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Simulating propagation of coherent light in random media using the Fredholm type integral equation

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ABSTRACT

Studying propagation of light in random scattering materials is important for both basic and applied research. Such studies often require usage of numerical method for simulating behavior of light beams in random media. However, if such simulations require consideration of coherence properties of light, they may become a complex numerical problems. There are well established methods for simulating multiple scattering of light (e.g. Radiative Transfer Theory and Monte Carlo methods) but they do not treat coherence properties of light directly. Some variations of these methods allows to predict behavior of coherent light but only for an averaged realization of the scattering medium. This limits their application in studying many physical phenomena connected to a specific distribution of scattering particles (e.g. laser speckle). In general, numerical simulation of coherent light propagation in a specific realization of random medium is a time- and memory-consuming problem. The goal of the presented research was to develop new efficient method for solving this problem. The method, presented in our earlier works, is based on solving the Fredholm type integral equation, which describes multiple light scattering process. This equation can be discretized and solved numerically using various algorithms e.g. by direct solving the corresponding linear equations system, as well as by using iterative or Monte Carlo solvers. Here we present recent development of this method including its comparison with well-known analytical results and a finite-difference type simulations. We also present extension of the method for problems of multiple scattering of a polarized light on large spherical particles that joins presented mathematical formalism with Mie theory.

Keywords: light scattering, Monte Carlo simulations, light coherence, numerical methods

1. INTRODUCTION

In recent years, studies on coherent light propagation in scattering media has drawn a lot of scientific attention. Some of the recent works were focused on developing numerical models and simulators that can facilitate further basic and applied research.[1–3] Despite the fact, that some of the applied models are based on the well-known ideas, numerical simulation of propagation of the coherent light beams in the scattering media may not be a trivial task.

Currently known numerical methods can be divided into two groups: (I) the ones based on the radiative transfer theory, in particular the Monte Carlo method for solving the radiative transfer equation[4] and (II) the ones based on numerical solving the wave equation describing propagation of the light beams.[1,5–7] The first approach allows to simulate propagation of the light inside the large volumes of the scattering media but it is difficult to incorporate wave and coherence properties of light in them.[1,7–9] The second approach intrinsicly deals with wave nature of the light but due to a large computation time and computer memory required, they allow to simulate propagation of the light only in small volumes of the scattering media.

In our recent work,[2] we have proposed a different approach based on solving the Fredholm type integral equation that describes scattering of the light. Our model is not based on the radiative transfer theory but can be directly derived from the Helmholtz equation. As such, it also intrinsicly incorporates the wave properties of the light. However, unlike such methods as finite difference time-domain (FDTD), the proposed method does

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not compute the electromagnetic wave distribution in entire simulation space but only its values in the positions of light-scattering particles. This allows to save computer memory and computation time, and thus to simulate propagation of the light in larger volumes of the scattering medium.

In this paper, we briefly describe the mathematical basis of our method in case of a scalar light wave, which was described in more details in our paper.[2] Also, we present extension of the presented model to the Mie scattering of the polarized light. At the end, we show numerical experiments veryfing the correctness of the model: computations of mean free path of photons in the scattering materials and total scattering cross-section of a group of a large scattering particles.

2. DISCRETE SCATTERERS MODEL

The entry point of the presented numerical method is the Helmholtz equation that governs the propagation of the light in the scattering medium in the Fredholm type integral form (also known as the Lippmann-Schwinger equation):[2]

$$E(\boldsymbol{r}) = E^i(\boldsymbol{r}) + \int G_0(\boldsymbol{r} - \boldsymbol{r'})F(\boldsymbol{r'})E(\boldsymbol{r'})d^3r', \tag{1}$$

where $E(\boldsymbol{r})$ is the complex amplitude of the electric field in point $\boldsymbol{r}$, E^i is the complex amplitude of the field from an external source, $G_0(\boldsymbol{r})$ is the Green function and F is the scattering potential.

The proposed numerical method is based on the discrete scatterers model (DSM), which consist on treating the scattering medium as an uniform medium with randomly positioned scattering particles, what allows to discretize the eq. (1) and solve it numerically. In presented research, it was assumed that all particles are spheres with unifom optical properties and are all of the same size. However, as we discuss in further part of this paper this is not a necessary requirement.

The DSM (and numerical methods based on it) can be compared e.g. with the discrete dipol approximation (DDA) method for simulating light scattering by the large particles.[2] The difference between the DDA method and our method is the fact, that DDA deals with a uniformly spaced, strongly coupled dipoles,[10] while our approach deals with a nonuniformly spaced, lously coupled particles that may not act as a single electric dipole.

2.1 Introduction to Discrete Scatterers Model

The basis of DSM can be found in our previous paper.[2] Here, we only introduce basic equations that are necessary for further discussion.

The numerical methods based on DSM compute electric field only in the position of the scattering particles. In particularly simple case of small particles, the light scattering process can be described using the equation:[2]

$$\begin{bmatrix} E_1 \\ \vdots \\ E_N \end{bmatrix} = \begin{bmatrix} E_1^i \\ \vdots \\ E_N^i \end{bmatrix} + \begin{bmatrix} g_{1,1} & \cdots & g_{1,N} \\ \vdots & \ddots & \vdots \\ g_{N,1} & \cdots & g_{N,N} \end{bmatrix} \begin{bmatrix} E_1 \\ \vdots \\ E_N \end{bmatrix}, \tag{2}$$

where E_n is the amplitude of electric field at n-th particle, E_n^i is the amplitude of incident electric field at n-th particle, N is the number of particles and coefficients $g_{i,j}$ describe propagation of light wave in free space between i-th and j-th particle.

Further, we will use the shorter notation:

$$\boldsymbol{E} = \boldsymbol{E^i} + \boldsymbol{GE}. \tag{3}$$

or

$$\boldsymbol{ME} = \boldsymbol{E^i}, \tag{4}$$

for $\boldsymbol{M} = \boldsymbol{I}_{N\times N} - \boldsymbol{G}$.

Extensions of eq. (2) that takes into account the changes of the polarization of light in Rayleigh scattering regime can be found in our previous paper.[2]

2.2 Extending the DSM to Mie scattering case

In case of Mie scattering, the complex amplitude of the scattered light depends on the angle between the incident and scattered light direction θ.[11] For in-plane scattering, this can be written in a form resembling Jones vector formalism:

$$\boldsymbol{E_s^{(0)}} = \boldsymbol{J}(\theta)\boldsymbol{E_i^{(0)}}, \tag{5}$$

$$\boldsymbol{J}(\theta) = \frac{e^{jkr}}{jkr}\begin{bmatrix} -S_2(\theta) & 0 \\ 0 & S_1(\theta) \end{bmatrix}. \tag{6}$$

where $S_1(\theta)$ and $S_2(\theta)$ can be derived from the Mie theory.[11]

In most situations, one need to consider scattering process outside the scattering plane. In such a case, the polarization state of the incident and scattered light amplitudes may be expressed in different coordinate systems dependent on the light propagation direction. The eq. (5) need to be then modified to a form:

$$\boldsymbol{E_s} = \boldsymbol{R_s^{-1}}\boldsymbol{J}(\theta)\boldsymbol{R_i}\boldsymbol{E_i}, \tag{7}$$

where matrices $\boldsymbol{R_s}$ and $\boldsymbol{R_i}$ rotate the coordinate frames of the scattered and indicent light respectively, to align them with the scattering plane coordinate system.

In the system containing N particles, one can define the complex amplitude of the light illuminating each of the particles as: $E_{i,j,\alpha}$, where:

- i is the number of the particle and $i = 1, ..., N$,
- j defines the direction of the light and $j = 0, 1, ..., N$. Value $j = 0$ means the direction of the light from the external source and the pair i, j for $j \neq i$ and $j > 0$ means wave propagating from particle j to particle i.
- α is the polarization component of the light and $\alpha = x, y$ or $\alpha = 1, 2$.

The scattering process can be described using the equation:

$$E_{i,j,\alpha} = E^{(i)}_{i,j,\alpha} + \sum_{k,\beta} g_{i,j,k,\alpha,\beta} E_{j,k,\beta}, \tag{8}$$

where $E^{(i)}_{i,j,\alpha}$ are complex electric field amplitudes of the light from the external source and coefficient $g_{i,j,k,\alpha,\beta}$ can be computed using the eq.(5) - (7).

Eq. (8) can be expressed in a matrix-vector form:

$$\boldsymbol{E} = \boldsymbol{E^i} + \boldsymbol{G}\boldsymbol{E}, \tag{9}$$

that might be solved using the standard linear algebra algorithms.

3. NUMERICAL SIMULATIONS

In this section, we present two numerical experiment veryfing correctness of the presented DSM model. The first experiment consists on computing mean free path of the photons in scattering media and second – on computing the differerential light scattering cross-section of the group of scattering particle with sizes corresponding to Mie scattering regime.

3.1 Computing photon mean free path using scalar DSM

The mean free path of photon in the scattering medium can be computed by comparing the Green function of three-dimensional wave equation in free space with the Green function in the scattering medium averaged over many realization of this medium.[12] This two functions are connected by the folowing equation:

$$\langle G(r) \rangle = G_0(r) \exp(-r/(2l)), \tag{10}$$

where r is the distance between two points in space, $G_0(r)$ is the Green function describing light propagation between these two points in the free space, $\langle G(r) \rangle$ is the Green function for light propagation in the scattering medium averaged over many different realizations of this medium and l is the photon mean free path.

Function $\langle G(r) \rangle$ can be computed using DSM by solving eq. (4). In order to do that one can use the following Monte Carlo scheme: (I) use Green function for free space as an external field distribution $\boldsymbol{E^i}$, (II) use solved values of electric field on the scattering particles $\boldsymbol{E}$ to compute electric field distribution in the analyzed volume, (III) repeat this process for various realization of the scattering medium (various positions of scattering particles), and (IV) average the results. Example of such a simulation are presented in Fig. 1.

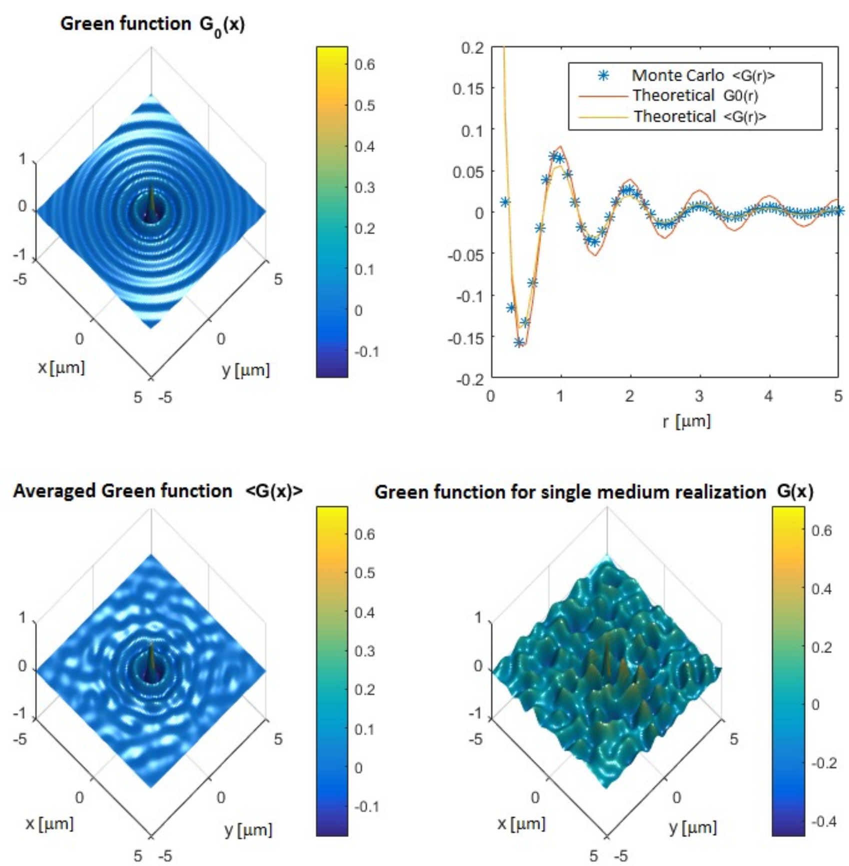

Figure 1. Example of photon mean free path computation using the proposed numerical method. In free space the wave propagates without loss of coherence. In case of light propagation inside the scattering medium, the phase of light beam contains not only coherent, but also noncoherent component. This noncoherent component is stronger further from the source causing the harmonic oscillation in the averaged (over the various realization of the scattering medium) Green function to be exponentially attenuated. This effect can be both qualitatively and quantitavely predicted using the described Monte Carlo algorithm.

3.2 Computing scattering cross-section of multi-particle system

In order to verify correctness of the DSM model for Mie scattering regime, we have compared it with the finite-difference time-domain simulation. For this purpose, the following numerical experiment was designed:

1. Set of eight spherical particles with refractive index 1.5 and radius 0.5 of light wavelength was considered.
2. The particles were placed in corners of the cube.
3. The length of the cube edge was varying between 1.5 to 2 wavelengths of the light.
4. For each edge, the far field pattern of the light scattered by the particles was computed using the FDTD simulation.
5. For each edge, the Jones vectors of light at each particle was computed by direct solving the DSM equations system. Then, the far field pattern of the scattered light was computed by adding patterns coming from each of the particles.
6. Computations from previous point were repeated by limiting the problem to the single scattering approximation (i.e. the vector $\boldsymbol{E}$ in right-hand side of eq. (9) was approximated by the vector $\boldsymbol{E^i}$).
7. Results of FDTD and DSM models were compared.

The sample simulation results are presented in Fig. 2 and 3.

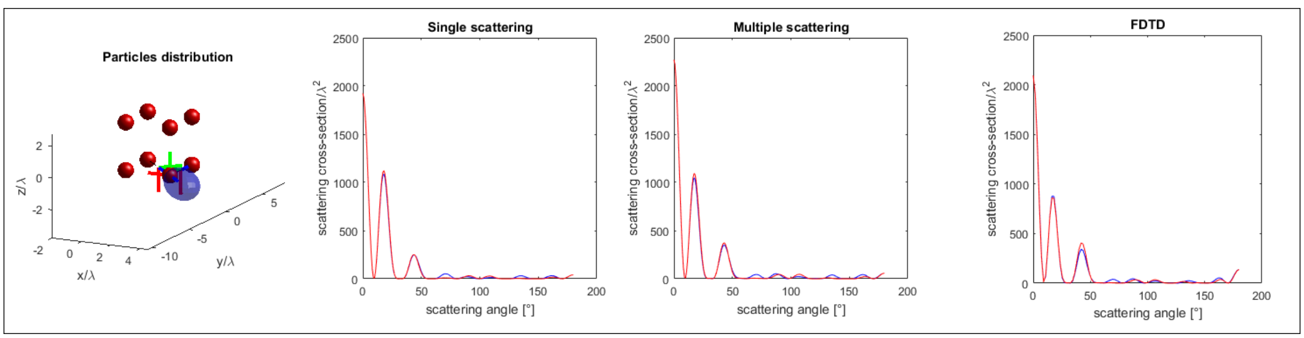

Figure 2. Comparison between FDTD and DSM models for scattering of light on a group of particles separated by 1.5 of light wavelength.

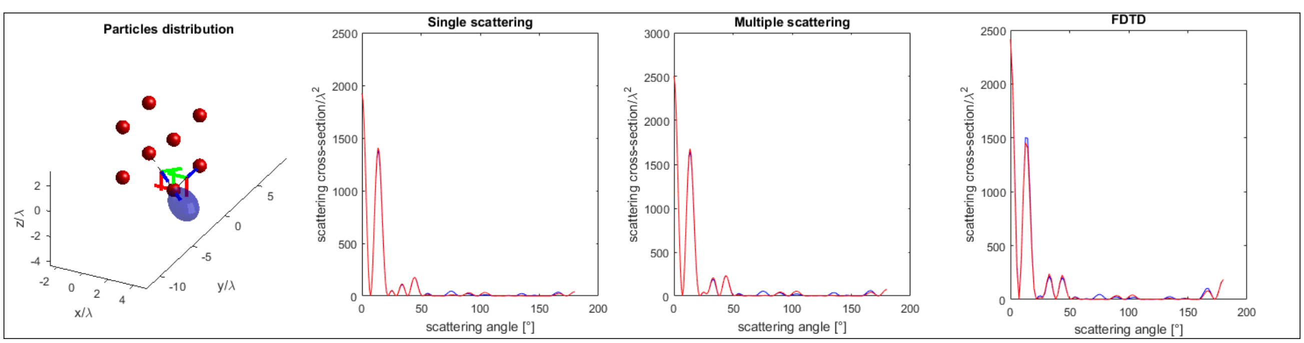

Figure 3. Comparison between FDTD and DSM models for scattering of light on a group of particles separated by 2 light wavelengths.

4. DISCUSSION

The presented research was focused on verification of the described numerical method and its extension to the Mie scattering regime. From the results presented in Fig. 1 one can see, that the proposed method predicts loss of light coherence during the propagation in scattering medium that is in agreement with the well established theories.

The extension of DSM model to Mie scattering regime is not an obvious task. One possible approach to solve this problem is usage of the numerical model presented in this paper. The performed simulation shows that it can correctly predict the scattering cross-section of a set of particles with sizes comparable with the light wavelength, what was confirmed by a FDTD simulations. Interesting feature of our DSM model is the fact it allows to easily reduce the problem to a single-scattering approximation (this is not possible e.g. in FDTD simulations) and thus analyze the impact of multiple light scattering on the observed physical phenomena.

The proposed approach to Mie scattering appears to fail in case of tightly-packed particles. For small distance between scattering particles (smaller than two ligth wavelengths), neither single nor multiple scattering case of DSM model did not provided results that were in agreement with FDTD model. The scattering cross-section of the entire particles group in the single scattering approximation was smaller than the reference FDTD result and the scattering cross-section in multiple scattering case was larger than the reference results. This can be attributed to the fact, that the proposed model does not include near-field effects in the scattered light wave that may play significant role in the scattering process when the particles are close to each other.

When particles separation exceeded two light wavelength, the results of multiple scattering case of DSM model were in good agreement with FDTD results. The comparison of single and multiple scattering cases showed that single scattering approximation was providing significantly lower value of scattering cross-section than the full multiple-scattering solution.

The proposed approach to Mie scattering simulation has one important drawback. Due to the fact that there is necessity to compute complex amplitude of electric field for each pair of scattering particles, the number of equations in the model grows very rapidly. Therefore, if systems with large amount of particles need to be analyzed, the different approach need to be developed and tested.

5. CONCLUSIONS

We presented the initial numerical verification of simulating the propagation of light in scattering medium using the Discrete Scatterers Model (DSM) by computing the averaged Green function of light in the medium and computing the photon mean free path. The numerical experiments are in agreement with well-known behaviour of light waves in turbid media. We also proposed a method for extending the DSM model to Mie scattering regime and compared this extension with FDTD simulations. The obtained results confirm validity of the proposed approach as long as the separation of scattering particles exceeds two wavelengths of the scattered light. The main drawback of the presented approach is the fact, that the required amount of computation time and memory rapidly grows with the number of scattering particles in the simulated system.

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