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Stirling engines – the state of technology development and computational models

Stirling engines represent a technologically important solution in combined heat and power systems. Their use enables the achievement of over 90 percent efficiency in the management of the primary energy source with a very high durability of the device, mainly due to the lack of contact of the working gas with external factors and a very small number of mechanical components. The use of a Stirling engine may be equally important when applying renewable energy sources or waste heat from other processes. The first part of the work presents an overview of available commercial Stirling engine solutions. The second part of the work presents an overview of numerical models of Stirling engine operation, which enable the correct selection of the main geometrical features of the devices and the improvement of the structure in order to maximize efficiency or power.

1. Introduction

Stirling engines are one of the possible technological solutions enabling the management of the renewable and waste energy sources [10]. Currently, their main commercial application, however, is the production of heat and electricity from conventional energy sources, such as natural gas, diesel oil [8, 11, 20]. One of the most important technical problems related to the use of Stirling engines to generate electricity from low-temperature energy sources is the need to meet two basic countercurrent requirements, minimizing the dead volume associated with transporting the working gas between the compression and expansion spaces, and maximizing the heat exchange surface to provide the right amount of heat to the system with a low-temperature difference [18, 21, 25]. The optimization of the Stirling engine design requires very precise mapping of the influence of the above-mentioned design features on the efficiency and power of the system. For this purpose, computational models with varying degrees of details and accuracy are used. The paper presents an overview of the technology development of these structures.

2. Preview of commercial solutions

Stirling engine is powered from external source (an external chamber engine). This fact allows using heat for two simultaneous processes - heating and electricity production. The ability of supplying from the technological process as well (i.e. chemical or metallurgy industries) leads to reduce emission of carbon dioxide and consumption of fossil fuels. The Stirling engine is able to support process of electricity production with solar energy. Commercial systems aim at only electricity production or cogeneration working. These systems might be supplied by solid fuels, gas fuels and liquid fuels. Some commercial constructions have been presented below (Table 1).

Company Genoa offers engines in two variants [11, 12]. In the first configuration engines have one cylinder (configuration beta) and reach 1 kW (Fig. 1).The second configuration is equipped with two cylinders (configuration alpha) of power 3 kW (Fig. 2). Nitrogen is a working medium in both engines. Biomass and natural gas is a source of

heat. These engines require 700°C to start up, the heat source should have at least 850°C.

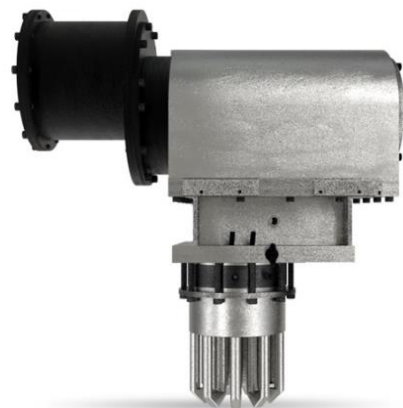Fig. 1. Two-cylinder Stirling engine Genoa, ML3000, power 3 kWe [13]

Fig. 2. One-cylinder Stirling engine Genoa ML1000, power 1 kWe [19]

Microgen products are based on one cylinder beta type Stirling engine. Available is setup with 1 kW and 2 kW electric power and heat might be generated in an external combustion chamber produced by burning natural gas or solid fuels. These engines belong to group free piston Stirling engines. Linear generator converts motion of piston on electric energy. This engine starts from about 180°C (working medium temperature). In Fig. 3 a view of Microgen Stirling engine has been presented.

Fig. 3. One-cylinder Microgen, power 1 kWe [5]

System of Inspirit Company includes generation of heat and electricity. In this solution the beta Stirling engine with free piston has been used. Overall efficiency equals to 92% including 16% of electric efficiency. In this case the heat of source is limited to natural gas. Version 3.1 is offered on the market. Previous versions of the engine generating lower power, i.e. Inspirit Charger 2.0 1–2 kWe and 5–10 kWt and Charger 3.0 1–3 kWe and 5–15 kWt. The last revision 3.1 offers up to 6.4 kWe and 15 kWt. In Fig. 4 Stirling engine of Inspirit (view on heater) has been presented.

Fig. 4. CHP Inspirit Stirling engine, power 1 kWe [8]

WhisperGen is distinguished by a lower electric power. In comparison with the engines previously presented, product of WhisperGen is prepared for working on land and sea. WhisperGen PPS 16 reaches 750 W of electric power and 5 kW of thermal power. The upper limit of the heat source is

set to 550°C. In Fig. 5 the WhisperGen PPS 16 has been presented.

Fig. 5. Stirling engine Whispergen PPS16, power 0.75 kWe [28]

Engines of Powergen series are based on the beta free piston Stirling engines with linear generators also in two variants of power, respectively 1.2 kWe and 5.65 kWe (Fig. 6). Produced heat exceeds 2 to 3 times electricity production. Electric efficiency is estimated on 30% (the highest value for analyzed construction). The working gas temperature in the heater can operate in the range 400–800°C. Engines of Powergen Company use helium as a working gas.

Fig. 6. Stirling engine Qnery PowergGen 1200/5650, power 1.2 kWe/5.65 kWe [15, 16]

In the Table 1 has been presented engines specification of Genoa and Microgen. In the Table 2 has been presented engines specification of WhisperGen, Qnergy and Inspirit Charger.

Table 1. Technical specification of Genoa Stirling and Microgen

Parameter	Unit	Genoa ML3000	Genoa ML1000	Microgen 1 kW	Microgen 2 kW
Electric power	kWe	3.3	1.1	1.05	2
Thermal power	kWt	18.6	10	4	8
Electric efficiency	%	14	14	15.5–23	15.5–23
Power supply	–	Gas or biomass	Gas or biomass	Pellet, briquette, wood, natural gas or heating oil	Pellet, briquette, wood, natural gas or heating oil
Working medium	–	Nitrogen (air)	Nitrogen (air)	Air	Air
Working temperature (hot side)	°C	850–900	850–900	180–525	180–525
Cooling medium	–	Water	Water	Air	Air
Working temperature inlet/outlet (cold side)	°C	15/60	15/40	Min. 6/max 70; nominal 30	Min. 6/max 70; nominal 30
Cooling medium flow	dm^3/mi	6.5	6.5	Nominal 8 (minimum 7)	Nominal 8 (minimum 7)

	n			

Table 2. Technical specification of WhisperGen, Qnergy and Inspirit

Parameter	Unit	WhispeGen PPS16	Qnergy PowerGen 1200	Qnergy PowerGen 5650	Inspirit Charger 3.1
Electric power	kWe	0.75	1.2	5.65	1–6.4
Thermal power	kWt	5	3–4.5	14–20	5–15
Electric efficiency	%	–	30	30	16
Power supply	–	Burning refined paraffin or diesel oil	Natural gas	Burning natural gas, LPG, propane	Natural gas
Working medium	–	Nitrogen	Helium	Helium	–
Working temperature (hot side)	°C	Up to 550	400–800	400–800	–
Cooling medium	–	Glycol	–	–	Water
Working temperature inlet/outlet (cold side)	°C	45/70	–	–	60/80
Cooling medium flow	dm^3/min	–	–	–	–

3. Numerical models of Stirling engine

3.1. Isothermal model

Mathematical model of Stirling engine was shown by Gustav Schmidt in 1871. Two basic assumptions of the model are constant temperature in each selected spaces and sinusoidal changing of volume working gas [1]. A heat included in the working gas is transformed to work. There is no heat transfer between environment and cooler, regenerator and heater. Heat transfer is realized in expansion and compression space. Based on these assumption, the engine works is a constant range of temperatures, the maximum temperature is reached in heater and expansion space, whereas the minimum temperature is observed in cooler and workspace called compression space. Those assumption cannot be met in a real engine because of the expected isothermal processing of the cycle, which requires infinity heat transfer surface in the heat exchangers [3].

It can mark out five work spaces of the engine: compression space c, cooler k, regenerator r, heater h and expansion space e. In Fig. 7 layout of temperature in work spaces was shown.
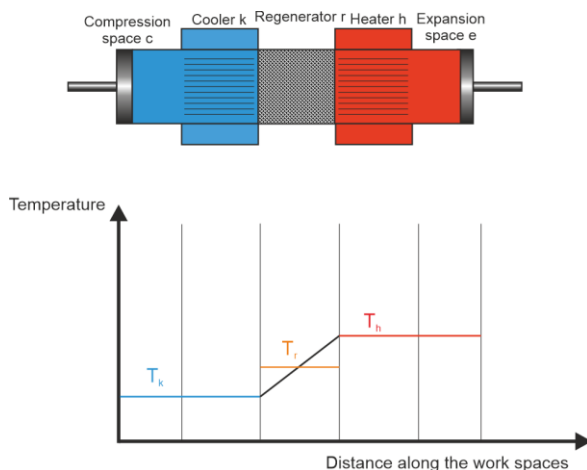

Fig. 7. Temperature profile in workspaces for isothermal model [23]

Assumptions for isothermal model are as follows [23]:
1. The constant gas temperature is over a surface of piston and in heat exchanger i.e. in compression space and in cooler (average temperature of lower heat source) are equal, that is $T_c = T_k$. The same rule is obeyed in the expansion space and the heater $T_e = T_h$.
2. Rotational speed is constant.

3. The total mass of the working gas is constant (no leakage) in accordance with the following equation (1):

$$M = m_c + m_k + m_r + m_h + m_e \qquad (1)$$

where M is the total mass of working gas in the system, m_c is the mass of working gas in the compression space, m_k is the mass of working gas in the cooler, m_r is the mass of working gas in the cooler, m_h is the mass of working gas in the heater and m_e is the mass of working gas in the expansion space.
4. Working gas is considered as an ideal gas.
5. The pressure of the working gas is the same in all spaces.
6. Volume change in the working spaces is sinusoidal.
7. Changes in kinetic and potential energy are negligible.

As a result of the usage of the Clapeyron equation, the total mass of gas equals:

$$M = \frac{P}{R}\left(\frac{V_c}{T_k} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_h}\right) \qquad (2)$$

where P is the working gas pressure, R is the individual gas constant of working substance, V is the volume of working gas and T is the absolute temperature of working gas. After transformation equation (2) the formula of the pressure in the system is defined as:

$$P = \frac{MR}{\left(\frac{V_c}{T_k} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_h}\right)} \qquad (3)$$

Temperature distribution in the regenerator is described by a linear function. In Fig. 8 the distribution of the temperature was shown. On the basis of the assumption there is no heat losses, the change of temperature is the same during heating and cooling process.
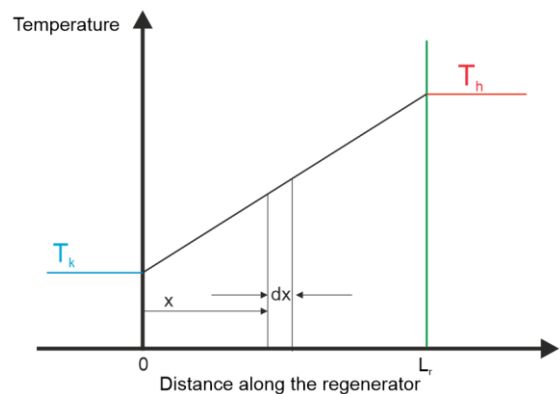

Fig. 8. Temperature distribution in ideal regenerator [24]

The change of temperature along the regenerator can be therefore defined as follows:

$$\frac{T(x)}{x} = \frac{T_h - T_k}{L_r} + T_k \qquad (4)$$

where $T(x)$ is the absolute gas temperature away of the inlet and L_r is the regenerator length.

By making transformations, we get:

$$T(x) = \left(\frac{T_h - T_k}{L_r}\right)x + T_k \qquad (5)$$

The mass of gas in the regenerator is determined by the following formula:

$$m_r = \int_0^{V_r} \varrho\,dV_r \qquad (6)$$

where m_r is the accumulated mass of gas in the workspace of regenerator, ϱ is the gas density and V_r is the volume of the regenerator workspace. The formula of gas density is define based on Clapeyron equation as:

$$\varrho = \frac{RT}{p} \qquad (7)$$

Volume of regenerator V_r is equal:

$$V_r = A_r L_r \qquad (8)$$

In differential form volume of regenerator equals:

$$dV_r = A_r dx \qquad (9)$$

Using equations (7)–(9) the final form of form of equation (6) is given by:

$$m_r = \frac{V_r P}{R} \int_0^{L_r} \frac{1}{T(x)l_r}\,dx \qquad (10)$$

Subsequently using equation (4) we obtain:

$$m_r = \frac{V_r P}{R} \int_0^{L_r} \frac{1}{(T_h - T_k)x + T_k L_r}\,dx \qquad (11)$$

and after integration:

$$m_r = \frac{V_r P}{R} \frac{\ln\left(\frac{T_h}{T_k}\right)}{T_h - T_k} \qquad (12)$$

or:

$$m_r = \frac{V_r P}{R T_r} \qquad (13)$$

Gas pressure is equal in all spaces, this fact allows calculation of mass based on gas volume and temperature. By summarizing formulas (13) and (7) we obtain:

$$T_r = \frac{T_h - T_k}{\ln\left(\frac{T_h}{T_k}\right)} \qquad (14)$$

Work of isothermal process in expansion space equals:

$$W_e = \oint P\,dV_e \qquad (15)$$

and respectively work in compression space is calculated by following formula:

$$W_c = \oint P\,dV_c \qquad (16)$$

Summarizing generated work in expansion (15) and compression space (16), leads to the total work of engine:

$$W = W_e + W_c = \oint P dV_e + \oint P\,dV_c \qquad (17)$$

According to first law of thermodynamics for open system [2]:

$$dQ + \left(C_p T_i m_i' - C_p T_o m_o'\right) = dW + C_v d(mT) \qquad (18)$$

where dQ is the heat transferred from the environment to workspace of cylinder, C_p is the specific heat by the constant pressure, T_i is the inlet gas temperature, m_i' is the inlet gas mass, T_o is the outlet gas temperature, m_o' is the outlet gas mass, dW is the work transferred into the environment and C_v is the specific heat by the constant volume. In the Fig. 9 the scheme of the cylinder workspace was shown.
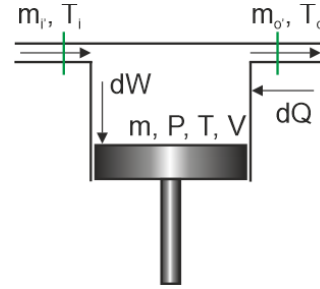

Fig. 9. Scheme of the cylinder workspace [7, 23]

Isothermal model is leading to simplification of formula (18), because the inlet temperature and the outlet temperature are equal ($T_i = T_o$). The formula describing difference between the inlet mass (m_i) and the outlet mass (m_o) is replaced by the parameter dm. Finally it can be described as:

$$dQ + C_p T dm = dW + C_v T dm \qquad (19)$$

Correlation between c_p and c_v is described as $c_P = c_v + R$, which reduces formula (19) to the form:

$$dQ = dW + RT dm \qquad (20)$$

Amount of the heat transferred into gas working during engine cycle equals integer dQ (20). After integration, the following equations are obtained:

$$Q_c = W_c, \; Q_e = W_e, \; Q_k = 0, \; Q_h = 0, \; Q_r = 0$$

where Q_c is the heat generated during gas compression, W_c is the work of compression process, Q_e is the heat generated during gas expansion, W_e is the work generated during gas expansion, Q_k is the heat derived from cooler into environment, Q_h is the heat rejected from environment, Q_r is the heat losses in regenerator.

The following conclusions can be drawn from the above considerations:
1. The heat exchange is processed only in compression and expansion space.
2. There is no heat losses in the regenerator.
3. In the heater and the cooler there is no heat transfer.

3.2. Adiabatic model

The novel approach in the field of modelling Stirling engine was caused by the adiabatic model developed by Finkelstein [26]. From the point of view of modelling process – compression and expansion cause a significant change. These processes are described by an adiabatic model. The heat transfer into cooler and heater is modelled by the isothermal model. Calculated efficiency based on the isothermal model is higher by about 20–30% in comparison to adiabatic model [9].

Isreaeli Urieli and Dave Berchowitz [26] developed quasi static model of ideal Stirling engine with adiabatic expansion and compression. The assumptions of model are as follows:

1. All processes are realized in a steady state.
2. The rotational speed is constant.
3. The working gas is treated as an ideal gas.
4. Process of compression and expansion are conducted without the heat transfer with the environment.
5. Instantaneous pressure in expansion and compression space is equal.
6. Temperature of gas in the cooler and heater is constant.
7. The change of temperature in regenerator is linear.
8. Kinetic and potential energy is negligible.
9. Heat transfer takes place only in the cooler and heater.
10. Total gas mass in the system is constant.
11. Heat loss as the result of flow between workspaces (compression and expansion) and also heat loss into the environment is negligible.
12. There is no gas leakage.

Split of workspaces was shown in Fig. 10, between workspaces implemented four domains (boundary areas): compression space-cooler ck, cooler-regenerator kr, regenerator-heater rh and heater-expansion space he.
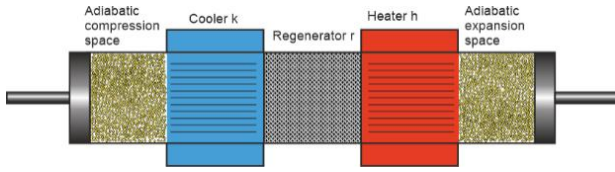

Fig. 10. Scheme of workspaces for adiabatic model [22]

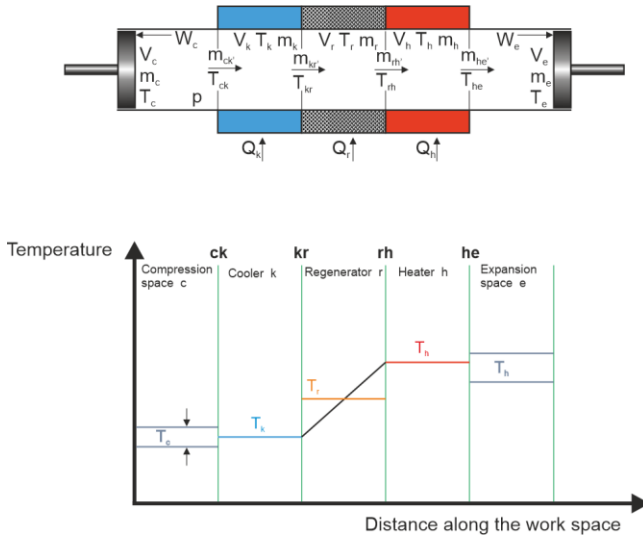Temperature profile in workspaces with split on domains was presented in Fig. 11.

Fig. 11. Temperature profile in workspaces for adiabatic model [22]

The values of temperature on the domain is conditioned by flow direction. Mass flow rate flowing through the domain ck equals m_{ck}'. Positive mass flow rate means gas flow to expansion space, whereas negative is correlated with flow in opposite direction. The value of temperature is

determined by direction flow with following equations (21)–(24) [3]:

$$T_{ck} = \begin{cases} T_c \text{ for } \dot{m}_{ck} > 0 \\ T_k \text{ for } \dot{m}_{ck} < 0 \end{cases} \tag{21}$$

$$T_{kr} = \begin{cases} T_k \text{ for } \dot{m}_{kr} > 0 \\ T_r \text{ for } \dot{m}_{kr} < 0 \end{cases} \tag{22}$$

$$T_{rh} = \begin{cases} T_r \text{ for } \dot{m}_{rh} > 0 \\ T_h \text{ for } \dot{m}_{rh} < 0 \end{cases} \tag{23}$$

$$T_{he} = \begin{cases} T_h \text{ for } \dot{m}_{he} > 0 \\ T_e \text{ for } \dot{m}_{he} < 0 \end{cases} \tag{24}$$

where T_{ck} is the gas temperature, m_{ck} is the mass of working gas on the edge compression space and cooler. In the adiabatic model gas mass is constant (similar isothermal model):

$$M = m_c + m_k + m_r + m_h + m_e \tag{25}$$

Working substance is considered as an ideal gas, according to Clapeyron equation in each compartment relationship with thermodynamical parameters are defined:

$$PV = mRT \tag{26}$$

After differentiating Clapeyron equation (26) has taken the following form:

$$\frac{dP}{p} + \frac{dV}{V} = \frac{dm}{m} + \frac{dT}{T} \tag{27}$$

Mass balance of system is written as:

$$dm_c + dm_k + dm_r + dm_h + dm_e = 0 \tag{28}$$

Total gas mass equals:

$$M = \frac{P}{R}\left(\frac{V_c}{T_k} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_h}\right) \tag{29}$$

Transforming formula (29), is the following formula has been obtained:

$$P = \frac{MR}{\left(\frac{V_c}{T_k} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_h}\right)} \tag{30}$$

Temperature and volume is constant in the heat exchangers (cooler, regenerator and heat) and therefore the formula (27) is reduced to:

$$\left.\frac{dm}{m}\right|_{k,r,h} = \left.\frac{dP}{P}\right|_{k,r,h} \tag{31}$$

Ideal gas equation in differential form is written as:

$$dm = dP\frac{V}{RT} \tag{32}$$

After substitution (31) in equation (28) mass balance equals:

$$dm_c + dm_e + dP\left(\frac{m_k}{p} + \frac{m_r}{p} + \frac{m_h}{p}\right) = 0 \tag{33}$$

Modification equation (33) by substitution of Clapeyron equation leads to notation:

$$dm_c + dm_e + \frac{dP}{R}\left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h}\right) = 0 \tag{34}$$

Separated section of analyzed spaces (compression space and cooler) was presented in Fig. 12.
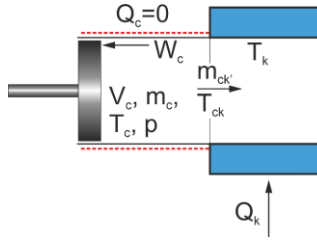
Fig. 12. Scheme of adiabatic compression space and cooler [22]

Compression space is insulated to environment and hence – form of change of internal energy took place only for work. In energy balance for compression space piece of equation related with gas flow in direction cooler occurred [6].

$$-C_p T_{ck} \dot{m}_{ck} = P dV_c + C_V d(T_c m_c) \tag{35}$$

The gas flow is realized from compression space to cooler thus expression (35) changed their form:

$$C_p T_{ck} m_c = P dV_c + C_V d(T_c m_c) \tag{36}$$

Using assumptions about ideal gas, relationship between specific heat by constant pressure and specific heat by constant volume is related with individual gas constant in next form [19]:

$$R = C_p - C_v \tag{37}$$

$$\kappa = \frac{C_p}{C_v} \tag{38}$$

where κ is the heat capacity ratio. Accumulated gas mass into compression space equals:

$$dm_c = \frac{P dV_c + \frac{V_c dP}{\kappa}}{R T_{ck}} \tag{39}$$

and in expansion space:

$$dm_e = \frac{P dV_e + \frac{V_e dP}{\kappa}}{R T_{he}} \tag{40}$$

Combining dependencies (37)–(40) a differential equation is obtained describing pressure in system:

$$dP = -\frac{P\kappa \left(\frac{dV_c}{T_{ck}} + \frac{dV_e}{T_{he}} \right)}{\frac{V_c}{T_{ck}} + \kappa \left(\frac{V_h}{T_h} + \frac{V_k}{T_k} + \frac{V_r}{T_r} \right) + \frac{V_e}{T_{he}}} \tag{41}$$

Determining equation of change of pressure (41) creates a system of nonlinear differential equations including parameters for compression space i.e. T_c, m_c and expansion T_e, m_e. Determination of pressure and mass allows gathering other parameters, volumes in workspaces dV_c, dV_k, dV_h and dV_e which are marked based on kinematic equations. Equation of state gas form compression space has got the following form:

$$T_c = \frac{P V_c}{m_c R} \tag{42}$$

After differentiating state equation (33) equals:

$$dT_c = \frac{V_c}{m_c R} dP + \frac{P}{m_c R} dV_c - \frac{P V_c}{R} \frac{1}{m_c^2} dm_c \tag{43}$$

hence:

$$dT_c = \frac{T_c}{p} dP + \frac{T_c}{V_c} dV_c - \frac{T_c}{m_c} dm_c \tag{44}$$

Final form equation of determining temperature in compression space:

$$dT_c = T_c \left(\frac{dP}{p} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c} \right) \tag{45}$$

In compression space gas temperature is obtained by formula:

$$dT_e = T_e \left(\frac{dP}{p} + \frac{dV_e}{V_c} - \frac{dm_e}{m_e} \right) \tag{46}$$

Using the energy balance equation for heat exchangers (constant temperature and volume of working gas) and equation state of an ideal gas (26), equation energy balance is written as:

$$dQ + \left(C_p T_i m_i' - C_p T_o m_o' \right) = C_v T dm \tag{47}$$

and:

$$dQ + \left(C_p T_i m_i' - C_p T_o m_o' \right) = V dP \frac{C_v}{R} \tag{48}$$

Amount of heat transferred in cooler equals:

$$dQ_k = V_k \frac{C_v}{R} dP - C_p (T_{ck} m_{ck}' - T_{kr} m_{kr}') \tag{49}$$

and in regenerator:

$$dQ_r = V_r \frac{C_v}{R} dP - C_p (T_{kr} m_{kr}' - T_{rh} m_{rh}') \tag{50}$$

also in heater:

$$dQ_h = V_h \frac{C_v}{R} dP - C_p (T_{rh} m_{rh}' - T_{he} m_{he}') \tag{51}$$

The temperature of gas is on the permanent level [3]:

$$\begin{cases} T_{kr} = T_k \\ T_{rh} = T_h \end{cases} \tag{52}$$

Total quantity of work is a sum of a produced work in each cylinder:

$$W = W_e + W_c \tag{53}$$

processes are described by adiabatic model hence work of cycle equals:

$$dW = p dV_e + p dV_c \tag{54}$$

3.3. CAFS – Combined Adiabatic–Finite Speed

Mathematical model developed by Hosseinzadeh and other [7] is an amplification adiabatic model [26], additional elements are mechanical losses and finite speed thermodynamic.

The idea of thermodynamic with finite speed was presented in 50's of 20th century [14]. The assumption of FST is the analysis of real processes which occurred in thermodynamical systems of piston engines towards optimization (increase power and efficiency) and it included irreversible phenomena.

The main task of thermodynamic with finite speed is to describe the interaction between piston (moving with speed w_p) and gas molecules located in the system (moving with average speed of sound c).

Petrescu and other [14] demonstrated that in consequence of piston motion gas pressure being over piston surface are different from instantaneous mean pressure in cylinder. This fact means that compression and expansion work are different to calculated work based on the classical thermodynamic. It causes that work generated by engine is

lower than received work by classical thermodynamic. On account of piston motion local disorder of gas placed in work space is generated leading to make an acoustic wave and as a consequence of this phenomena pressure drops [4]. The propagation of wave took place with the speed of sound [7].
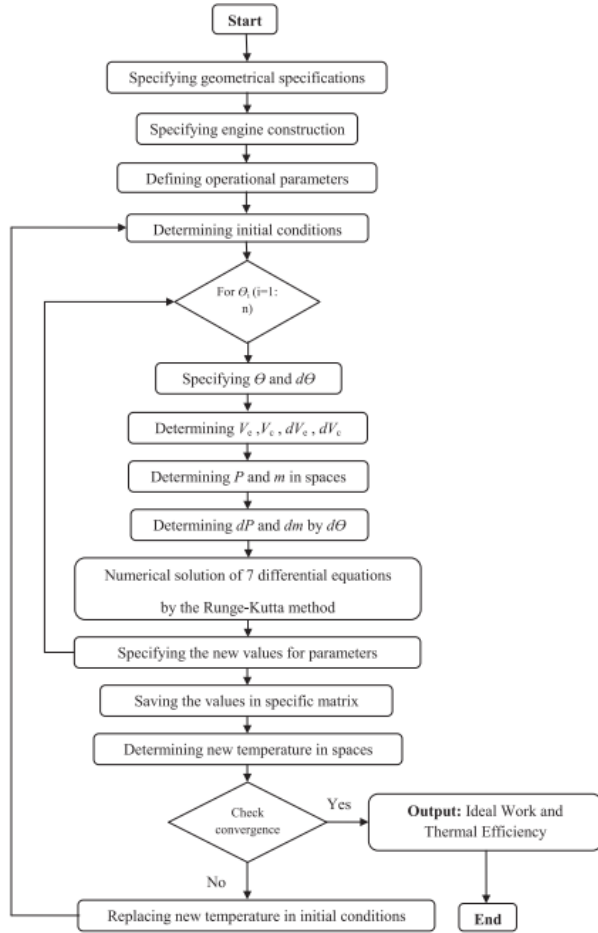

Fig. 13. Flow chart of the algorithm of ideal adiabatic model [7]

The first law of thermodynamics for process with finite speed in a closed system is written as (the scheme of workspace is presented in Fig. 9):

$$dU = \delta Q - p\left(1 \pm \frac{aw_p}{c} \pm \frac{fr\Delta p_f}{p_m}\right)dV \qquad (55)$$

where dU is the change of internal energy, δQ is the transferred heat between cylinder and environment, p is the instantaneous mean pressure over piston, a is the coefficient $a = \sqrt{3\kappa}$ [4], w_p is the piston speed, c is the speed of sound $c = \sqrt{3RT}$ [4]. Friction factor fr is a range of from 0 to 1. For fr = 0 heat transfer took place to environment (compression space), heat is keep in the system while fr = 1.

One of the assumption is that the present model is adiabatic process during expansion and compression, hence expression $\delta Q = 0$. Based on equation (58) work is equal change of internal energy with negative sign:

$$dU = -p\left(1 \pm \frac{aw_p}{c} \pm \frac{\Delta P_f}{p_m}\right)dV \qquad (56)$$

hence equation (56) is written:

$$mC_v dT = -p\left(1 \pm \frac{aw_p}{c} \pm \frac{\Delta P_f}{p_m}\right)dV \qquad (57)$$

Work of irreversible process is obtained:

$$\delta W_{FST} = p\left(1 \pm \frac{aw_p}{c} \pm \frac{\Delta p_f}{p_m}\right)dV \qquad (58)$$

where δW_{FST} is calculated irreversible work with using FST. Sign (+) is applicated in equations (55)–(58) for describing expansion process, sign (–) occurs while compression process.

Drop pressure ΔP_f is caused by mechanical losses (friction) ΔP_f. it is computed by equation [4]:

$$\Delta P_f = \frac{(0.4 + 0.0045w_p)10^5}{3\zeta}\left(1 - \frac{1}{cr}\right) \qquad (59)$$

where ζ is the constant value [5]:

$$\zeta = 1 - \frac{1}{cr} \qquad (60)$$

where cr is the compression ratio. Total work of engines (with throttle losses) equals [5]:

$$\delta W_{total} = \delta W_{FST} + \Delta P_{th}dV \qquad (61)$$

where δW_{total} is generated work by engine with considered finite piston speed, mechanical friction losses and throttle phenomena in heat exchangers, ΔP_{th} is the drop pressure in heat exchangers, V is the instantaneous gas volume.

Drop pressure in heater and cooler are significantly lower than in regenerator. According to Darcy-Weisbach drop pressure equals [27]:

$$\Delta P = fr\frac{\varrho}{2}\frac{L}{D_H}u^2 \qquad (62)$$

where fr is the friction coefficient, ϱ is the gas density, L is length, D_H is hydraulic diameter and u is the speed of gas flow.

Drop pressure of throttling is obtained in result of summarizing drop pressure in regenerator, cooler and heater:

$$\Delta P_{th} = \Delta P_r + \Delta P_k + \Delta P_h \qquad (63)$$

The efficiency regenerator is calculated based on number of thermal unit (NTU) [7]:

$$\varepsilon = \frac{NTU}{1 + NTU} \qquad (64)$$

where ε is the regenerator efficiency, NTU is the number thermal unit is determined based on formula:

$$NTU = \frac{StL_r}{D_{hr}} \qquad (65)$$

where St is Stanton number, L_r is length of regenerator and D_{hr} is regenerator hydraulic diameter. Stanton number is determined based on Reynolds and Prandtl number [13]:

$$St = 0.023Re^{-0.2}Pr^{-0.6} \qquad (66)$$

Hydraulic diameter is calculated in the following way [7]:

$$D_{hr} = \frac{4\Pi}{\Phi(1 - \Pi)} \qquad (67)$$

where Π is the porosity regenerator, Φ is the coefficient shape is related with volumen and surface:

$$\Phi = \frac{A_{wg}}{V_{mr}} \qquad (68)$$

where A_{wg} is the wetted surface and V_{mr} is the volume of wires regenerator. Gas temperature in heater and cooler is received by solving equations [7]:

$$T_h = T_{wh} - \frac{fr[Q_h + Q_r(1-\varepsilon_r)]}{\alpha_h A_h} \tag{69}$$

$$T_k = T_{wk} - \frac{fr[Q_k + Q_r(1-\varepsilon_r)]}{\alpha_k A_k} \tag{70}$$

where T_h is the gas temperature in heater, T_{wh} is the wall temperature in heater, fr is engine frequency, Q_h is heat supplied to the heater, Q_r is heat exchanged in regenerator, ε_r is regenerator efficiency, α_h is heat transfer coefficient in heater, A_h is heat transfer surface in heater, T_k is gas temperature in heater, Q_k is heat removed from the cooler, α_k is the heat transfer coefficient in cooler and A_k is the heat transfer surface in cooler. Heat transfer coefficient is obtained using the following equation [7]:

$$\alpha = \frac{0.0791 \mu C_p Re^{0.75}}{2 D_h Pr} \tag{71}$$

where α is heat transfer coefficient, μ is dynamic viscosity, C_p is the specific heat and D_h is hydraulic diameter.

3.4. PSVL – Polytropic model Stirling with various losses

Babaelahi and other [1] developed their model based on the adiabatic model. Energy balance according to first law of thermodynamic for open system is as follows [1]:

$$dQ - dQ_{poli} - dQ_{sh} + (m_i C_{p,i} T_i - m_o C_{p,o} T_o) =$$
$$= dW + C_V d(mT) \tag{72}$$

where dQ is transferred heat between working gas and environment, dW is network generated by engine, dQ_{poli} is heat emitted in compression and expansion space by polytropic model and dQ_{sh} is transferred heat by displacer from expansion to compression space shuttle effect of piston [1]:

$$dQ_{sh} = \frac{\pi S^2 \lambda_t D_{dis}(T_e - T_c)}{8 J L_{dis}} \tag{73}$$

where S is stroke, λ_{dis} is the displacer thermal conductivity D_{dis} is diameter of displacer, J is size of the gap between the displacer and the cylinder, L_{dis} is length of displacer. The fourth piece of left side equation (73) is related with enthalpy change of the working substance. The second piece of right side energy balance equation concerns change of internal energy in volume. Transferred heat into environment dQ_{poli} in compression and expansion chamber is obtained [8]:

$$Q_{poli} = m C_n (T_0 - T) \tag{74}$$

where C_n is the specific heat polytropic, T_0 is the ambient temperature and T is the temperature in controlled space. Differential form of equation energy balance is written as:

$$dQ_{poli} = C_n(T_0 - T)dm - m C_n dT \tag{75}$$

Polytropic specific heat is determined in the following way [17]:

$$C_n = C_V \frac{n-k}{n-1} \tag{76}$$

According to the polytropic equation:

$$PV^n = const \tag{77}$$

whereas differential form of polytropic equation equals:

$$P(nV^{n-1}dV) + nV^n dP = 0 \tag{78}$$

hence polytropic exponent:

$$n = -\frac{VdP}{PdV} \tag{79}$$

After modification we get a formula describing the change of mass gas:

$$dm_c = \frac{\left[\left(\frac{PdV_c + \frac{V_c}{\kappa}dP}{RT_{ck}}\right) - \left(\frac{C_{nc} m_c}{C_p T_{ck}}dT_c\right) - \left(\frac{Q_{sh}}{C_p T_{ck}}\right)\right]}{\left[\left(\frac{C_{nc}}{C_p}\right)\left(\frac{T_0 - T_c}{T_{ck}}\right) + 1\right]} \tag{80}$$

where C_{nc} is the polytropic exponent in compression space and C_{ne} is the polytropic exponent in expansion space, thus $dm_c = \dot{m}_{ck}$. An analogous description is retained for the expansion space

$$dm_e = \frac{\left[\left(\frac{PdV_e + \frac{V_e}{\kappa}dP}{RT_{he}}\right) - \left(\frac{C_{ne} m_e}{C_p T_{ck}}dT_e\right) - \left(\frac{Q_{sh}}{C_p T_{he}}\right)\right]}{\left[\left(\frac{C_{ne}}{C_p}\right)\left(\frac{T_0 - T_e}{T_{he}}\right) + 1\right]} \tag{81}$$

By ordering the expressions (80) and (81), the equation defining the change in pressure dP is obtained:

$$dP = \frac{-\kappa\left[\frac{\frac{PdV_e}{T_{he}} - \frac{RC_{ne}m_e}{C_p T_{he}}dT_e + \frac{RQ_{sh}}{C_p T_{he}}}{B_1} + \frac{\frac{PdV_c}{T_{ck}} - \frac{RC_{nc}m_c}{C_p T_{ck}}dT_c + \frac{RQ_{sh}}{C_p T_{ck}}}{B_2}\right] + Rm_w}{\frac{V_c}{T_{ck}B_1} + \frac{V_e}{T_{he}B_2} + \kappa\left(\frac{V_k}{T_k} + \frac{V_h}{T_h} + \frac{V_r}{T_r}\right)} \tag{82}$$

where:

$$B_1 = \left(\frac{C_{nc}}{C_p}\right)\left(\frac{T_0 - T_c}{T_{ck}}\right) + 1 \tag{83}$$

$$B_2 = \left(\frac{C_{ne}}{C_p}\right)\left(\frac{T_0 - T_e}{T_{he}}\right) + 1 \tag{84}$$

Working gas temperature in compression space is calculated based on following expression:

$$dT_c = T_c\left(\frac{dP}{p} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c}\right) \tag{85}$$

Gas temperature in expansion space is computed in a similar way:

$$dT_e = T_e\left(\frac{dP}{p} + \frac{dV_e}{V_c} - \frac{dm_e}{m_e}\right) \tag{86}$$

Transferred heat into the heat exchanger equals respectively [9]:

$$dQ_k = V_k \frac{C_V}{R}dP - C_p(T_{ck}\dot{m}_{ck} - T_{kr}\dot{m}_{kr}) \tag{87}$$

$$dQ_h = V_h \frac{C_V}{R}dP - C_p(T_{rh}\dot{m}_{rh} - T_{he}\dot{m}_{he}) \tag{88}$$

$$dQ_r = V_r \frac{C_V}{R}dP - C_p(T_{kr}\dot{m}_{kr} - T_{rh}\dot{m}_{rh}) \tag{89}$$

Generated work by gas in compression space:

$$dW_c = pdV_c \tag{90}$$

and expansion space:

$$dW_e = pdV_e \tag{91}$$

Overall work was produced by engine is a sum of work made by gas in compression and expansion space.

$$W = \int dW_c + \int dW_e \tag{92}$$

Engine efficiency is quotient of work engines (92) and delivered heat into system (88):

$$\eta_{netto} = \frac{W}{Q_h} \qquad (93)$$

The assumption of nonideal heat transfer in regenerator means that efficiency is lower than 100 percent. Regenerator efficiency ε is defined as its ability to change gas enthalpy that is a real change of enthalpy to the theoretical maximum change of gas enthalpy. Included heat loss, amount of heat delivered to the heater and taking from cooler equals:

$$Q_h = Q_{hid} + Q_{rl} = Q_{hid} + Q_{rid}(1-\varepsilon) \qquad (94)$$

$$Q_k = Q_{kid} - Q_{rl} = Q_{kid} - Q_{rid}(1-\varepsilon) \qquad (95)$$

Element $Q_{rid}(1-\varepsilon)$ represents heat loss in the regenerator. In comparison with formulas (94)–(95) in this analyze it is regarded regenerator efficiency ε. Regenerator efficiency with assumption linear profile of temperature [26] as function inlet and outlet temperature [1]:

$$\varepsilon = \frac{1}{1+\frac{\delta T}{T_{h_i}-T_{h_o}}} \qquad (96)$$

where ε is the regenerator efficiency, δT is the difference temperature between flowing gas from heater in regenerator and flowing gas from cooler, T_{h_i} is the inlet gas temperature to regenerator from heater and T_{h_o} is the outlet gas temperature from regenerator. According to the equation (96) the increase of regenerator efficiency occurs when outlet gas temperature T_{h_o} reaches lower temperature (supplying regenerator from heater side – in regenerator more heat is located). Heat balance of the flowing gas equals $\dot{m}C_p(T_{h_i} - T_{h_o}) = \alpha A_{wg}\delta T$. In consequence regenerator efficiency is defined as:

$$\varepsilon = \frac{1}{1+\frac{\dot{m}C_p}{\alpha A_{wg}}} = \frac{1}{1+\frac{1}{NTU}} = \frac{NTU}{1+NTU} \qquad (97)$$

where $\dot{m}$ is the mass flow rate, C_p is the specific heat of gas, α is heat transfer coefficient, A_{wg} is the wetted surface by gas.

It is impossible to implement the isothermal model in reality. Hence this model might be only used in order to determine maximum efficiency for the examined engine. This model is based on algebraic equation, this fact allows calculation in an easy way. In other words, results of the isothermal model is upper limit in regarding reached values of the power and efficiency.

Adiabatic model is closer to real engines than isothermal one because in expansion and compression space temperature change occurs but in the heat changers there is still a constant temperature. In the real engines heat transfer appears mainly in the heat exchanger, in significantly small scale in cylinder, so this model is discrepant from real conditions. Adiabatic model is basic for modern models.

In Combined Adiabatic model with Finite Speed an assumption is included that the change of temperature in heat exchangers, nonideal regenerator and mechanical (friction) losses, pressure losses (throttling) and thermodynamics

with finite speed. Polytropic model is development of this approach.

One of serious restriction for CAFS and PVSL is necessary awareness of exact specification. Full technical specification often is restricted by producers. CAFS and PVSL (and other models with nonisothermal processes in heat exchangers and cylinders) might be used for modelling real machines.

Isothermal and simple adiabatic model might be only applicable for tentative calculations – estimating power and efficiency for set parameters.

Conclusions

So far devices with Stirling engines offered nowadays on market are mostly not designed to supply with low-temperature heat sources, usually they are equipped with a combustion chamber allowing supply with conventional fuels such as compressed natural gas or mixture of propane and butane. Heaters of such kind of engines c cannot be directly used to supply with contaminated flue gas due to the small cross-sectional area of slots, which the gas passes.

Few examples of devices that enable the use of waste energy, require proper operation of the temperature of energy source no lower than 500°C. Such restrictions have two main reasons: the first one, of a technical nature, heavy mechanical losses in the SE caused by high pressure of working gas, often exceeding 60 MPa [29]. The second reason is of economical nature, SE supplied with low temperature heat source will have extremely long period of return on investment (» 20 years).

The key improvements, which must be done before commercialization of SE supplied with low temperature renewable sources are lowering the mechanical losses in the SE, rising performance of the flue gases – working gas heat exchanger and developing low cost manufacturing technology that ensures low cost of production.

The analysed numerical models of SE operation have a different level of complexity and describe the processes taking place in a real device with a different degree of idealization. Depending on the stage of development of structures powered by low-temperature energy sources, the presented models can be effectively used to determine the direction of changes in the design parameters of the device. The first of them the isothermal model (section 3.1) enables the analysis of the main design parameters of the device without the necessity to take into account the empirical coefficients that are difficult to collect at the initial design stage of design. In engines powered by low-temperature energy sources, it is extremely difficult to design a structure, in which the heat is delivered in the expansion space and not through an external heat exchanger. Hence, the adiabatic model (section 3.2) will be characterized by a higher accuracy, but still does not require the use of hard-to-obtain empirical coefficients. This kind of model is unfortunately more computationally demanding. The next CAFS model – Combined Adiabatic-Finite Speed (section 3.3) should be applied, when the structural details of the designed device are known, the computational base is the adiabatic model, however, a number of hydraulic losses are also taken into account. Experimental coefficients are required for the calculations. The last model, with the most

complexity, PSVL – Polytropic model Stirling with various losses (section 3.4), enables a more detailed description of the phenomena occurring in the area of compression and expansion by using a polytropic model instead of the adia-batic one. It takes into account a similar model of hydraulic losses as previously described, and at the same time requires the greatest number of experimental factors.

Nomenclature

CAFS	Combined Adiabatic Finite Speed
FST	Finite Speed Thermodynamics
NTU	Number of transfer units

PSVL	Polytropic model Stirling with Various Losses
SE	Stirling Engine

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