of dispatches to each destination over the year. The variables sought for in our decision problem are represented by the values of the warehouse's longitude and latitude. To begin with (before optimization takes place), any coordinates chosen for the facility location can be used. Figure 3 includes data computed for the following inputs: latitude – 50.95455486 and longitude – 19.96042017.

	A	B	C	D	E	F	G	H	I
6		It.	City	Lat (N)	Long (E)	Dispatch (thou. pcs/y)	Distance (km)	Dispatch dist. total	
7		1	Łódź	51,770	19,459	12	69,88	838,57	
8		2	Radom	51,399	21,159	10	93,32	933,18	
9		3	Kielce	50,889	20,649	2	50,49	100,99	
10		4	Lublin	51,240	22,570	10	191,64	1916,36	
11		5	Zamość	50,729	23,260	8	241,43	1931,45	
12		6	Rzeszów	50,049	21,999	9	162,84	1465,54	
13		7	Krosno	49,700	21,759	14	160,08	2241,14	
14		8	Kraków	50,060	19,959	15	65,30	979,54	
15		9	Wieliczka	49,990	20,090	5	71,05	355,23	
16		10	Katowice	50,259	19,020	15	85,39	1280,82	
17		11	Tarnów	50,009	20,990	8	102,05	816,37	
18		12	Sandomierz	50,689	21,740	7	131,35	919,43	
19		13	Przemyśl	49,790	22,780	9	222,69	2004,25	
20		14	Opole	50,679	17,940	4	148,86	595,42	
21		15	Gliwice	50,310	18,669	6	105,36	632,18	
22							Total distance	17010,47	km

Figure 3. Inputs for the company's warehouse location problem
Source: Author's own analyses.

The following formula computing the distance is a key to the model design:

$$= 73 * \sqrt{(lat_1 - lat_2)^2 + (long_1 - long_2)^2}, \quad (4)$$

and it has been entered in cells G7:G21. It is based on formula (1), which is used for distance calculation in the Euclidean metric and will be used for computing the distance between two locations[5] in our model, based on their geographic coordinates. The constant in formula (4), $\alpha = 73$, represents the number of km equivalent to 1 degree. It is 111 km[6] at the equator, in USA – 69 km and in Poland – about 73 km. Hence, the distance computing formula (for cell G7) shall take the following form:

=73* SQRT((D7-G4)^2+(E7-H4)^2) (5)

[5] Between the delivery destination town and the space point of the warehouse's optimum location.
[6] A quotient of two operands referring to the circumference of Earth expressed in kilometres (40,076 km) and geographic degrees (360°).

Through addition formula:

$$= \text{TOTAL(H7:H21)} \qquad (6)$$

cell H22 allows calculation of the total length of all distances covered by deliveries. It should be stressed that it is the objective function at the same time and needs to be minimized by the changing cell values to be found (decision variables), i.e. by the value of latitude and longitude of the central warehouse location. When Solver is activated and the non-linear GRG option is chosen, the program calculations will be performed in a manner leading to finding such warehouse location coordinates, where objective cell H22 achieves the minimum value. After clicking the *Solve* button in the Solver *Parameters* dialogue box, the solution presented in Figure 4 is obtained.

LOCATION	
Lat.	Long.
50,35458716	20,96038626

It.	City	Lat (N)	Long (E)	Dispatch (thou. pcs/y)	Distance (km)	Dispatch dist. total
1	Łódź	51,770	19,459	12	150,63	1807,52
2	Radom	51,399	21,159	10	77,61	776,08
3	Kielce	50,889	20,649	2	45,15	90,30
4	Lublin	51,240	22,570	10	134,11	1341,06
5	Zamość	50,729	23,260	8	170,08	1360,66
6	Rzeszów	50,049	21,999	9	79,03	711,29
7	Krosno	49,700	21,759	14	75,38	1055,32
8	Kraków	50,060	19,959	15	76,20	1142,98
9	Wieliczka	49,990	20,090	5	68,89	344,44
10	Katowice	50,259	19,020	15	141,82	2127,30
11	Tarnów	50,009	20,990	8	25,32	202,56
12	Sandomierz	50,689	21,740	7	61,93	433,49
13	Przemyśl	49,790	22,780	9	139,08	1251,71
14	Opole	50,679	17,940	4	221,76	887,03
15	Gliwice	50,310	18,669	6	167,30	1003,82
					Total distance	14535,56 km

Figure 4. Solution to the warehouse location problem
Source: Author's own analyses.

The calculations show that the warehouse should be located at latitude of 50.35458716 N and longitude of 20.96038626 E (cells G4:H4). After entering the location coordinates in Google maps, an answer is received that the central warehouse should be located in the area near the village of Świniary (Figure 5).

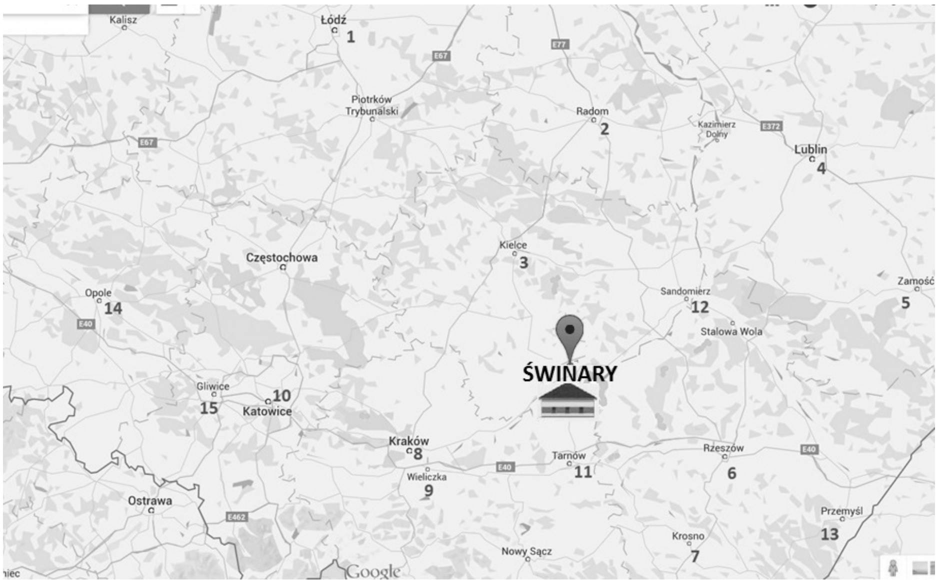

Figure 5. Problem solution visualised on the map
Source: Author's own analyses.

FINDINGS AND DISCUSSION

As a result of Solver-aided calculations, the village of Świniary was indicated as the optimum location for the Company's warehouse meeting the pre-set conditions. Świniary is a village in the Świętokrzyskie region (near the town of Pacanów), not far from national road no. 79, which intersects national road no. 73[7] nearby. The location can be regarded as the right choice for three main reasons:

- the accessibility of the transport infrastructure;
- with regard to its accelerative function, which may contribute to the economic development of the region in the future;
- due to land prices of the rural area which may be attractive for the investor.

The decision where to site the company's logistics facilities depends on the set of strategic decisions. Long-term effects of this decision may determine the organisation's competitive advantage. It should be stressed that the siting decision is a complex process, which can be affected by many different factors (criteria), more or less valid. They vary over time and many of them are qualitative in their nature, thereby being difficult to estimate, due to the subjective choice made by the decision maker when locating the specific facility. Problems of this type may be solved using the Multiple Criteria Decision Making methods (MCDM). They are represented by a large group of procedures, including: PROMETHEE (Preference Ranking Organization Method for

[7] National road no. 73 is a strategic one and an element of the country's main road system connecting the north with the south. The road plays a vital role in supporting the economic growth.

Enrichment Evaluations), ELECTRE (*Elimination Et Choix Traduisant la REalite*), TOPSIS (Technique for Order Performance by Similarity to Ideal Solution), AHP (Analytic Hierarchy Process), ANP (Analytic Network Process).

The objective of this study is to select an optimum location for the Company's warehouse using Solver. As it has already been mentioned, this tool enables the user only to solve single-criterion optimization problems. In our case study, the conditions of investing in a warehouse are very similar all over the considered geographic area (south-eastern Poland), therefore the cost of transport could be considered as the main – and, in this case, only – determinant when solving the optimization problem using Solver for choosing the Company's warehouse location (Krawczyk 2001, p. 181). The location identified in the solution to the problem should be regarded as an indication for the potential site, an approximation of the place to be chosen for the project.

To be certain that the site identified as a result of the exercise is a fully rational choice, the process should be followed by the analysis of additional specific criteria covering both quantitative data (land prices, cost of infrastructure access, etc.) and qualitative features (the condition of the available transport infrastructure, the existing development plans, the quality and availability of land, potential threats or conflicts of interest as well as legal and environmental aspects).

As a result of supporting the selection of an optimum location for a logistics facility with Solver, the consecutive stages of the process are software-aided and, in consequence, automated, thereby significantly reducing the time needed to make the final decision.

Owing to the approach based on the Euclidean metrics, as presented in the paper, the location data do not need to be re-calculated, since geographic coordinates expressed in kilometres can be easily converted into decimal degrees, which, when entered in digital maps, show the point on the map in no time at all.

By visualising the solution on digital maps one gains an additional opportunity to analyse other data pertinent to the location, while facilitating the process of identifying infrastructural limitations of the area at the same time.

To sum it up, indicating the location by means of the method presented in the paper is a good foundation for the final analysis of conditions of the logistics facility siting. The company should thoroughly check any regional and local determinants that may affect the decision to be made. When choosing the facility location one should bear the highest customer service standards in mind, while performing a detailed cost analysis at the same time.

CONCLUSION

The purpose of the case study presented here was to use the single-criterion optimization when selecting the location for the company's warehouse. The objective was achieved using the Solver add-on, based on Alfred Weber's classic location theory, which minimizes the cost of transport.

The solution to the location problem presented in the paper offers many benefits:

- the Solver-aided optimization reduces the decision-making process duration significantly;

- laborious computations are automated in the spreadsheet and thereby performed in a precise and error-free manner;
- with the Euclidean metric used, the geographic location expressed in kilometres is easily convertible to decimal coordinates;
- after entering the geographic coordinates in digital maps, the point to be found is instantly visualised on the map;
- this allows efficient analysis of additional data pertinent to the location.

Yet, to evaluate all aspects of the proposed approach, its weak points should be also mentioned. These include the following:

- the real course of roads is not shown, including any obstacles (e.g. bridges), therefore the method is not entirely flawless;
- potentially, the method may provide rather an unacceptable solution (e.g. facility location in a densely built-up area or within a nature reserve).

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