



## Stability of an Imperfect Truss Loaded by Wind

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The present paper is devoted to the numerical research of stability of a truss loaded by wind and stiffened by elastic supports located at the top chord. The lateral braces or lateral and torsional braces were taken into account. In this paper, the linear buckling analysis results for the beam and shell model were presented. Two different shapes of initial geometric imperfections were considered in the non-linear static analysis performed for the shell model of the structure. As a result, the truss buckling and the limit load were found to be related to the truss bracing stiffness and the threshold bracing condition, necessary to provide maximum buckling resistance of the truss, was obtained.

**Key words:** elastic supports, buckling load, limit load.

### 1. INTRODUCTION

Steel trusses have a much greater strength and stiffness in their plane than out of their plane, and therefore should be braced against lateral deflection and twisting. The problem of bracing requirements necessary to provide lateral stability of compressed members is presented in the design code [1]. The calculation models for assessment of the lateral supporting of the bottom truss chords as the flexible restraint in the roof trapezoidal sheets were presented in [2]. Usually the lateral (translational) brace stiffness is considered. However, the rotational stiffness of braces caused by the interaction between the torsional stiffness of the truss top chord and the bending stiffness of the roof elements (purlins, sandwich panels, trapezoidal sheet) should be taken into account.

The experimental and numerical analysis for out-of-plane buckling of trusses was presented in [3] and [4]. On the basis of analysis results the full bracing condition can be defined as the minimum bracing stiffness that causes the maximum buckling load of the truss.

In recent years, the authors' research was devoted to the stability analysis of truss girders subjected to gravity loading [4]. However, the numerical analysis results for the structure loaded by symmetric upward loading were presented in [5]. The present analysis is a continuation of the author's previous research [5], and new problems were taken into account. In this research, the non-symmetric truss loading due to wind [6] was considered (Fig. 1a). In addition, two different types of boundary conditions at the truss supports at their ends were investigated. The nonlinear analysis results (geometric and material nonlinear imperfection analysis-GMNIA) for the structure with initial geometric imperfection in the form of buckling mode were presented and the magnitudes of reaction forces in elastic braces were calculated. The truss without braces at the compressed bottom chord was considered.

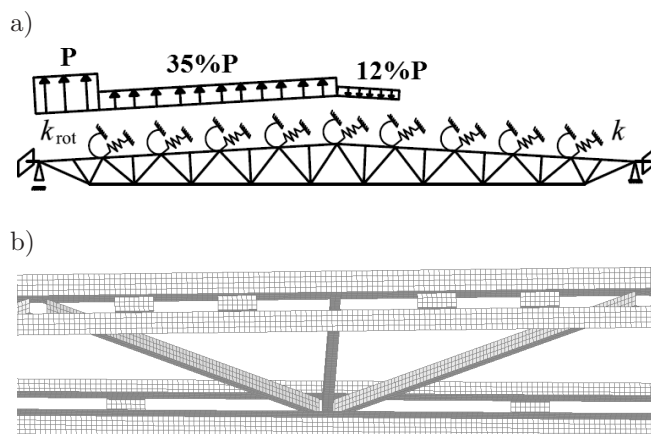


FIG. 1. Model of the truss: a) static schema, b) shell model detail.

## 2. DESCRIPTION OF THE TRUSS

The steel ( $f_y = 235$  MPa) truss length was  $L = 24.0$  m and the depth was  $h = 1.6$  m in the middle of the span (Fig. 1). The built-up cross-section of the top chord ( $2 \times L90 \times 90 \times 9$ ) was battened every 0.4 m and the bottom chord ( $2 \times L80 \times 80 \times 8$ ) every 0.8 m. The battens were made of C65 rolled profiles (length 0.15 m). Also, the diagonals and verticals were made of C65 profiles besides two diagonals near the end supports ( $2 \times L65 \times 65 \times 7$  – battened every 0.4 m). The truss was braced at the top chord by elastic supports of translational  $k$  [kN/m] and rotational  $k_{rot}$  [kN·m/rad] stiffness. The distance between braces was equal to 2.4 m. The structural models with prevented or free torsion at the truss supports at their ends were considered.

In the shell model of the structure about 63000 four-node shell elements ( $QUAD4$ ) [7] were used. The braces were modeled by means of degree-of-freedom



(DOF)-spring elements with lateral (translational) or rotational stiffness only. The loading was applied in the form of concentrated forces at the braced joints (arc-length method). In the beam model [8] of the structure, the standard one-dimensional (1D) elements (six degrees of freedom at node) were used. In the 1D modified model, each angle profile for the top and bottom chord cross-section was modeled separately.

### 3. NUMERICAL ANALYSIS RESULTS

In each case, the buckling load (obtained from the linear buckling analysis – LBA) increased with an increase of brace stiffness (Figs. 2 and 3). The threshold

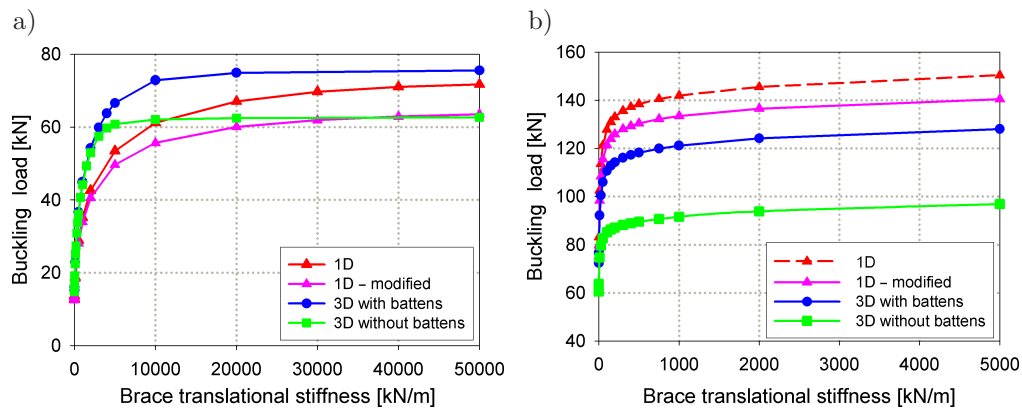


FIG. 2. The relation between buckling load and brace translational stiffness in 1D and three-dimensional (3D) models ( $k_{\text{rot}} = 0 \text{ kN} \cdot \text{m}/\text{rad}$ ) for: a) torsion free, b) torsion blocked at the marginal supports.

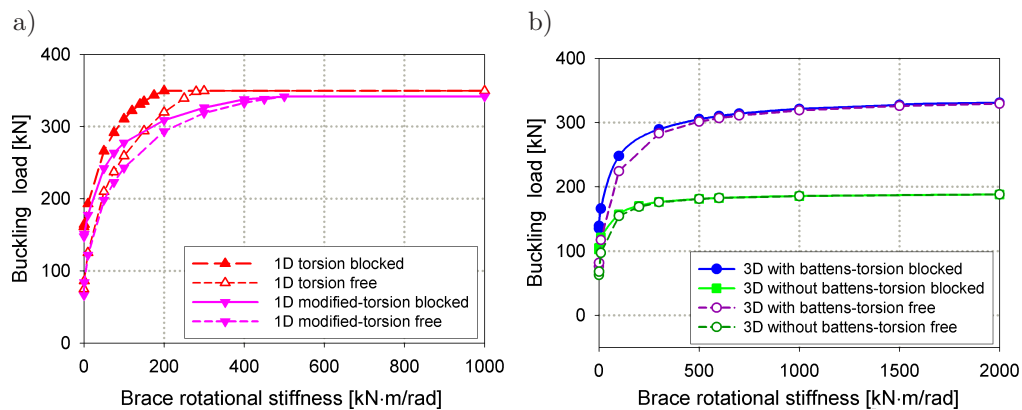


FIG. 3. The relation between buckling load and brace rotational stiffness ( $k = 10^6 \text{ kN}/\text{m}$ ) with respect to boundary conditions on the marginal supports in: a) 1D model of the truss, b) 3D model.



stiffness for the shell model of the structure (with battens) can be defined as  $k = 1000 \text{ kN/m}$  ( $k_{\text{rot}} = 0 \text{ kN} \cdot \text{m/rad}$  – torsion blocked at the marginal supports) or  $k = 600 \text{ kN} \cdot \text{m/rad}$  for the rigid lateral (translational) braces. For these magnitudes of brace stiffness, the buckling load was only about 10% lower in comparison to the structure with rigid intermediate supports.

For the structure with free torsion at the marginal supports (Fig. 2a) the buckling load obtained from the 1D models (or 1D modified – with battens) were lower than for the 3D models ( $k_{\text{rot}} = 0 \text{ kN} \cdot \text{m/rad}$ ). On the basis of buckling modes (Fig. 4) it was observed that in these cases the torsion of the truss top chord was partially stopped for the shell model. The explanation for this

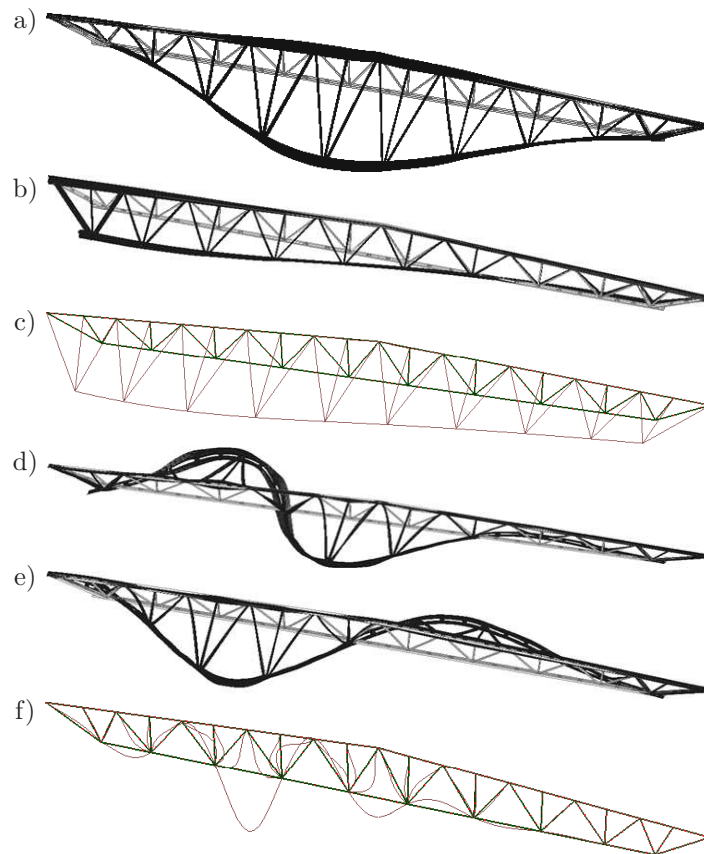


FIG. 4. Buckling modes for the truss with braces of stiffness: a)  $k = 10^6 \text{ kN/m}$ ,  $k_{\text{rot}} = 0 \text{ kN} \cdot \text{m/rad}$ , torsion blocked, shell model, b)  $k = 10^6 \text{ kN/m}$ ,  $k_{\text{rot}} = 0 \text{ kN} \cdot \text{m/rad}$  torsion free, shell model, c)  $k = 10^6 \text{ kN/m}$ ,  $k_{\text{rot}} = 0 \text{ kN} \cdot \text{m/rad}$  torsion free, beam model, d)  $k = 10^6 \text{ kN/m}$ ,  $k_{\text{rot}} = 600 \text{ kN} \cdot \text{m/rad}$ , torsion blocked, shell model, e)  $k = 10^6 \text{ kN/m}$ ,  $k_{\text{rot}} = 100 \text{ kN} \cdot \text{m/rad}$ , torsion blocked, shell model, f)  $k = 10^6 \text{ kN/m}$ ,  $k_{\text{rot}} = 200 \text{ kN} \cdot \text{m/rad}$ , torsion blocked, beam model.

might be the different performance of elastic braces in the numerical models. The elastic DOF-spring elements (braces) [7] were joined with several nodes on the top chord angles walls  $\text{kN} \cdot \text{m}/\text{rad}$  (webs). In the 1D models, the elastic supports are joined with the structure only at one node (center of gravity). For the structure with torsion blocked at the truss supports at their ends the buckling load magnitudes for the 3D models were lower than the results for the 1D models (Fig. 2b). The reason for these differences was the battened cross-section designed for some of the truss members and the fact that in the 1D models the warping was neglected.

In the nonlinear analysis (GMNIA) carried out for the shell model of the structure, two types of initial geometric imperfections were considered (maximum magnitude  $L/500$ ). The first one – Imperfection I can be described as arch curvature of the compressed bottom chord due to code requirements (Fig. 4a) [1]. The second – Imperfection II was implemented to the structure on the basis of buckled mode of the truss (obtained for the rigid braces) and it is presented in Fig. 4d. The analyses were performed for different boundary conditions at the marginal supports (torsion blocked or free). In each case, the limit load (maximum magnitude of loading) increased with an increase of brace rotational  $k_{\text{rot}}$  [ $\text{kN} \cdot \text{m}/\text{rad}$ ] stiffness (lateral stiffness  $k = 10^6 \text{ kN/m}$ ). The truss bearing capacity depended on the initial imperfection shape (Fig. 5a).

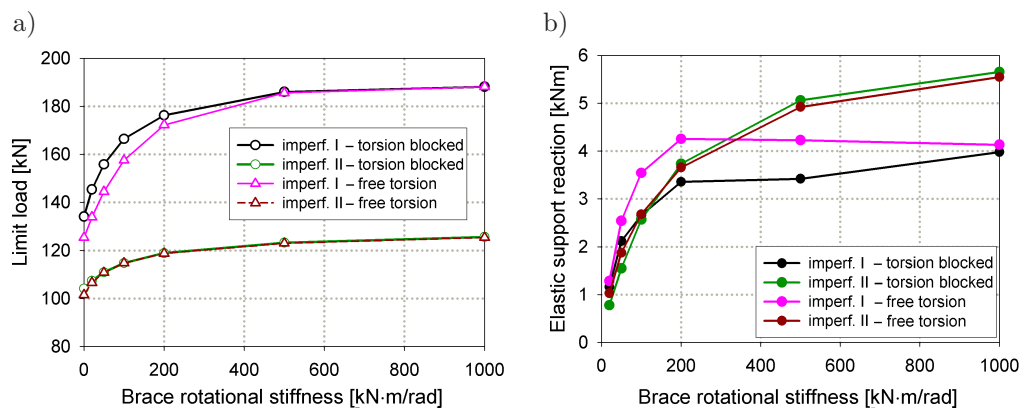


FIG. 5. The nonlinear analysis result ( $k = 10^6 \text{ kN/m}$ ): a) limit load for the truss due to the brace rotational stiffness, b) elastic support reaction due to the brace rotational stiffness.

The magnitude of elastic brace reaction (bending moment for the most strained brace – Fig. 5b) depended on truss deformation (Fig. 6). The rotation (torsion) of the top chord was larger for the structure with Imperfection II and it caused higher magnitudes of reaction forces in elastic supports.

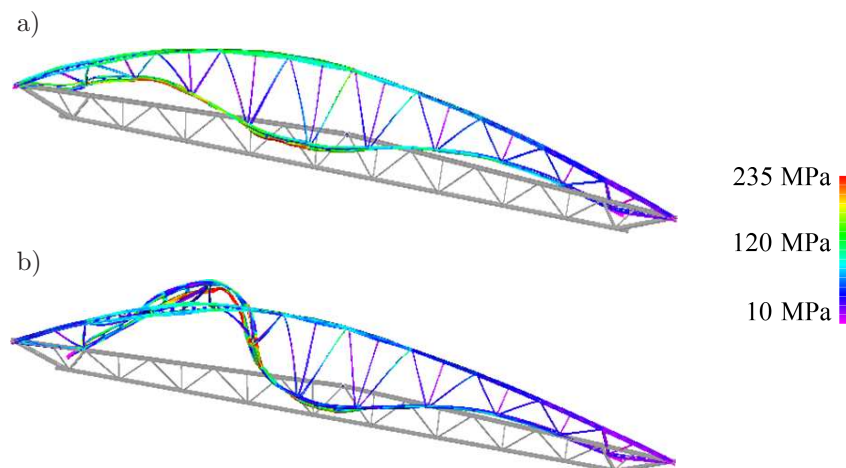


FIG. 6. Deformation at the limit state (GMNIA) for the truss with braces of stiffness:  
 a)  $k = 10^6$  kN/m,  $k_{rot} = 100$  kN · m/rad – imperf. I,  $k = 10^6$  kN/m,  $k_{rot} = 100$  kN · m/rad  
 – imperf. II.

#### 4. THE TRUSS BEARING CAPACITY DUE TO CODE REQUIREMENTS

The truss bearing capacity calculated on the basis of code requirements was compared to the GMNIA results obtained for the structure with initial geometric Imperfection II (Table 1). On the basis of the LBA results the normal force at the bottom chord ( $N_{cr}$ ), related to the buckling load ( $P_{cr}$ ), was calculated. On the basis of chord capacity  $N_{brd}$  the maximum magnitude for truss loading  $P_{max}$  was obtained.

**Table 1.** The comparison between truss maximum loading due to the code EC3 requirements and GMNIA results,  $\lambda$  – slenderness,  $\chi$  – buckling factor,  $P_{lim}$  – limit load GMNIA.

| Brace stiffness<br>$k$ [kN/m], $k_{rot}$ [kN · m/rad] | $P_{cr}$<br>[kN] | $N_{cr}$<br>[kN] | $\lambda$ | $\chi$ | $N_{brd}$<br>[kN] | $P_{max}$<br>[kN] | $P_{lim}$<br>[kN] |
|---|------------------|------------------|-----------|--------|-------------------|-------------------|-------------------|
| $k = 10^6$ , $k_{rot} = 0$ , torsion free             | 75.9             | 186.1            | 1.76      | 0.26   | 149.4             | 60.9              | 101.5             |
| $k = 10^6$ , $k_{rot} = 10^6$ , torsion free          | 341.1            | 836.6            | 0.83      | 0.68   | 393.4             | 160.4             | 128.2             |
| $k = 10^6$ , $k_{rot} = 0$ , torsion blocked          | 135.2            | 331.5            | 1.32      | 0.41   | 236.4             | 96.4              | 104.2             |
| $k = 10^6$ , $k_{rot} = 10^6$ , torsion blocked       | 342.1            | 839.2            | 0.83      | 0.68   | 393.9             | 160.6             | 128.4             |

#### 5. CONCLUSIONS

For the spatial models of truss girders subjected to wind loading the normal force corresponding to the flexural-torsional buckling of the compressed bottom chord can be calculated on the basis of buckling load coefficient (LBA). More-

over, the threshold (minimum) brace stiffness (translational –  $k$  [kN/m] and rotational –  $k_{\text{rot}}$  [kN · m/rad]), which ensures the maximum buckling resistance of the truss, can be precisely determined. These stiffness magnitudes may be useful during the design process. The possibility of conducting linear buckling analysis (LBA) is often present in structural analysis software.

The application of the battens to the members with built-up cross-section significantly increased the buckling resistance of the structure loaded by wind. In this case, the additional braces placed at the bottom truss chord, which are often designed, may not be necessary.

For the specific analyzed truss girder, often built in reality, the limit load obtained from the GMNIA depended on the imperfection shape (differences up to 35%).

Bearing capacity of the truss calculated according to code requirements was up to 20% lower in comparison to the nonlinear analysis results for the structure with initial geometric imperfection in the form of first buckling mode (calculated from LBA for rigid braces). In the code procedures, the acceptable imperfections of various elements should be defined.

It is worth noting that for the trusses with sloping top chords and rigid lateral braces the rotation of the structure is stopped and therefore the boundary conditions at the end supports (torsion free or restrained) had a small impact on the truss stability subjected to wind loading.

The structure bearing capacity with non-symmetric wind loading was up to 20% lower in comparison to the results presented in [5] for the symmetric upward loading.

The experimental research of the truss stability under upward loading is planned in the future.

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