STRUCTURAL RELIABILITY ASSESSMENT OF CORRODED TANKER SHIP BASED ON EXPERIMENTALLY ESTIMATED ULTIMATE STRENGTH

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ABSTRACT

This work deals with the reliability assessment of a tanker ship hull structure subjected to a vertical bending moment and corrosion degradation. The progressive collapse and ultimate load carrying capacity are estimated based on experimentally tested scaled box-shaped-specimens. The translation of the strength estimate of the scaled specimen to the real tanker ship hull structure is performed based on the dimensional theory developing a step-wise linear stress-strain relationship. The load-carrying capacity is considered as a stochastic variable, and the uncertainties resulted from the scaled-specimen to the real-structure strength translation, and the subjected load of the real ship are also accounted for. A sensitivity analysis concerning the stochastic variables, included in the ultimate limit state function is performed. The partial safety factors, in the case of a scaled specimen and real structure, are also identified, and conclusions are derived.

Keywords: ship structures; reliability assessment; corrosion; ultimate strength; tanker ship

INTRODUCTION

Many studies have been dedicated to the ultimate strength of ship structures in the last decades. The study of the ultimate strength of ship structures was performed by Caldwell in 1965 [1], where a simplified direct formulation taking into account material yield and buckling was introduced. Alternatively, in 1977, the incremental-iterative approach was developed by Smith [2], considering the longitudinal elasto-plastic response of individual structural components.

Nowadays both improved, direct and progressive approaches are part of the IACS Common Structural Rules [3]. The progressive collapse method discretises the ship cross section into appropriate elements, usually stiffened plates (stiffener with attached plating) and rigid corners.

The structural behaviour of each component is predicted in the form of a load-shortening curve. Next, the incremental procedure is introduced to obtain the moment-curvature response of the ship hull subjected to a combined horizontal and vertical bending.

The progressive collapse method has been widely employed to predict the load-carrying capacity of intact [4] and damaged [5] ship sections. However, it was discovered that non-uniform corrosion degradation is the factor that can significantly affect the load carrying capacity [6–8] mainly due to the cross-sectional area reduction and mechanical properties changes [9].

Paik et al. [10] investigated the influence of pitting corrosion on the ultimate strength using the finite element method. The reduction observed was significant, the smallest
cross-sectional area governs the ultimate strength of a plate under axial compressive load.

Saad-Eldeen et al. [11] tested experimentally three corroded box girders subjected to pure vertical bending and studied the initial and post-collapse plate deflections. As a conclusion, crossing specific slenderness ratio the initial shape governs the post-collapse shape. The analysis was further developed into the ultimate strength assessment of the tanker ship with the use of the dimensional theory [12].

The influence of a random corrosion thickness distribution on the ultimate strength of rectangular steel plates was also investigated by Silva et al. [13]. Applying the Monte Carlo Simulation, the plate thickness distribution was generated considering the degree of degradation. Furtherly, they used to perform a reliability analysis [14].

The reduction of mechanical properties of corroded steel small-specimens was investigated by Garbatov et al. [9]. Tensile strength tests have been performed for various corrosion degradation levels. The observed reduction of mechanical properties (modulus of elasticity, yield stress, tensile strength and others) was significant. Garbatov et al. [15] also investigated numerically and experimentally the ultimate strength of stiffened plates with different corrosion degradation levels. The ultimate strength reduction was significant. The experimental results were further compared with the Finite Element analysis by Woloszyk et al. [16].

To determine the uncertainty level in the ultimate strength assessment due to various governing parameters, reliability methods are employed. One of the first attempts to employ probability-based methods in the field of ship structural design was by Mansour [17, 18] and Mansour & Faulkner [19]. The first applications dealt with the safety of ship hulls subjected to a wave-induced bending load.

The methodologies for single structural components were developed too. An oil tanker hull girder was analysed by Saydam et al. [20] considering different speeds, headings and sea states. The reliability index was determined for the intact and six damaged hull cases, where the limit state function was based on the hull girder ultimate strength at a midship section.

Zayed et al. [21] performed a reliability analysis of ship hulls subjected to corrosion and maintenance, where ship loading uncertainties, random variables and inspection events were considered. The advanced uncertainty analysis was performed by Teixeira et al. [22], where the approach to assess the ultimate strength of plates considering the random initial distortions, material and geometrical properties and random corrosion degradation was presented.

The paper deals with the reliability assessment of corroded tanker ship subjected to a compressive load employing the result of tested scaled small box specimens. The target reliability function considers the ultimate hull girder capacity, which is derived based on the experimental and numerical estimates. A sensitivity analysis concerning the stochastic variables, included in the ultimate limit state function is also performed.

**TIME-DEPENDENT ULTIMATE STRENGTH**

Based on the experimental and numerical results [15,16] the ultimate time-dependent strength of stiffened plate is derived. The specimens analysed in [15] are taken from the box girder subjected to the incrementally increasing bending moment [23] with depth, length and breadth of 1,400 mm, 800 mm and 600 mm respectively. Using the dimensional theory, the box girder can represent the midship section of the single-skin hull tanker with a length, breadth and depth of 108 m, 16 m and 12 m respectively and specimens correspond to the stiffened plate of the deck.

The analysis of the similarity between the model and real structure concerning the ship hull ultimate capacity was conducted in [12], and more information about the dimensional theory may be found in [24,25]. The scaling factors are as follows:

- the linear dimensions:

  \[
  \frac{x}{x'} = \frac{y}{y'} = \frac{z}{z'} = \frac{l}{l'} = C_l
  \]  

- stresses:

  \[
  \frac{\sigma_x}{\sigma_x'} = \frac{\sigma_y}{\sigma_y'} = \frac{\tau_{xy}}{\tau_{xy'}} = \frac{\sigma}{\sigma'} = C_\sigma
  \]  

- strains:

  \[
  \frac{\varepsilon_x}{\varepsilon_x'} = \frac{\varepsilon_y}{\varepsilon_y'} = \frac{\gamma_{xy}}{\gamma_{xy'}} = \frac{\varepsilon}{\varepsilon'} = C_\varepsilon
  \]  

- displacements:

  \[
  \frac{u}{u'} = \frac{v}{v'} = \frac{w}{w'} = C_u
  \]  

where \(l, \sigma, \varepsilon\) and \(u\) are the dimensions of real structure and \(l', \sigma', \varepsilon'\) and \(u'\) are for the model respectively. The thickness term need to be defined independently of the plane stress terms:

\[
C_t = \frac{t}{t'}
\]  

The relations between the ship hull (real structure) and the model are as follows:

- length

  \[
  C_l = \frac{L_{\text{structure}}}{L_{\text{model}}} = \frac{B_{\text{structure}}}{B_{\text{model}}} = \frac{D_{\text{structure}}}{D_{\text{model}}} = 20
  \]
– thickness
\[ C_t = \frac{t_{\text{structure}}}{t_{\text{model}}} = 3 \]  

(7)

– subjected force
\[ C_p = \frac{P_{\text{real}}}{P_{\text{model}}} = 60 \]  

(8)

– bending moment
\[ C_m = \frac{M_{\text{structure}}}{M_{\text{model}}} = C_p C_t = 1,200 \]  

(9)

resulting in the same stresses in the model and real structure:
\[ \sigma_{\text{structure}} = \sigma_{\text{model}} \frac{C_m}{C_p C_t} = \sigma_{\text{model}} \]  

(10)

Based on that, the ultimate stress is the same for the model and real structure.

Since the thickness scale is 3 and it is assumed that the corrosion depth follows the same time dependency in real structure and specimen, the degree of degradation (DoD) of the tanker ship model is:
\[ t_{\text{corroded}} = t'_{\text{corroded}} \]  

(11)

\[ t = t_{\text{netto}} + t_{\text{corroded}} \]  

(12)

\[ \text{DoD} = \frac{t_{\text{corroded}}}{t} = \frac{t'_{\text{corroded}}}{C_t} = \frac{\text{DoD}'}{C_t} \]  

(13)

Since the stress scale is 1, the DoD-dependent ultimate-strength of the tanker ship is defined as:
\[ \sigma_u(\text{DoD}) = \sigma_u \left( \frac{\text{DoD}'}{C_t} \right) = \sigma_u \left( \frac{\text{DoD}'}{3} \right) \]  

(14)

The degree of degradation of the tanker ship plate is a function of the time, which can be derived from the corrosion degradation model as presented in [26]. The corrosion depth versus time is shown in Figure 1, where the coating life is not considered.

Based on that, the ultimate time-dependent strength of both model and tanker ship can be derived. The ultimate time-dependent strength of the tanker stiffened plate based on the experimental and numerical investigations is presented in Figure 2.

For the current analysis, the assumption is made that the ultimate strength is equal to initial member collapse strength (the collapse of the weakest stiffened plate in the cross-section which initiates the progressive collapse). However, after the collapse of the first stiffened plate, the hull girder section still has some capacity [27] making the current assumption conservative one.

Bottom and deck of the analysed tanker ship are made from high tensile steel with a yield stress of 315 MPa, and the Young modulus is 206 GPa.
STRENGTH ASSESSMENT

Based on the dimensional theory, the section modulus at the deck and bottom of the tanker ship is estimated using small-scale box girder specimens as presented in [12]. The deck section modulus is $Z_d = 3.64\ m^3$ and the bottom section modulus is $Z_d = 3.32\ m^3$.

The still water and wave-induced bending moments are estimated from the Common Structural Rules [3] in hogging and sagging condition respectively as follows:

$$
M_{sw,h} = 156,043\ kNm
$$

(15)

$$
M_{sw,s} = -121,714\ kNm
$$

(16)

$$
M_{w,v,h} = 246,686\ kNm
$$

(17)

$$
M_{w,v,s} = -259,537\ kNm
$$

(18)

In the Common Structural Rules [3], the ultimate strength limit state is formulated as:

$$
\gamma_{sw}M_{sw} + \gamma_{w,v}M_{w,v} \leq \frac{M_U}{\gamma_R}
$$

(19)

where $M_U$ is the hull girder ultimate bending capacity and $\gamma_{sw}, \gamma_{w,v}, \gamma_R$ are the partial safety factors.

The ultimate capacity is estimated based on the experimental or numerical results and dimensional theory as:

$$
M_U = \left( \frac{\sigma_U}{\sigma_Y} \right)_{\text{experimental/FEM}} Z_{d/b} Re
$$

(20)

where $\left( \frac{\sigma_U}{\sigma_Y} \right)_{\text{experimental/FEM}}$ is the normalized ultimate capacity based on the experimental or numerical results, $Z_{d/b}$ is the section modulus of deck or bottom and $Re$ is the yield strength of material.

RELIABILITY ANALYSIS

To analyse the impact of the uncertainty in the ultimate strength assessment due to various governing parameters, the FORM [28] method is used here employing own algorithm.

The term formulated deterministically in Eqn (19) can be transformed into a limit state function:

$$
g = \bar{x}_U M_U - \bar{x}_{sw} M_{sw} - \bar{x}_w \bar{x}_s M_{w,v}
$$

(21)

where $M_U, M_{sw}, M_{w,v}$ are the bending moments described as random variables, $\bar{x}_U$ is the model uncertainty related to the ultimate strength, $\bar{x}_{sw}$ is the uncertainty in the model of predicting the still water bending moment, $\bar{x}_w$ is the uncertainty in the wave-induced bending moment due to the linear sea keeping analysis and $\bar{x}_s$ takes into account nonlinearities in the sagging loading condition.

The statistical descriptors of the uncertainty coefficients presented in Eqn (21) are assumed to follow the Normal distribution function [29,30]:

$$
\bar{x}_U \sim N(1.1; 0.1)
$$

(22)

$$
\bar{x}_{sw} \sim N(1.0; 0.1)
$$

(23)

$$
\bar{x}_w \sim N(1.0; 0.1)
$$

(24)

$$
\bar{x}_s \sim N(1.0; 0.1)
$$

(25)

The First Order Reliability Method, FORM [28] is used to estimate the reliability index of the ultimate limit state. Since the ultimate capacity depends on time – the reliability index is also time-dependent.

The still water bending moment can be fitted to the Normal distribution as stipulated in [31] where the statistical descriptors are defined by regression equations based on about 2000 data points as a function of the length and the dead-weight ratio ($W = DWT/\text{Full Load}$). The estimated coefficients are shown in Table 1.

Tab. 1. Statistical descriptors of Still water bending moment [31]

<table>
<thead>
<tr>
<th></th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean($M_{sw,max}$)</td>
<td>114.7</td>
<td>-105.6</td>
<td>-0.154</td>
</tr>
<tr>
<td>StDev($M_{sw,max}$)</td>
<td>17.4</td>
<td>-7</td>
<td>0.035</td>
</tr>
</tbody>
</table>

The mean value and standard deviation of still water bending moment are estimated as:

$$
\text{Mean}(M_{sw}) = \frac{\text{Mean}(M_{sw,max}) M_{sw,CSR}}{100}
$$

(26)

$$
\text{StDev}(M_{sw}) = \frac{\text{StDev}(M_{sw,max}) M_{sw,CSR}}{100}
$$

(27)

where the $M_{w,CSR}$ are the still water bending moments calculated with the formulas provided in Common Structural Rules [3].

The dead-weight ratio of the analysed tanker ship is 0.82 in the full-load condition, 0.61 in the partial load condition and 0.41 in the ballast condition.

The still water bending moments for the analysed tanker ship under different loading conditions are shown in Table 2.
As can be noticed, the positive mean value indicates that the maximum bending moment is in a hogging loading condition and it has to be pointed out that the analysed ship is a single-hull oil tanker. Due to that, the hogging condition is furtherly analysed.

The distribution of the extreme values of the wave-induced bending moment at a random point of time, over a specified time period can be represented as a Gumbel distribution as proposed by Guedes Soares et al. [32], considering that the wave-induced bending moment given by the Common Structural Rules [3] may be modelled as a Weibull distribution with a probability of exceedance of $10^{-8}$.

The Gumbel distribution, for the extreme values of the vertical wave-induced bending moment, over the reference period $T_w$ is derived based on the shape, $h$ and scale, $q$ factors of the Weibull distribution function as [32]:

$$
\alpha_m = q (\ln(n))^h
$$

(28)

$$
\beta_m = \frac{q}{h} (\ln(n))^{\frac{1-h}{h}}
$$

(29)

where $\alpha_m$ and $\beta_m$ are the parameters of the Gumbel distribution, $n$ is the mean number of load cycles, expected over the reference time period $T$, for a given mean value period $T_w$. It is assumed that $T_w = 8$ sec. and $T_w$ is equal to 1 year. The number of load cycles $n$ is calculated as:

$$
n = \frac{p_i \cdot T_r \cdot 365 \cdot 24 \cdot 3600}{T_w}
$$

(30)

where $p_i$ is the partial time in which the ship is in seagoing conditions.

The partial time factors and extreme values of the vertical wave-induced bending moment in hogging are shown in Table 3.

### Table 3. Statistical descriptors wave-induced bending moment $M_{wv}$

<table>
<thead>
<tr>
<th>Loading condition</th>
<th>$p_i$ [-]</th>
<th>$\alpha_m(M_{wv})$</th>
<th>$\beta_m(M_{wv})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-load</td>
<td>0.4</td>
<td>341,542</td>
<td>9,246</td>
</tr>
<tr>
<td>Partial</td>
<td>0.1</td>
<td>303,302</td>
<td>9,378</td>
</tr>
<tr>
<td>Ballast</td>
<td>0.4</td>
<td>341,542</td>
<td>9,246</td>
</tr>
<tr>
<td>Harbour</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The CoV of the ultimate capacity is based on the previous results [15,16] and is assumed as 0.0208 for the experimental one and 0.0363 for the FEM estimates. Additionally, due to the differences between experimental and FEM results the bias factor is introduced:

$$
x_{FEM} = \frac{\sigma_{U,FEM}}{\sigma_{U,exp}}
$$

(31)

The bias factor is modelled as a random variable following the Normal probability function, $x_{FEM} \sim N(0.893; 0.052)$, since both experimentally and numerically obtained capacities are fitted to the Normal probability function. The $x_{FEM}$ is assumed constant for different corrosion degradation levels and the mean value of the distribution is calculated as the mean of the fractions $\sigma_{U,FEM}/\sigma_{U,exp}$ for different time of exploitation. The standard deviation is calculated as the square root of the sum of squares of differences between the mean value and particular fractions. The limit state function for the ultimate capacity based on the FEM results is:

$$
g = x_{FEM} \cdot \alpha_g M_{U} - x_{FEM} M_{SW} - x_{FEM} \cdot \alpha_g M_{WW} - \sigma_g
$$

(32)

All random variables are considered here as non-correlated ones. When the reliability analysis is performed, the estimated reliability index needs to be compared with the target one. The target level is related to the failure cause and according to DnV [33] is equal to $\beta = 3.09$ ($P_f = 10^{-5}$) for the less serious and $\beta = 3.71$ ($P_f = 10^{-4}$) for serious consequences of failure. The targeted reliability level is considered here as $\beta_{target} = 3.71$. The required Beta index is based on the sum of probabilities of failure for different loading conditions:

$$
\beta_{target} = -\Phi^{-1}(\Sigma P_f)
$$

(33)

### RESULTS AND DISCUSSION

For the analysed tanker ship, a more severe loading is the hogging case, due to the contribution of the still water bending moment comparing to the sagging condition. The calculations performed for sagging condition showed that the probability of failure is significantly smaller compared to the hogging condition.

The reliability safety indices for different load cases and the total Beta index are shown in Table 4.

### Table 4. Reliability safety indices for different load cases, experiment

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Load Beta index [-]</th>
<th>Beta index [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full load</td>
<td>Ballast load</td>
</tr>
<tr>
<td>0.0</td>
<td>5.201</td>
<td>4.315</td>
</tr>
<tr>
<td>0.7</td>
<td>5.041</td>
<td>4.141</td>
</tr>
<tr>
<td>8.0</td>
<td>4.644</td>
<td>3.715</td>
</tr>
<tr>
<td>12.8</td>
<td>4.424</td>
<td>3.481</td>
</tr>
</tbody>
</table>
The results of the reliability safety index versus time for the ultimate capacity based on the experimental results are presented in Figure 3.

![Figure 3. Reliability safety index of analysed tanker ship, experimental results](image)

As it can be noticed, after eight years of exploitation, without accounting for the coating life, the required reliability index is not satisfied. After this time, some maintenance action needs to be done. The most critical condition is the ballast one, and as it can be noticed, full and partial load conditions have almost no influence on the resulting reliability safety index.

Figure 4 shows the sensitivity analysis concerning the influence of different random variables. As it can be noticed, the most influencing variable is the model uncertainty of the ultimate capacity.

![Figure 4. Sensitivity analysis](image)

The reliability safety indices obtained based on the FEM is shown in Table 5. Very similarly to the experimental results, the most critical condition which governs the resulting Beta index is the ballast one.

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Load Beta index [-]</th>
<th>Beta index [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full load</td>
<td>Ballast load</td>
</tr>
<tr>
<td>13.9</td>
<td>4.377</td>
<td>3.431</td>
</tr>
<tr>
<td>14.5</td>
<td>4.353</td>
<td>3.406</td>
</tr>
<tr>
<td>23.1</td>
<td>4.042</td>
<td>3.077</td>
</tr>
<tr>
<td>37.3</td>
<td>3.682</td>
<td>2.700</td>
</tr>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

![Figure 5. Reliability safety index of analysed tanker ship, numerical results](image)

The service time, where the Beta index crosses the target value is very similar to the results obtained based on the experiment, and it is about eight years.

Figure 6 shows the sensitivity analysis concerning reliability based on the FEM. Similarly, to the experimental data, the most influencing parameters are the model uncertainty of the ultimate capacity and FEM calculations.

![Figure 6. Sensitivity analysis](image)
The partial safety factors for the initial value of the plate thickness are presented in Table 6.

<table>
<thead>
<tr>
<th>Partial Safety Factors</th>
<th>FEM</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{Mw}$</td>
<td>1.10</td>
<td>1.09</td>
</tr>
<tr>
<td>$\gamma_{Msw}$</td>
<td>1.67</td>
<td>1.29</td>
</tr>
<tr>
<td>$\gamma_{Mu}$</td>
<td>0.97</td>
<td>0.98</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

The objective of this work was to perform a structural reliability assessment based on the experimentally and numerically estimated ultimate strength.

The results show that it is possible to estimate the ultimate strength of the ship cross section based on the experimental results and dimensional theory. Additional modelling uncertainties need to be taken into account for proper estimation of reliability safety index considering the FE model. The reliability safety index was calculated as a function of the time, and the assumed target reliability safety index was crossed after around eight years. The study revealed that the reliability analysis conducted in hogging condition, considering only the ballast load case for the present oil tanker, would give satisfying results.

The presented methodology is flexible and can be furtherly used in more complex systems.

**REFERENCES**


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