STOCHASTIC MODEL OF THE LOAD SPECTRUM FOR MAIN ENGINES OF SEA-GOING SHIPS

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Abstract

The paper presents possibility of applying the theory of semi-Markov processes for probabilistic description of load spectrum for diesel engines employed for ship propulsion, so for main engines. The considerations include output power characteristics for this kind of engines. The characteristics have enabled formulating a four-member set of states of the process of load on the engines. The theory of semi-Markov processes has been applied for describing the real process of loads on the mentioned engines. This theory has made possible building a model of the engine loads in form of a continuous-time semi-Markov process with a four-state set of values. Properties of a Darboux continuous function, which allow considering the engine loads as the discrete-states and continuous-time processes, have been used to build the model. In consequence a limiting distribution of the occurring process of successive states of engine loads could be determined. The distribution is the probabilities of staying a sea-going ship main engine in successive states of load. The distribution is characterized by the spectrum of loads on this kind of engines. The presented model can be developed by taking into account many states of the process of main engines loading as the need is to make such a probabilistic description of the process that provides possibility of rational control of the operating process.

Keywords: *semi-Markov process, load, diesel engine, main engine, sea ship*

1. Introduction

Empirical researches show that values of load on any engine installed in propulsion system of a sea-going ship, so - on main engine, are not possible to predict exactly [1, 7, 10, 12, 13]. Thus, the following hypothesis (H_1) can be formulated: „engine load is a random variable because its values from successive measurements can be predicted only under determined probability" [3]. Therefore, considering the load on any engine of this kind, at any time of operation, the hypothesis (H_2) can also be stated as true: „process of engine loading is a stochastic process because the values of engine loads, assigned to a moment in time are random variables" [3]. From the theory of stochastic processes results that a set of mentioned moments is a set of parameters of the process. In this situation, in order to recognize the characteristics of loads in any moment of main engine operating time the load spectrum needs to be determined. Operating tests on this kind of engines show that they are loaded in continuous way and they work for longer time under strictly determined loads imposed in accordance with valid standards of their operation [7,10]. Transition from one load to another is made at time much more shorter than the duration of each such load is. Transition of loads proceeds randomly. This means that it should be described in form of a random process $\{B(t): t \geq 0\}$ of continuous, positive and limited realizations making a set W. Formulation of a model of the process requires digitizing its values W [2, 4, 8, 9].

2. Determination of limiting distribution of engine load

As a result of digitizing the values W of the process $\{B(t): t \geq 0\}$ the following sequence of intervals can be obtained:

$$\{W_k = [w_{k-1}, w_k)\}, k = 1, 2, \ldots, K,$$

which will be a division of the set W. This indicates that the following relations proceed:

$$W = \bigcup_{k=1}^{K} W_k \quad \text{and} \quad W_i \cap W_j = \varnothing \ \text{ for } i \neq j.$$

In consequence the process $\{B^*(t): t \geq 0\}$ is obtained, of which values belong to the set $C = \{c_1, c_2, \ldots, c_K\}$ and which is determined by the formula:

$$B^*(t) = c_k \ \text{ for } t \in B^{-1}(W_k), k = 1, 2, \ldots, K.$$

From the properties of a Darboux continuous function results [8] that the process $\{B^*(t): t \geq 0\}$ is a random function with the graph of states transition presented in Fig. 1.

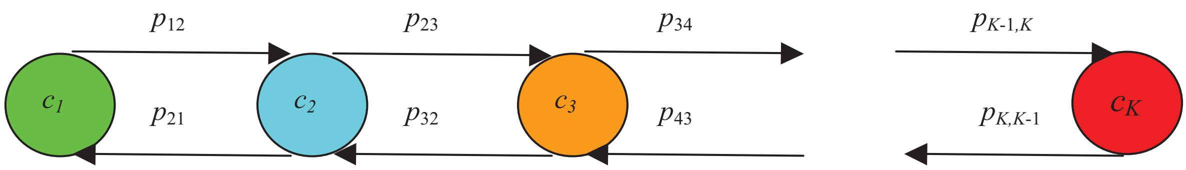

Fig. 1. Graph of states transition of the proces {B(t): t ≥0}*

In case of ship main engines it can be accepted that the most significant are these loads which make the set [10, 11]:

$$C = \{c_1, c_2, c_3, c_4\} \tag{1}$$

with the following interpretation of its members [….]:

c_1 – engine load according to the partial output power characteristic,
c_2 – engine load according to the operating output power characteristic,
c_3 – engine load according to the rated output power characteristic,
c_4 – engine load according to the maximum output power characteristic.

Distinguishing these types of loads is very important because they result from principles of operation of ship main engines and the associated need of precise determination of a fuel strip position which corresponds to particular load.

In this case it can be accepted that the stochastic process $\{D(t): t \geq 0\}$of which intervals are identical (constant) and realizations are continuous on the right side, is equivalent in stochastic terms to the process $\{B^*(t): t \geq 0\}$, [5, 6]. Lengths of intervals $[\tau_0, \tau_1)$, $[\tau_1, \tau_2)$, $[\tau_2, \tau_3)$, …, $[\tau_n, \tau_{n+1})$, …, in which the process $\{D(t): t \geq 0\}$ takes constant (identical) values, are random variables of positive distributions. In case of loads on the mentioned engines the assumption can be made [6] that duration of the state $c_i \in C$ (1) occurred at the moment τ_n and the state occurred at the moment τ_{n+1} do not depend stochastically on states occurred before or intervals of their duration. Thus, it can be accepted that the process $\{D(t): t \geq 0\}$ of transitions of loads on these engines is a semi-Markov process. This means that the process $\{D(t): t \geq 0\}$ is a model of main engine load changing in a random way, with a set of the states $C = \{c_1, c_2, c_3, c_4\}$ and a graph of states transition presented by Fig. 2.

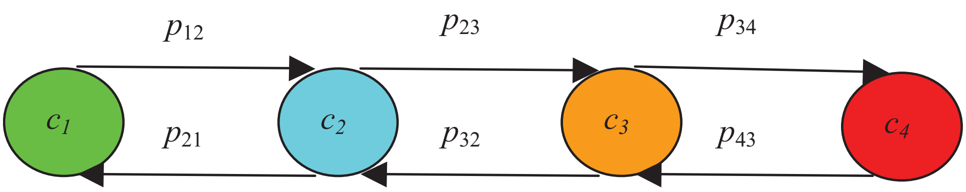

Fig. 2. Graph of states transition in the process{D(t): t ≥0}

The process is totally determined (defined) when a functional matrix is determined

$$Q_{ij} = \lfloor Q_{ij}(t) \rfloor, \qquad i, j = 1, 2, 3, 4, \tag{2}$$

where:

$$Q_{ij}(t) = P\{D(\tau_{n+1}) = c_j, \tau_{n+1} - \tau_n < t | D(\tau_n) = c_i\}$$

and initial distribution is:

$$P_i = P\{D(0) = c_i\}, \quad i = 1, 2, 3, 4. \tag{3}$$

The distinguished states $c_i \in C$ (i = 1, 2, 3, 4) of the process $\{D(t)$: $t \geq 0\}$ of any main engine can be recognized with help of proper diagnosing systems (SDG) [7, 10]. Example of the process $\{D(t)$: $t \geq 0\}$ realization has been presented in Fig.3.

Transition of states belonging to the set $C = \{c_i$; i = 1, 2, 3, 4$\}$ can be considered during operation of each main diesel engine, as the process $\{D(t)$: $t \geq 0\}$ with constant (identical) realizations in particular time intervals continuous on the right side [5, 6, 8, 9]. Lengths of the intervals in which the process $\{D(t)$: $t \geq 0\}$ takes constant (identical) values are random variables T_{ij} that determine duration of the state $c_i \in C$ of the process on condition that the successive state is $c_j \in C$, where i,j = 1, ..., 4 and $i \neq j$. These variables are independent random variables with finite expected values $E(T_{ij})$ and have positively concentrated distributions. Additionally, the process is characterized by the property that the duration of the state c_i occurred at the moment τ_n and the state occurred at the moment τ_{n+1} do not depend stochastically on states occurred before or on intervals of their duration [5, 6, 8, 9]. Thus, it can be accepted that forward states (situations) depend only on the current situation. This means that the process $\{D(t)$: $t \geq 0\}$ can be, as it has already been stated above, accepted as a semi-Markov process. In order to determine this process its initial distribution P_i and functional matrix **Q(t),** are required to be defined.

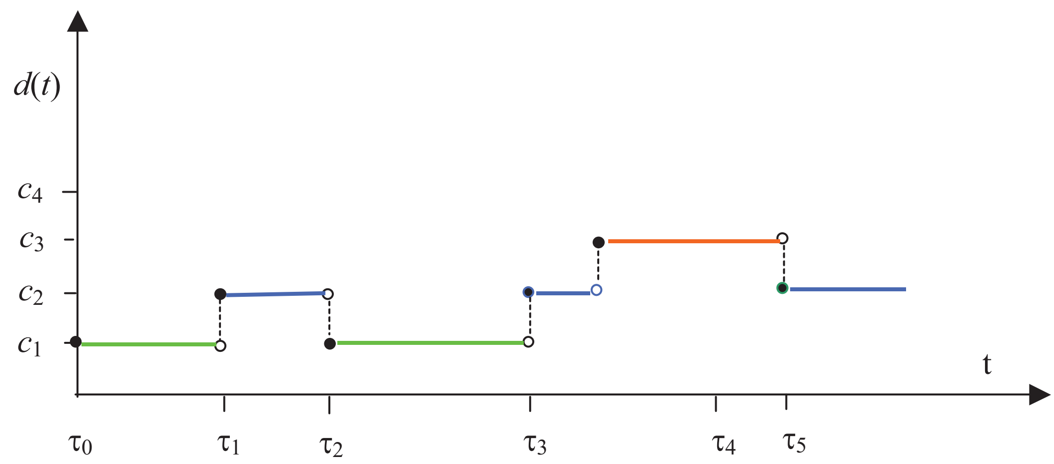

Fig. 3. Example of the process {D(t): t ≥ 0} run in one-dimensional form

Initial distribution of the process $\{D(t)$: $t \geq 0\}$is as follows:

$$P_i = P\{D(0) = c_i\} = \begin{cases} 1 & dla \quad i = 0 \\ 0 & dla \quad i = 1,2,...,3 \end{cases}, \tag{4}$$

Functional matrix in accordance with the graph of states transition, presented in Fig. 2, is of the following form:

$$Q(t)=\begin{bmatrix} 0 & Q_{01}(t) & 0 & 0 \\ Q_{10}(t) & 0 & Q_{12}(t) & 0 \\ 0 & Q_{21}(t) & 0 & Q_{23}(t) \\ 0 & 0 & Q_{32}(t) & 0 \end{bmatrix}. \tag{5}$$

Elements of the matrix (5) are non-decreasing functions of a variable t, being the probabilities of the $\{D(t)\colon t \geq 0\}$ process transition from state c_i into state c_j (c_i, $c_j \in C$; $I,j = 1, 2, 3, 4$; $I \neq j$) at time not longer than t, determined as follows []:

$$Q_{ij}(t) = P\{D(\tau_{n+1}) = c_j,\ \tau_{n+1} - \tau_n < t \,|\, D(\tau_n) = c_i\} = p_{ij}F_{ij}(t), \tag{6}$$

where:

$$\text{ci, cj} \in \text{C(I, j = 1, 2, 3, 4; I} \neq \text{j)},$$

where p_{ij} – probability of transition by one step in the embedded homogeneous Markov chain of the process $\{D(t)\colon t \geq 0\}$, and $F_{ij}(t)$ – distribution of a random variable T_{ij}, determining the duration of the state c_i of the process $\{D(t)\colon t \geq 0\}$ on condition that the successive state is c_j.

Probability p_{ij} is interpreted as follows:

$$P_{ij} = P\{D(\tau_{n+1}) = c_j \,|\, D(\tau_n) = c_i\} = \lim_{t\to\infty} Q_{ij}(t)\,. \tag{7}$$

In this situation the solution of the stated problem consists in finding a limiting distribution of the process $\{D(t)\colon t \geq 0\}$, of the following interpretation:

$$P_j = \lim_{t\to\infty} P\{D(t) = c_j\}, \quad j = \overline{1,4}\,.$$

The distribution can be determined using the formula [2, 4, 8. 9]:

$$P_j = \frac{\pi_j E(T_j)}{\sum_{k=0}^{3} \pi_k E(T_k)}, \quad j = 1,\ 2,\ 3,\ 4, \tag{8}$$

where $\pi_j = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} P\{D(\tau_n) = c_j | Y(0) = c_i\}$ and $[\pi_j;\, j = 1, 2, 3, 4]$ is a stationary distribution of the embedded Markov chain $\{D(\tau_n)\colon n \in N\}$ of the process $\{D(t)\colon t \geq 0\}$.

This distribution satisfies the system of equations (9) and (10) [8, 9]:

$$\sum_{i=1}^{4} \pi_i p_{ij} = \pi_j; \quad i, j = 1,2,3,4, \tag{9}$$

$$\sum_{i=1}^{4} \pi_i = 1, \tag{10}$$

Matrix (5) is a stochastic one, so the matrix of transition probabilities $\mathbf{P} = [p_{ij}]$, $i,j = 1, 2, 3, 4$ is:

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ 0 & p_{32} & 0 & p_{34} \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{11}$$

In that case the equations (9) and (10) characterizing the limiting distribution π_j, j = 1, 2, 3, 4 of the embedded Markov chain $\{D(\tau_n): n = 0, 1, 2, ...\}$of the process $\{D(t): t \geq 0\}$,can be presented in the form of the system of equations [8]:

$$\left.\begin{array}{l} [\pi_1, \pi_2, \pi_3, \pi_4] \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ 0 & p_{32} & 0 & p_{34} \\ 0 & 0 & 1 & 0 \end{bmatrix} = [\pi_1, \pi_2, \pi_3, \pi_4] \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \end{array}\right\} . \tag{12}$$

Solving the system of equations (12) the following dependences are obtained according to the formula (8):

$$P_1 = \frac{p_{21} p_{32} E(T_1)}{M}; \quad P_2 = \frac{p_{32} E(T_2)}{M}; \quad P_3 = \frac{p_{23} E(T_3)}{M}; \; P_4 = \frac{p_{23} p_{34} E(T_4)}{M}, \tag{13}$$

where:

$$M = p_{21}p_{32}E(T_1) + p_{32}E(T_2) + p_{23}E(T_3) + p_{23}p_{34}E(T_4).$$

where:

p_{ij} - probability of the {D(t): t ≥ 0}process transition from state c_i into state c_j (c_i, $c_j \in C$; i, j = 1, 2, 3, 4; $i \neq j$),

$E(T_j)$ - expected value of a random variable T_j(j = 1, 2, 3, 4) determining duration of the state $c_j \in C$(j = 1, 2, 3, 4) of the process $\{D(t): t \geq 0\}$ independently on state which the processtransits from it to.

Expected values $E(T_j)$ depend on expected values $E(T_{ij})$ and probabilities p_{ij} in the following way:

$$E(T_j) = E(T_i) = \sum_j p_{ij} E(T_{ij}), \quad i, j = \overline{1,4}; \;\; i \neq j\,. \tag{14}$$

Particular probabilities P_j(j = 0, 1, ..., 3) are of the following interpretation:

$$P_1 = \lim_{t\to\infty} P\{D(t) = c_1\},\; P_2 = \lim_{t\to\infty} P\{D(t) = c_2\},\; P_3 = \lim_{t\to\infty} P\{D(t) = c_3\},\; P_4 = \lim_{t\to\infty} P\{D(t) = c_4\}.$$

Expample of the $\{DY(t): t \geq 0\}$ process distribution (engine load spectrum) in a graph form, is shown in Fig 4. The distribution has been formulated on the base of engine operating data from which the following results are obtained:

$$P_1 = P\{c_1 = [0\text{kW}, 1000\text{kW})\} = 0{,}16;\; P_1 = P\{c_2 = [1000\text{kW}, 2000\text{kW})\} = 0{,}44;$$

$$P_1 = P\{c_1 = [2000\text{kW}, 3000\text{kW})\} = 0{,}32;\; P_1 = P\{c_2 = [3000\text{kW}, 4000\text{kW}]\} = 0{,}08.$$

P_1 - can be considered as probability of loading an engine according to partial power characteristic,

P_2 - as probability of loading an engine according to operating power characteristic,

P_3 - as probability of loading an engine according to rated power characteristic,

P_4 - as probability of loading an engine according to maximum power.

In order to obtain (in approximation, of course) values of the probabilities P_j(j = 1, 2, 3, 4) estimation of p_{ij} and $E(T_j)$ is required.

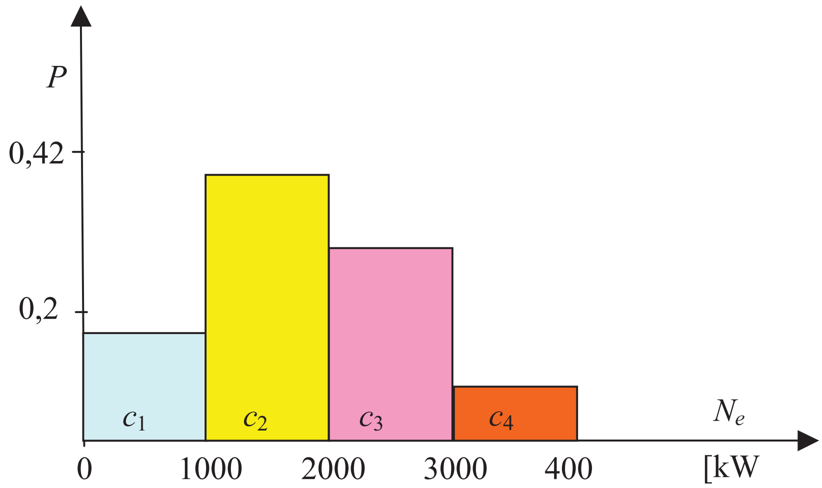

Fig. 4. Example of distribution of power load on a main engine of a sea-going ship: $\tilde{P}$ - probability of occurrence of the event, c_k (k = 1, 2, 3, 4), N_e - engine output power

The estimation of probabilities p_{ij} and expected values $E(T_j)$ is possible after obtaining realizations $d(t)$ of the process $\{D(t): t \geq 0\}$ in adequately long interval of testing time, so for $t \in [0, t_b]$, where $t_b >> 0$. It is possible then to estimate the quantities $n_{ij}(i,j = 0, 1, ..,3; i \neq j)$, which are numbers of transitions from state c_i into state c_j at properly long time.

The estimator of the highest reliability of transition probability p_{ij} is the statistics [8]

$$\hat{P}_{ij} = \frac{N_{ij}}{\sum_j N_{ij}}, \quad i \neq j; \quad i, j = 0, 1, ..., 3, \tag{15}$$

of which the value $\hat{p}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$ is the estimation of unknown probability p_{ij} of $\{D(t): t \geq 0\}$ process transition from state c_i into state c_j.

Realizations $t_j^{(m)}$, $m.$ = 1, 2, ..., n_{ij} of random variables T_j .can also be obtained from the mentioned $d(t)$ run. Application of point estimation allows for easy determination of $E(T_j)$ as an arithmetic average value of realization $t_j^{(m)}$ [8, 9].

The load spectrum model presented above can be developed by using interval estimation of expected values $E(T_j)$, $j = 1, 2, 3, 4$.

3. Summary

The paper presents a semi-Markov model of load spectrum, being a limiting distribution of the process $\{D(t): t \geq 0\}$at a four-state set of states $C = \{c_1, c_2, c_3, c_4\}$ and interpretation determined by the formula (1). The model has been formulated with regards to loads on sea-going ship engines employed in propulsion systems of the ships, so on main engines.

The model is of essentials meaning in practice because it is easy to determine estimators of transition probabilities p_{ij} $(i, j = 1, 2, 3, 4; i \neq j)$, the formula (15), and empirical distributions being approximation of distributions $F_{ij}(t)$, what is indispensable to estimate the elements $Q_{ij}(t)$ of the functional matrix $\mathbf{Q}(\boldsymbol{t})$, the formula (5). It is also easy to determine expected values $E(T_{ij})$ of random variables T_{ij}, enabling estimation of expected values $E(T_j)$ of random variables T_j, the formula (14).

The presented above proposal of a model can be developed by taking into account so many states of the process of main engines loading as the need is to make such a probabilistic description of the process that provides possibility of rational control of the operating process.

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