

1 Structured deformation of granular material in the state of active earth pressure

2 *Danuta Leśniewska¹, Antoinette Tordesillas², Magdalena Pietrzak³, Shuo Zhou⁴ and Michał Nitka⁵*

3 *¹Institute of Hydro-Engineering, Polish Academy of Sciences, Kościarska 7, 80-328 Gdańsk, Poland,*

4 *corresponding author, e-mail: danuta.lesniewska@ibwpan.gda.pl,*

5 *²School of Mathematics and Statistics, The University of Melbourne, Melbourne, Australia,*

6 *³Koszalin University of Technology, Koszalin, Poland,*

7 *⁴School of Computing and Information Systems, The University of Melbourne, Melbourne, Australia,*

8 *⁵Gdańsk University of Technology, Gdańsk, Poland*

9

10 **keywords:** structured deformation, retaining wall, physical state space, displacement state space, local intrinsic
11 dimensionality, digital image correlation

12

13 **Abstract**

14 The paper focuses on the ability of granular materials to undergo structured deformation by analysing the data
15 from the retaining wall model tests and discrete element simulations. The structured deformation means the
16 movement of a granular material which produces a stable, regular pattern of multiple shear bands. The paper's
17 primary purpose is to study this kind of deformation for the selected data representing the state of active earth
18 pressure of granular materials. The locations of high and negligible shear strains (shear zones and 'dead' zones)
19 in the displacement fields are determined using the shear strains definition. A recently introduced metric called *s*-
20 LID, expressing the perspective of collective grain motion, is applied to the same data. The *s*-LID analysis finds
21 the detailed structure of the localisation pattern directly from displacement data without using the continuum
22 mechanics concept of strains. It is entirely consistent with the digital image correlation analysis in the areas of
23 significant displacement. It expands the knowledge of the deformation structure in small displacement areas,
24 where the digital image correlation method loses its capability. Low *s*-LID and point-like representations in the
25 displacement state space identify nearly rigid zones in the area of high displacements.

26

27

28

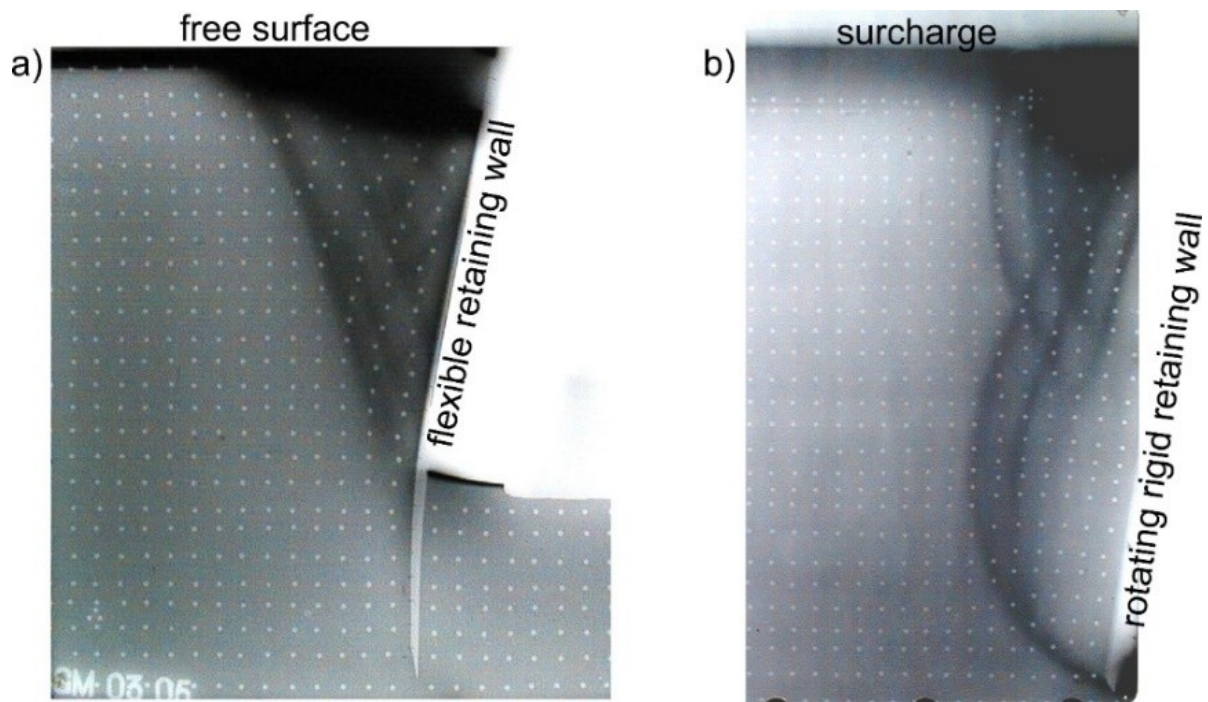
1
2
3

29 **1 Introduction**

30 Earth-retaining structures are one of the most important examples of extensive soil-structure
31 interaction. They are recently used in modern transportation systems (Greco, 2013; Xu et al.,
32 2019; Yang & Deng, 2019; Hossein et al., 2022; Hu et al., 2022; Khosravi et al., 2022),
33 ground protection against vibrational and seismic excitations (Ke et al., 2020; Ren et al.,
34 2022), deep excavations (Guan et al., 2022), offshore protection (McGovern et al., 2023),
35 tailing dams (Franks et al., 2021), and other applications. Accurate and realistic assessment of
36 performance and risk of failure of earth retaining structures is an integral part of geotechnical
37 design. The primary role of any retaining structure is the lateral support of the soil or backfill
38 to withstand the imposed horizontal and vertical stresses and ensure overall ground stability.
39 As a result, a vital element in the design of retaining structures is the reliable estimation of the
40 value, distribution and possible evolution of the earth's pressure.

41

42 The problem is that the mechanism of the earth pressure still needs to be fully recognized due
43 to the complex, multi-scale nature of the soil response to various loading and environmental
44 factors. Due to it, basic experimental and theoretical research on retaining structures are still
45 being carried out (e.g. Schmüdderich et al., 2020; 2022; Lai et al., 2022; Schweiger &
46 Tschuchnigg, 2021; Hu et al., 2022; Hegde& Murthy, 2022; Khosravi et al., 2016; 2022;
47 Wang et al., 2022; Fathipour et al., 2021). One of the features not accounted for in classical
48 earth pressure theory is *structured deformation*, addressed in this paper. The ability of
49 granular materials to undergo this kind of deformation has already been observed in past X-
50 rays experiments (e.g. James, 1965; James & Bransby, 1971; Milligan, 1974; Milligan, 1983),
51 but to our knowledge, it needs a systematic study.



52

53 **Fig.1.** Radiographs made at Cambridge University, showing examples of soil failure patterns (*structured*
 54 *deformation*) behind flexible (a) and rigid (b) model retaining walls. White dots on the radiographs are images of
 55 lead shot markers.

56 We define *structured deformation* as the movement of a granular material producing a stable,
 57 regular pattern of multiple shear bands. Fig.1 shows two examples of such phenomenon,
 58 recorded by X-rays (Milligan, 1974).

59

60 The patterns in Fig.1 correspond to the final stages of the walls' movement in small-scale
 61 tests on retaining structures. They are visible due to the reduction of the granular material
 62 density caused by dilation, characteristic of shear bands. Multiple shear bands in Fig.1a
 63 (excavation of a flexible wall) and Fig.1b (rigid wall rotating about its toe in out of soil
 64 direction) form a regular network, including characteristic rhomboidal elements created by
 65 intersecting bands (Leśniewska, 2001). The bands also appear in traditionally considered rigid
 66 areas (e.g. soil wedge behind the flexible wall in Fig.1a, usually referred to as the Coulomb
 67 wedge, Coulomb, 1776).

68

7
 8
 9

69 Leśniewska and Mróz (2000) noticed the issue of regular shear bands in granular materials
70 and proposed an analytical explanation of the most simple discrete pattern consisting of
71 parallel straight shear bands. Introducing the elastic wall parameter into classic Coulomb
72 wedge analysis was the solution in this case; however, such a simple approach becomes
73 inefficient in explaining more complex patterns, like the ones in Fig.1.

74

75 The traces of structured deformation also exist in some finite element solutions of a retaining
76 wall problem or DEM simulations. However, they are often treated with some suspicion as
77 potential numerical artefacts. An example of such a shear band pattern is shown in the paper
78 by Loukidis and Salgado, 2013. They found the developed discrete patterns of shear strains
79 resembling the real ones and noticed the change of the earth pressure distribution from
80 smooth to developing peaks and valleys. They concluded that it is a consequence of the shear
81 banding inside the sliding mass. It confirms our assumption that understanding the earth
82 pressure development behind a retaining wall requires studying the structured deformation;
83 however, the direct quantitative relation between structured deformation and the earth
84 pressure evolution appears challenging to establish at this research stage.

85

86 Based on Fig. 1, we assume that the structured deformation on a macro scale appears as
87 ordered, alternating areas of intense and negligible shear strains resulting from the different
88 local organisation of grain displacements. Such an internal division of motion should be
89 detectable not only by the material density changes, like in the case of X-rays, but also in the
90 displacement field, provided that an appropriate tool is employed to analyse the collective
91 movement of many data points. A recently introduced metric called s-LID, already applied by
92 Tordesillas et al. (2022) to demonstrate the structural deformation of a granular sample in the
93 elemental biaxial test, was selected.

10

11

12

94 It is a promising tool for analysing more complex deformation mechanisms characteristic of
95 boundary value problems, an example of which is the retaining structure. *s*-LID is a metric
96 based on the *Local Intrinsic Dimensionality* (LID) concept initially proposed by Houle
97 (2017). In essence, *s*-LID accesses the intrinsic dimensionality of a reference point to its *s*
98 nearest neighbours (*s*NN). That is the minimal number of latent factors required to represent a
99 given neighbourhood. It can serve as a quantitative measure of how outlying a grain's motion
100 is (depending on the *s*-LID score) to capture regions with abnormal or negligible motions.

101

102 Our paper uses the *s*-LID method to investigate the deformation structure of granular material
103 in the state of active pressure. Two characteristic displacement fields, one coming from the
104 experimental model test and the other simulated by the discrete element method (DEM), are
105 analysed to achieve this goal. The test and DEM simulation are described in detail by
106 Leśniewska and Muir Wood, 2009, 2010; Muir Wood and Leśniewska, 2011 and Leśniewska
107 et al., 2020. Since the *s*-LID method has not been widely used for soil mechanics problems,
108 some initial verification with the commonly accepted tools is recommended. That is why the
109 *s*-LID analysis is preceded by a preliminary one, performed on the selected displacement
110 fields, using corresponding shear and volumetric strains, calculated with DaVis8 discrete
111 image correlation (DIC) software. The purpose of this preliminary analysis is a simplified
112 recognition of the selected elements of the deformation structure, facilitating the interpretation
113 and verification of a complete structure obtained later in the paper using the *s*-LID method.

114

115 The DIC (or PIV) method of calculating strains from displacement fields has become a
116 fundamental tool in the experimental analysis of granular materials. It is a non-destructive
117 optical method to determine granular material displacements, where grains serve as tracers.



118 High-resolution deformation monitoring can be achieved by processing successive digital
119 images (Rechenmacher & Finno, 2004; Słominski et al., 2007; Stanier et al., 2016; White et
120 al., 2003, 2005).

121

122 The paper is arranged as follows. Section 2 discusses the displacement data sources – the
123 model test on a retaining wall and its discrete element (DEM) simulation. Section 3 presents
124 the selected displacement fields and their characteristics in 2D physical state space (PSS) and
125 displacement state space (DSS). In PSS, the measurement points (e.g. grains) are described by
126 their x and y coordinates and in DSS, by horizontal and vertical displacements v_x and v_y (a
127 scatter plot). Section 4 discusses the results of the preliminary analysis of the selected
128 displacement fields. In Section 5, the s -LID method is introduced, and the results of the final
129 analysis of the displacement fields are given and compared to standard DIC analysis results.
130 The conclusions are summarized in Section 6.

131

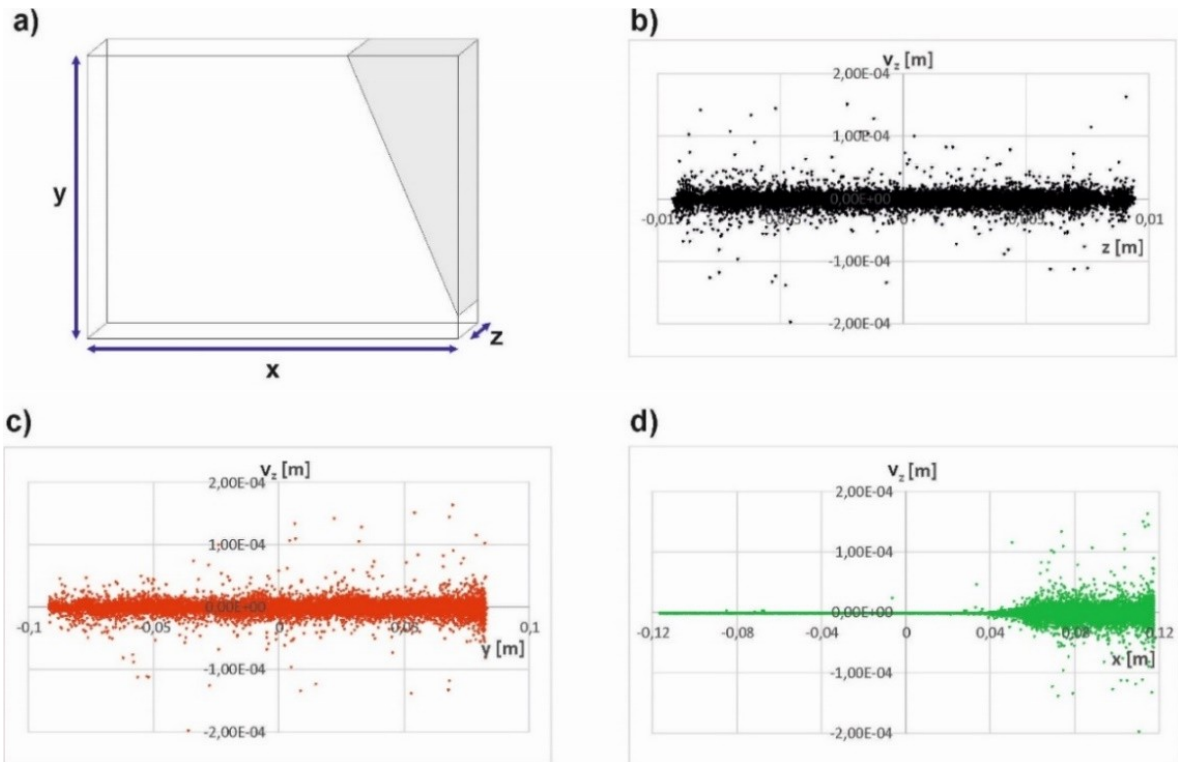
132 **2 Sources and types of the selected displacement data**

133 *2.1. The type of test being analysed*

134 We use the numerical and experimental displacement fields taken from the small-scale **P-**
135 **3D** model test (test number 070528 in our database) and its simulation, discussed in detail by
136 Leśniewska et al. (2020). P-3D testing involves relatively large granular samples placed in
137 rectangular boxes of finite depth, which is significantly smaller than the other two dimensions
138 (see the sketch in Fig.2a). Small (compared to the depth of the test box) 3D grains are usually
139 used, and any external loading is uniformly distributed along the (z) dimension. Such tests are
140 called pseudo-3D because, despite their actual 3D geometry, they create plane strain
141 conditions on a macro scale - deformation of a granular sample along its depth (z) is
142 negligible compared to deformation in the other two directions (x, y plane).

16
17
18



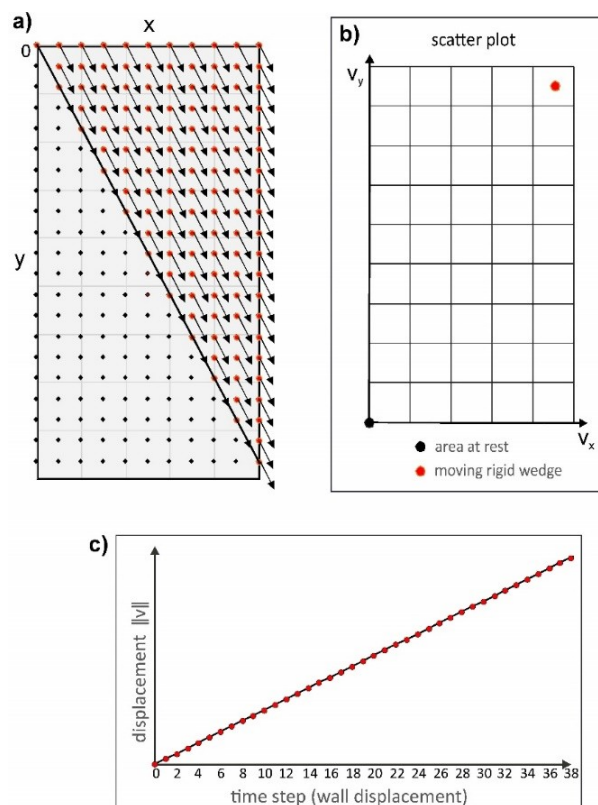


143
 144 **Fig.2.** The out-of-plane grain displacement v_z in 3D DEM simulation of a P-3D test. a) schematic geometry of a
 145 sample in a P-3D test, b), c), d) – v_z as a function of z , y and x .
 146
 147

148 However, in the micro-scale, some out-of-plane grain movements are possible because the
 149 sample consists of more than one non-flat ‘layers’, and grains have a chance to rearrange also
 150 in (z) direction. Fig.2 illustrates this effect - it contains the results of a 3D DEM simulation of
 151 the test with the same geometry as the test 070528 (Fig.2a), using about 100 000 grains, two
 152 times larger than in the actual test.

153
 154 Fig.2b-d shows that some grains show non-zero displacements in the out-of-plane direction,
 155 which are a function of x , y and z : they are evenly distributed over the entire depth of the
 156 sample (Fig.2b), show the increasing scatter with its height (Fig.2c) and appear only in the
 157 deformation wedge typical for retaining structures (Fig.2d), schematically marked in grey in
 158 Fig.2a. It is worthy of note that the maximum displacement v_z is about 0.2 mm, i.e. about 10%
 159 of the D_{50} grain diameter, equal to 2.0mm.

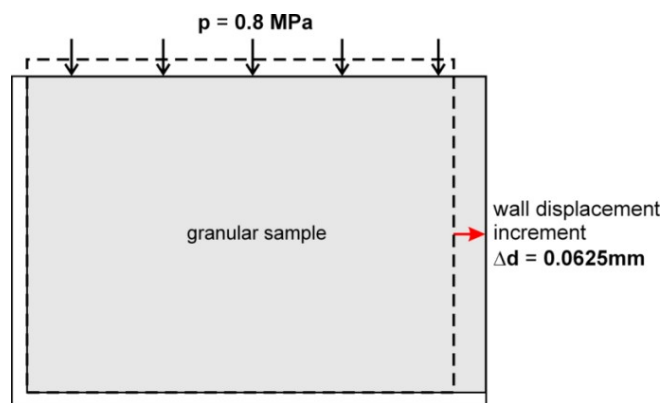
160 The existence of the out-of-plane grain motion, confirmed by 3D DEM, makes it difficult to
 161 directly compare some of the experimental and simulated results. The reason is that the 2D
 162 images recorded during the experiment are made in transmitted light, thus representing a
 163 superposition of information from different layers of the sample (they are 'averaged' over the
 164 depth). It is possible because transparent grains were used. Multiple 'layers' also allow for
 165 some competing failure mechanisms to be created in the sample at different test stages - the
 166 predominant mechanism is only established in the final stage at significant wall
 167 displacements. The simplest failure mechanisms can also be observed at the initial test stages.
 168 Examples of the competing failure mechanisms behind a rigid retaining model wall can be
 169 found, for example, in Leśniewska and Muir Wood, 2011 (Fig.19). It is difficult and time-
 170 consuming to create similar 'transmission' images based on 3D DEM results. The other
 171 solution, accepted in this paper, is to use the corresponding 2D simulations, bearing in mind
 172 that it cannot reproduce any out-of-plane grain movements.



173

22
 23
 24

174 **Fig.3.** Idealized case - a rigid wedge sliding on an inclined plane: a) discretized displacement field, b) scatter
175 plot, c) evolution of the wedge displacement as a function of the number of wall displacement increments (time
176 steps).



177
178 **Fig.4.** The layout of the retaining wall model test and its boundary conditions: the incremental horizontal
179 translation of the right boundary and constant external loading of the top boundary (through five adjusting rigid
180 blocks, Leśniewska et al., 2020).
181

182 2.2. Idealised case - rigid wedge sliding on an inclined plane

183 The following part of the paper will discuss the representation of the displacement field
184 behind a retaining wall in the physical space (PSS) and kinematical displacement space
185 (DSS). We start from the idealised case of a rigid wedge motion along an inclined plane,
186 corresponding to the classic Coulomb approach (Fig.2a).

187
188 A limitation on the wedge displacement rate is assumed to agree with the test scheme (Fig.4).

189 Fig.3a shows the example of discretised displacement field that can represent this type of
190 motion in the PSS - displacements of constant value and direction only inside the wedge.
191 Therefore, this field, presented in DSS (as a scatter plot, Fig.3b), consists of two points only:
192 the first at the beginning of the coordinate system, representing all points with zero
193 displacements outside the wedge, and the second (red circle) corresponding to all points of
194 constant displacement inside the wedge.

195

196 The position of the red point on the scatter plot depends on the value and direction of the
197 wedge displacement. The fact that points belonging to a particular rigid area form one point in
198 a scatter plot helps to interpret more complex displacement fields later in this paper.

199 The other observation, helpful in studying more complex kinematics, is that for any rigid
200 body motion and the corresponding discretized displacement field, the displacement of any
201 focal point (central for a given neighbourhood) selected in this field must be equal to the
202 average displacement in its vicinity:

$$203 \quad d_k = d_{N-av} \quad , \quad (1)$$

204
205 where d_k is the total displacement of a focal measuring point after k time steps, and $d_{N.av}$ is the
206 corresponding mean displacement of the neighbourhood consisting of N measuring points.

207
208 Therefore, Eq.(1) is fulfilled in any area experiencing rigid body motion. This area can be
209 found by simply calculating the mean displacement in a particular neighbourhood and
210 comparing it with the focal grain displacement. If both are equal or very close, the
211 neighbourhood moves as rigid or close to a rigid body. If the evolution of the wedge
212 displacement in time is considered, the total wedge displacement increases linearly with the
213 displacement of the wall, as shown in Fig.3c.

214

215 *2.3. Experimental model test*

216 Fig.4 shows the test layout and its stress and displacement boundary conditions. The
217 dimensions of the granular sample were 23.5x18.5x2.0cm, with the total number of grains in a
218 range of 10^6 . Full-field displacements were obtained using standard digital photography and
219 2D image analysis by digital image correlation (DIC). Horizontal translation of the model
220 wall was applied in 0.0625mm increments (to be compared with D_{50} grain diameter equal to

221 1.0mm) at constant external stress 0.8MPa. The final cumulative wall displacement reached
 222 about 3.5mm. The wall movement was independent of the sample. The test is described in
 223 detail by Leśniewska et al. (2020).

224 **Table 1:** Material constants used in DEM simulations

E_c [GPa]	ν_c [-]	μ [°]	β [-]	η [-]	R [m]	ρ [kg/m ³]	α [-]
2.4	0.3	20	0.3	0.005	0.001	2550	0.08

226
 227 *2.4. Numerical model test*

228 The so-called soft-particles approach was employed to simulate the experimental test,
 229 allowing the particles to overlap to account for their deformation (Frost et al., 2002; Gu et al.,
 230 2017, Luding, S., 2004; Nitka & Grabowski, 2021; Salazar et al., 2015; Zhao et al., 2018). The
 231 computations were performed using the open-source 3D numerical code YADE (Kozicki &
 232 Donze, 2008; Chèvremont et al., 2020; Hartmann et al., 2022; Kozicki et al., 2022; Maurin,
 233 2018; Thoeni et al., 2014; Thoeni, 2021). Calculations by YADE consisted of two steps: (1)
 234 computing the interaction forces between elements (grains) in contact and (2) computing the
 235 resulting acceleration of each element (grain) using Newton's second law of mechanics. The
 236 time integration of its acceleration gave the new position of the element (grain).

237
 238 DEM model, including rolling resistance, proposed by Kozicki and Donze (2008), was
 239 accepted in simulations, characterized by five main local material parameters: E_c
 240 (the modulus of elasticity of grain contact), ν_c (Poisson ratio of the grain contact), μ (the
 241 inter-particle friction angle), β (coefficient of rolling stiffness) and η (the rolling coefficient).
 242 Besides, the particle radius R , the particle density ρ and the numerical damping parameter α
 243 are required. The detailed model's equations are given in Leśniewska et al. (2020). Table 1
 244 contains the material constants accepted in the simulation. They were found based on

245 numerical, three-dimensional (3D) triaxial compression tests with rigid walls on a 10×
246 10×10cm³ sample of small spheres. Numerical results have been verified by the actual triaxial
247 compression data published by Cui et al. (2017). The entire calibration process is reported by
248 Leśniewska et al. (2020).

249 The rolling coefficient $\bar{\eta}$ resulting from the calibration procedure, is extremely low ($\eta = 0.005$)
250 compared to that of sand particles ($\eta = 0.4$, Nitka & Grabowski, 2021) – it is justified by the
251 round, but not ideally spherical shape of the glass beads. The effect of the damping parameter,
252 if $\alpha \leq 0.08$, was insignificant in quasi-static calculations (Kozicki et al., 2013).

253

254 Only a single layer of grains was modelled to facilitate finding the potential deformation
255 structure and to spare the calculation time. As a result, a single-layer sample was created,
256 consisting of 45 300 grains.

257

258 The boundary conditions adopted in the numerical simulations are consistent with Fig.4: the
259 incremental horizontal translation of the right boundary (0.0625mm displacement increment)
260 and the constant external load (0.8MPa) applied to the top boundary through five adjacent
261 rigid blocks. Simulations started with setting the external load and were continued by
262 applying horizontal translation to the vertical wall, thus imposing the active earth pressure
263 state. The wall translation rate was low enough to ensure the quasi-static conditions.

264

265 *2.5. The basis for the selection of displacement fields*

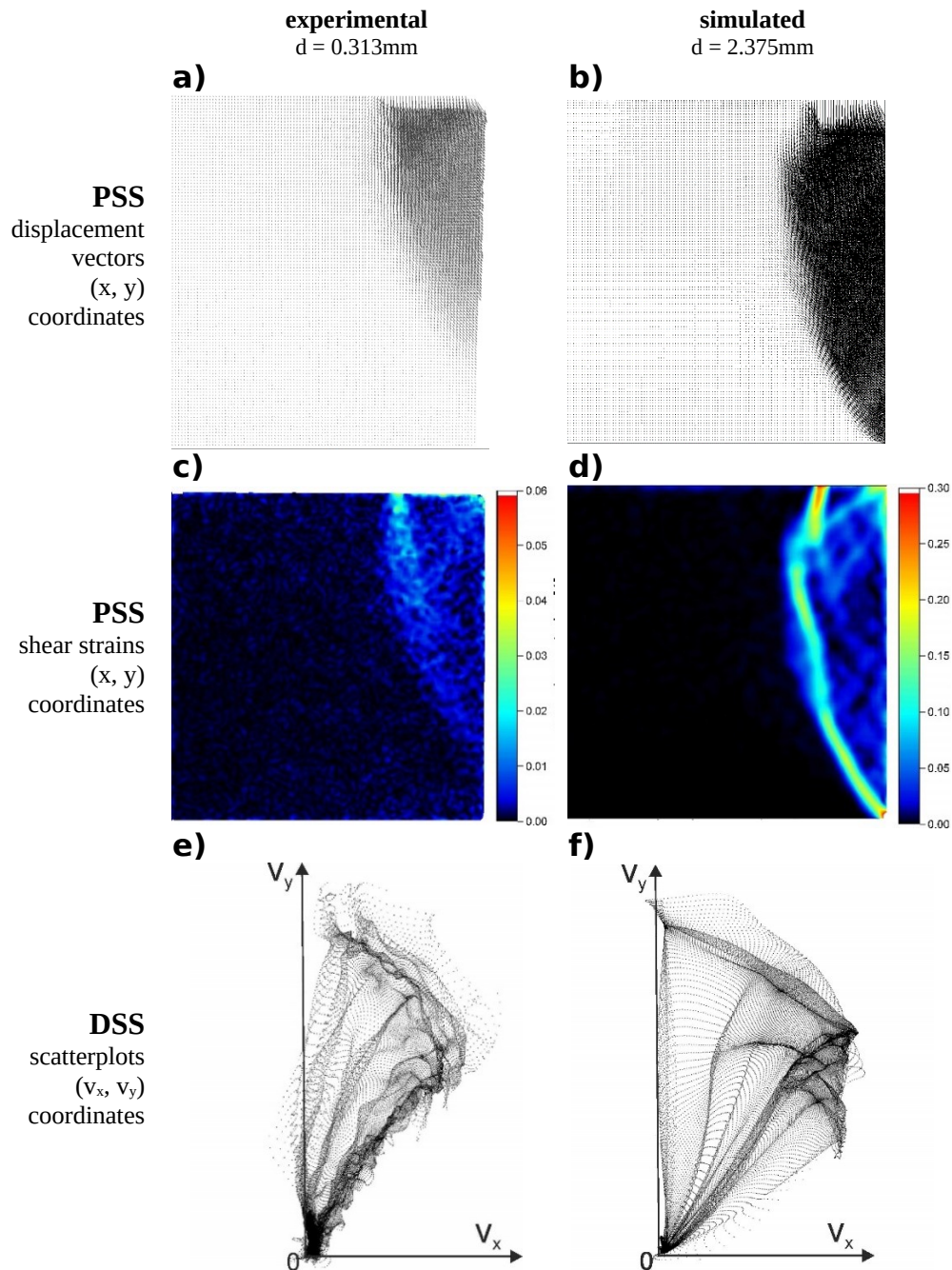
266 The two examples of displacement fields were selected for the analysis, one experimental and
267 the other simulated, corresponding to the wall displacement of 0.313mm and 2.375mm,
268 respectively (Fig.5). The basis for their selection is a similar simple failure mechanism, not
269 the same wall displacement.

34
35
36



270

271 Such a mechanism consists of a dominant shear band (Fig.5c, d), cutting off a triangular
272 wedge in the displacement field (Fig.5a, b). It can be observed mainly in the initial phase of
273 the experiment, where no competing failure mechanisms appear, as mentioned in Section 2.1.



274 **Fig.5.** Displacement data, representing the initial stage of the experiment (left column) and the final stage of the
275 DEM simulation (right column): **(a), (b)** - displacement vectors, **(c), (d)** - the corresponding shear strain maps
276 (PSS), **(e), (f)** - displacement fields (a) and (b) as scatter plots (DSS).

277 The experimental verification of the DEM simulation was already done by Leśniewska et al.
278 (2020). The present paper aims to investigate the specific structures in the displacement fields
279 behind a retaining wall. It will be demonstrated in the proceeding sections that the 2D
280 simulation more clearly shows this structure; therefore, it was chosen as the primary data
281 source to apply the s-LID analysis.

282 The experimental displacement field was only added to show that the deformation structure
283 revealed in the simulations is not just a numerical artefact but can also be found in actual data,
284 albeit less clearly.

285

286 **3 Representation of the selected displacement fields in PSS and DSS space**

287 The displacement fields analysed in this paper were obtained by image analysis (DIC
288 method). The experimental images of size 2560x1920 pixels were acquired using Sony
289 Digital Still Camera DSCF717. The distance of 1mm, representing D_{50} grain diameter,
290 corresponds to about 12 pixels in these images. The images of the DEM simulation are
291 screenshots of size 1920x1080 pixels.

292

293 Displacement vector fields were calculated using a patch size of 41x41 pixels, with a step size
294 of 4 pixels for the experimental test and 2 pixels for the DEM simulation. It created about
295 21000 and 35000 displacement measurement points in sub-grain resolution corresponding to
296 about 6 measuring points per grain in the experimental and about 12 in the simulated
297 displacement field.

298

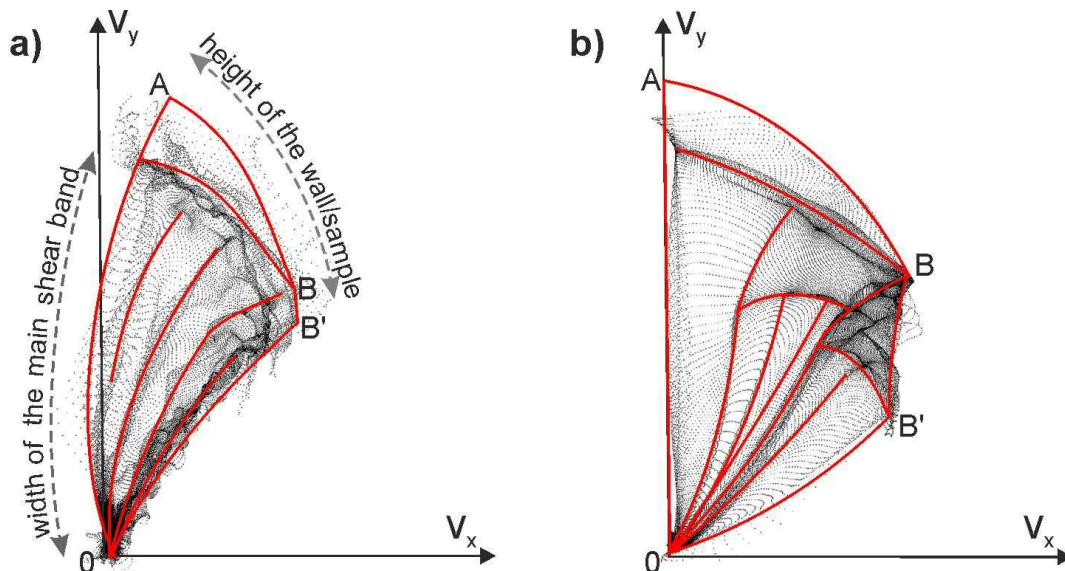
299 Figs 5a and b show the two selected examples of displacement fields. Figs 5c, d contain the
300 corresponding shear strain maps, calculated by DIC, where the high and low deformation
301 areas are colour-coded (red and yellow - high, blue and dark blue - low). Both maps show the

40

41

42

302 strong localization of shear strains, cutting off some Coulomb-like curvilinear wedges. Inside
 303 the wedge, in contrast to the classic image of the Coulomb wedge as a rigid body, weaker
 304 ('micro') bands are visible, indicating that the interior of the wedge is undergoing some
 305 deformation.
 306
 307 The image of the primary shear band is more apparent in the case of DEM simulation
 308 (Fig.5d). Some ordering of the micro-shear bands inside the wedge can be seen, not observed
 309 in Fig.5c.

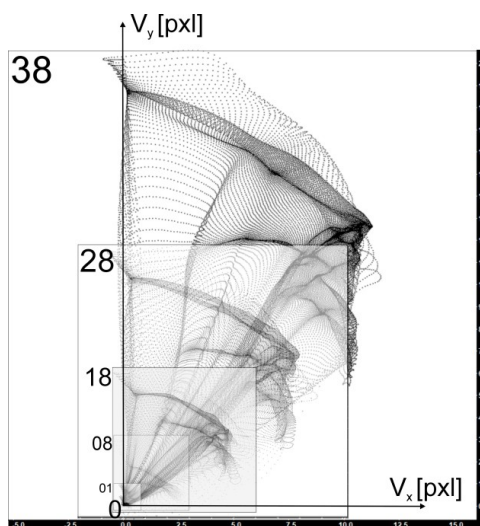


310
 311 **Fig.6.** Approximate division of the scatter plots, representing: **a)** the initial stage of the experiment (Fig.5e), **b)**
 312 the final stage of the DEM simulation (Fig.5f).

313
 314 Figs 5a and b are a standard presentation of the displacement field in physical-state space
 315 PSS. Figs 5e, f show the same data as scatter plots, where v_x and v_y are displacement
 316 components in x and y direction.

317
 318 Fig.6a shows that most of the scatter plot points in the case of the retaining wall test fill the
 319 interior of a particular curvilinear triangle OAB, with one of its vertices at the origin of the
 320 coordinate system. The triangle is subdivided by radial lines originating at point O and
 321 defining the regular structure of the displacement field. The existence of the structure is

322 difficult to guess when looking at Fig.5a. Similar structure is present in the simulation (Figs
 323 5f and 6b); however, in this case, the lines of the scatter plot's divisions are much more
 324 smooth. The most obvious difference is the area OBB', well developed and containing much
 325 detail in Fig.6b and reduced and devoid of precise details in Fig.6a. The reason for more
 326 randomness in the case of the experimental data is probably the earlier mentioned fact that the
 327 simulation is made for one layer of grains only. The experimental sample consists of some 20
 328 layers, giving the grains additional degrees of freedom in an out-of-plane direction. Some
 329 physical explanations for the divisions shown in Figs 5e, f and 6 will be given later in the
 330 paper.



331 **Fig.7.** Scatter plots of displacements after steps 01, 08, 18, 28 and 38 of the DEM simulation.

332
 333
 334 Despite the apparent differences between Fig.5e and 5f, both scatter plots have enough
 335 common features to be analysed in the same way. Therefore, in the next chapter, the
 336 significance of individual elements of the scatter plot structure and its evolution over time will
 337 be investigated.

338 **4 Characteristics of scatter plots for a retaining wall problem**

339 *4.1. DEM simulation – preliminary analysis of the displacement field*

340
 46
 47
 48

341 DEM simulation of the translating retaining wall (Fig.4) consisted of 38 wall displacement
342 steps of 0.0625mm and reached the total wall displacement $d = 2.375\text{mm}$. Fig.7 represents the
343 evolution of the displacement's scatter plot during the whole course of the simulation. Each
344 step's scatter plot consists of about 300 000 points, so for picture clarity, only 5 displacement
345 steps are shown (01, 08, 18, 28 and 38).

346

347 Fig.7 shows that the visible structure of scatter plots does not change significantly during the
348 simulation, and the increase in the total wall displacement only causes its expansion. It is
349 assumed that any observations concerning the structure of displacement fields described in the
350 following chapters are valid regardless of the considered time step.

351 The above finding also applies to the experiment, provided that the stages corresponding to a
352 single failure mechanism are considered separately (as noted in Section 2.1, in the case of a
353 multi-layer sample in P-3D tests, some competing failure mechanisms may coincide,
354 complicating the displacement field).

355

356 Fig.1 suggests that two types of elements may constitute the deformation structure: shear
357 bands (dark zones), where the granular material deforms extensively, as evidenced by changes
358 in the position of the lead shot markers (white dots), initially located in the nodes of the
359 square-meshed net, and areas of low deformation (approximately retaining the initial marker
360 pattern) closed between the intersecting shear bands. Such zones are referred to by Tordesillas
361 et al., 2022 as shear zones (SZ) and dead zones (DZ), respectively. This study aims to confirm
362 the coexistence of these two types of zones in the displacement field behind the retaining wall
363 and to characterize their kinematics using the s-LID analysis described in Section 5. In order
364 to correctly interpret the future s-LID results, firstly location and kinematics of the selected



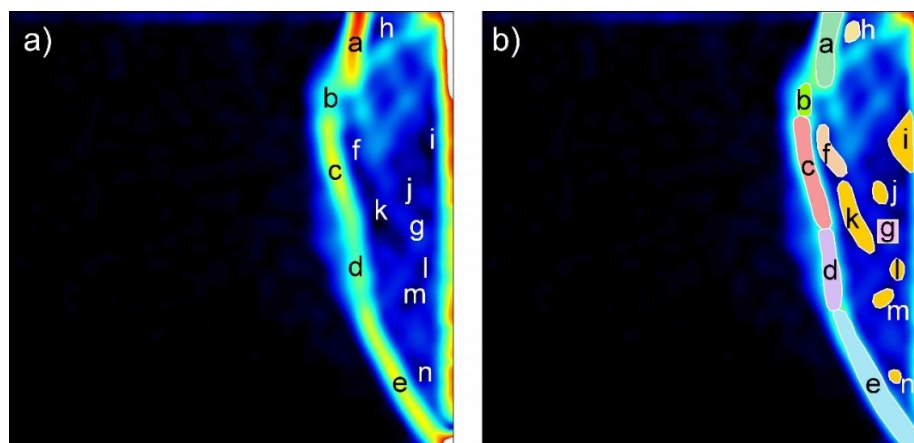
365 fragments of SZ and DZ zones in the PSS and DSS will be determined directly from the
366 definition of shear strains in 2D:

367
$$\varepsilon_{xy} = \frac{1}{2} \left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) \quad , \quad (2)$$

368 where v_x and v_y denote displacement components in x and y directions.

369

370 Fig.5d shows a typical 2D shear strain map for a retaining wall. This map will explain the
371 origins of the division lines in Fig.5f (the same lines are marked red in Fig.6b). We will also
372 demonstrate which separated areas or lines may correspond to SZ or DZ. For this purpose, the
373 shear band in Fig.5d was divided into equal squares with a dimension corresponding to the
374 approximate width of the shear band, estimated from the strain map assuming a shear strain
375 value greater than 10%, which corresponds to the light blue colour in Fig.5d.

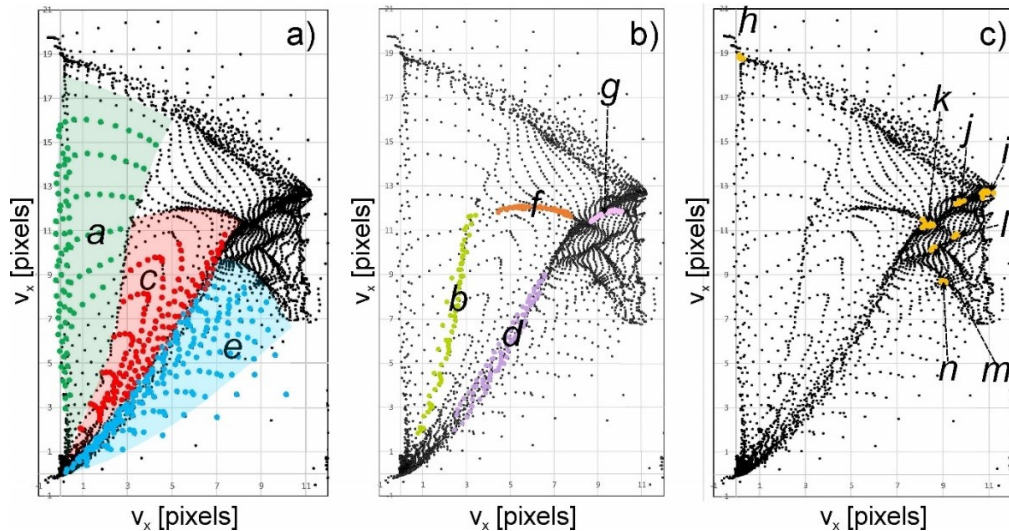


376
377 **Fig.8.** Areas of different shear strain values selected for data interpretation in DSS: **a)** – the approximate location
378 on the shear strain map, **b)** – extension and colour code.

379 Next, the shear strain and displacement data were double-filtered in the following way:

- 380
- 381 · first, several characteristic regions with high or low strain values have been identified from the shear strain map (Fig.8a),
 - 382 · second, the ranges of shear strain values for these areas and the corresponding ranges of coordinates (x, y) of the measurement points belonging to them were read from the
383 DIC numerical files,
- 384

- 385 · third, the (x, y) coordinates were used as filters on the original displacement data to
- 386 split them into subsets related to the selected ranges of shear strains,
- 387 · finally, the original and filtered displacements were plotted together as a scatter plot.



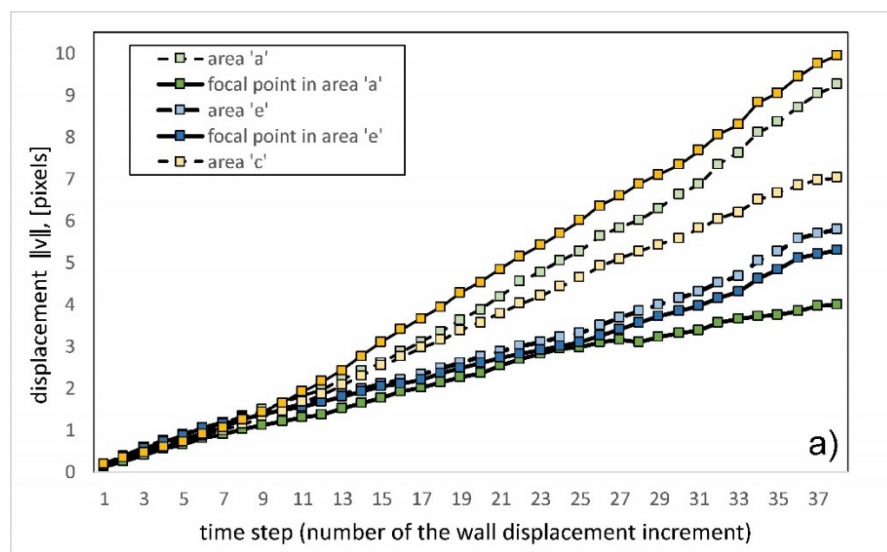
388
 389 **Fig.9.** Scatter plots with the reduced number of points showing locations corresponding to the areas from Fig.8b
 390 in DSS space.

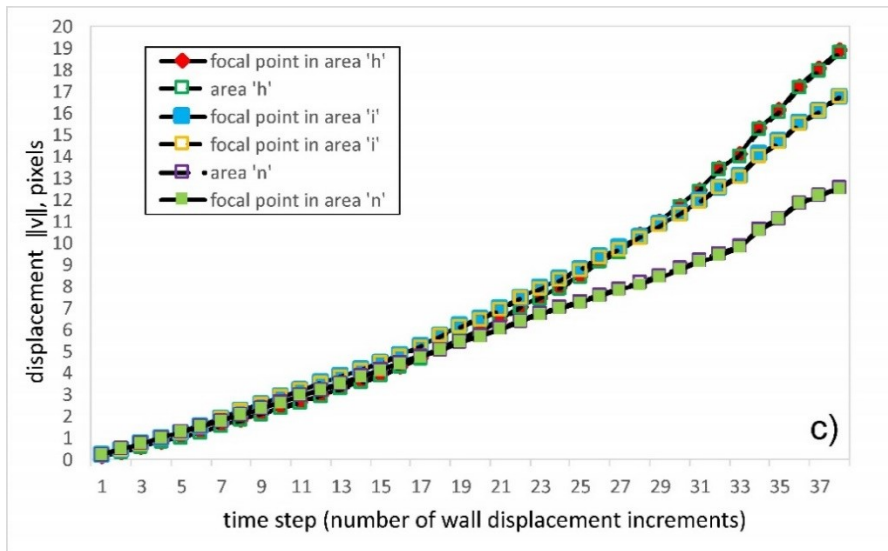
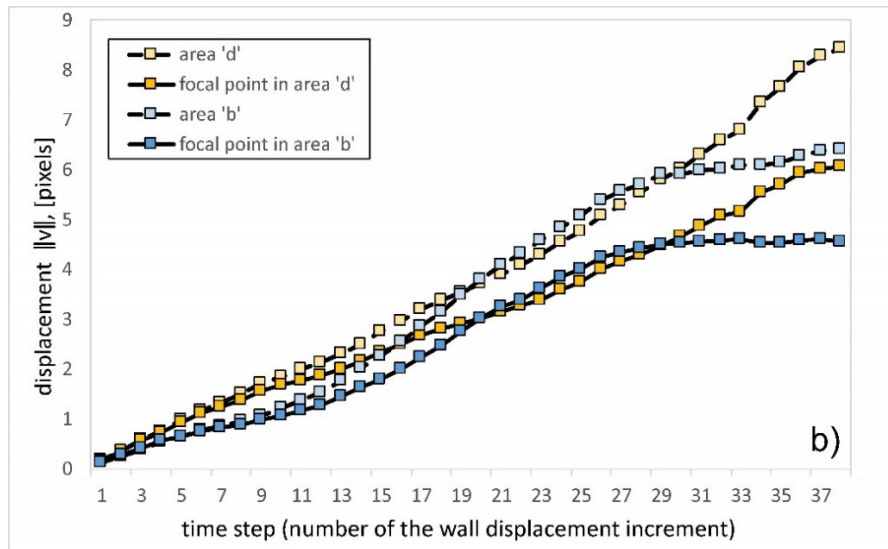
391 The regions ('samples') in Fig.8 belong to the primary shear band (**a-e**) or lay inside the
 392 curvilinear wedge delimited by it (**f-n**). The shear band seems non-uniform in terms of the
 393 shear strains – the areas **a**, **c** and **e** present visibly higher strains than **b** and **d**. Shear strains in
 394 the areas **h-m**, marked yellow in Fig.8b, are the lowest and can be regarded as parts of
 395 potential DZ, while the areas **f** and **g** represent the intermediate strains.

396
 397 The following approximate ranges of shear strains were filtered out: 0.12 - 0.18 for the areas
 398 **a**, **c** and **e**, 0.09-0.12 for **b** and **d** and 0.002-0.007 for **h-m**. Fig.9 presents the results of the
 399 double filtering procedure as scatter plots using the colour code from Fig.8b. For clarity, only
 400 every sixteenth point of the complete data set from Fig.5f is shown.

401
 402 Fig.9a confirms that the shear zone, commonly considered a single shear band, consists of
 403 three kinematically distinguished segments: **a**, **c**, and **e**, physically separated in the PSS space
 404 and adjacent in the DSS space. The material in these segments can move in different

405 directions but within limits defined by the opening of each part of the scatter plot 'fan'. This
 406 opening and the distances between measurement dots in **a**, **c**, and **e** mean that grains from
 407 close neighbourhoods move in different directions with different displacement rates, as
 408 commonly expected of SZ. As shown in Fig.6, going from segment **a** in Fig.9a,
 409 through **c** to **e**, we move from the top to the bottom of the strain localisation, i.e. from the top
 410 to the bottom of the analysed retaining structure. Moving up from the point (0,0) of
 411 segment **a** (for example), we cross the shear band from left to right.
 412
 413 The area of the scatter plot covered in Fig.9a by **a**, **c** and **e** confirm the common knowledge
 414 that it is in the strain localization that the most significant changes in the kinematics of the
 415 granular material occur.





416 **Fig.10.** Displacement value evolution for the selected focal points compared to their closest neighbourhood's
 417 average displacement in the areas: **a)** *a, e* and *c*, **b)** *b* and *d*, **c)** *h, i* and *n* (Fig.8b).

418 Fig.9b shows the displacement distributions in areas **b** and **d** – interestingly, they form
 419 borderlines between areas **a-c** and **c-e** from Fig.9a, respectively. Their linearity suggests some
 420 additional constraints imposed on grains' motion, compared to **a, c** and **e** – the grains there
 421 move approximately in one direction, however, by different displacements. Similar linear
 422 distributions characterize areas **f** and **g**. The rest of the selected regions (**h-m**) show another
 423 worth to note feature in DSS (Fig.9c) – each contains from a dozen to several dozen
 424 measurement points; however, in the scatter plot, they are very close to each other. Such
 425 concentration represents close to rigid motion (Section 2.2), so the existence of DZ in the

426 displacement field is confirmed. Each selected quasi-rigid area has its specific displacement
427 vector (a different position on the scatter plot), meaning they are kinematically independent.
428 The question is whether they are permanent or temporary. Several focal points were selected
429 to check it, representing regions **a-n** (Fig.8) and their closest neighbourhoods, covering about
430 20 measurement points.

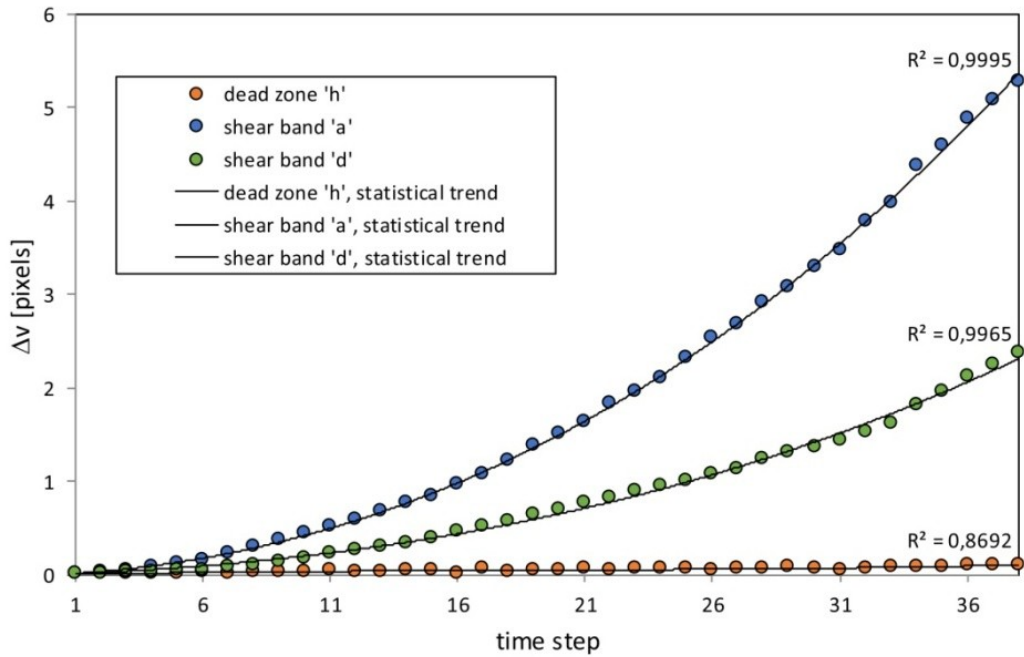
431

432 Fig.10 compares the changes in the displacement of individual focal points with the mean of
433 the immediate neighbourhood as a function of time steps. Starting from the areas **a**, **c** and **e**
434 (Fig.10a), the clear difference between d_k and d_{N-av} (Eq.(1)) appears around the 9th time step
435 and is growing quasi-linearly. It is the largest in **a** (light and dark green markers) and the
436 slightest, but detectable in **e** (yellow and orange). In the case of DZ (Fig.10c), preselected in
437 Fig.9c, d_k and d_{N-av} almost perfectly coincide from the first to the last time step, confirming
438 their motion is permanently close to that of a rigid body.

439

440 The evolution of d_k and d_{N-av} in the areas **b** and **d** (Fig.10b) at the final stage of simulation
441 seems to reach the state of constant displacement; however, with different values – this
442 suggests a way of movement that is difficult to classify as SZ or DZ, what Fig.11 best
443 illustrates.

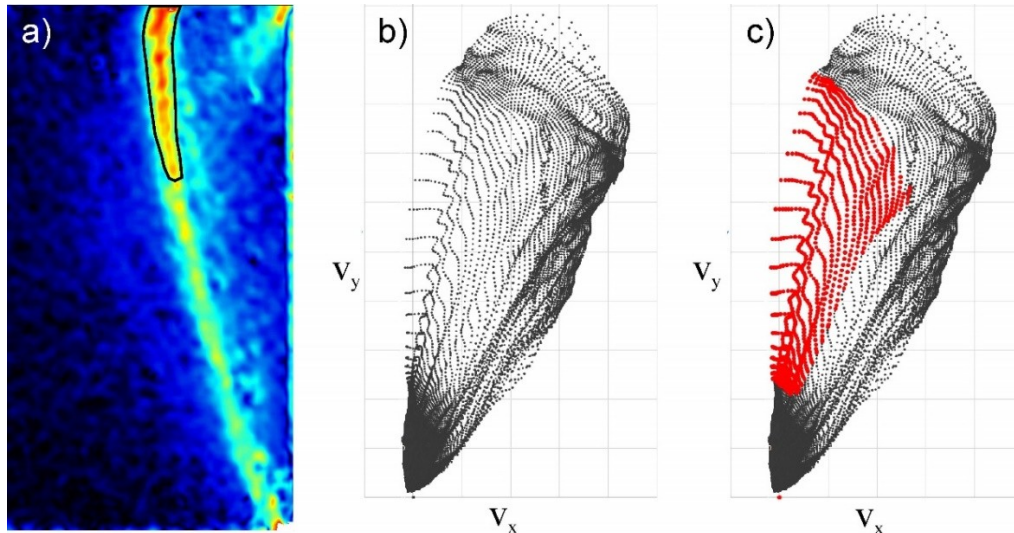




444
445 **Fig.11.** The statistical trend of the difference between the selected focal points' displacements and the average
446 displacement of their neighbourhoods, in areas **a**, **d** and **h** in Fig.8b.

447
448 This figure shows the statistical trend of the absolute difference Δv between displacements of
449 the selected focal points and the average displacement of their immediate neighbourhood, as a
450 function of the time step, for the data from Fig.10.

451
452 In the area **h** (orange circles), Δv takes a value close to zero from the beginning to the end of
453 the simulation, confirming that it is subject to permanent close-to-rigid motion, expected for
454 DZ. In contrast, in the area of **a** (blue circles), it grows significantly, which proves the
455 decreasing coordination of grain movement, characteristic of SZ. The Δv plot for area **d**
456 (green circles) suggests a way of movement of the granular material inconsistent with neither
457 DZ nor SZ. Looking at Fig. 9b, it becomes clear that the material motion in region **d** is
458 represented in the scatter plot by some well-defined curve, not an extensive 2D area or
459 neighbourhood of a point as in the case of **a** or **h**. The linearity of the area **d** in the scatter plot
460 suggests grain movement approximately in one direction; however, not with constant velocity
461 (like in a laminar flow), less constrained than in DZ, but not as free as in SZ.



462
 463 **Fig.12.** The example of the structure in the experimental data (the final stage of the retaining wall model test: **a**)
 464 shear strain map, calculated by DIC, **b**) corresponding scatter plot of the displacement field, **c**) location on the
 465 scatter plot of the area circled in black in (a).

466
 467 *4.2. Experimental test*

468 Fig.12 shows the data from the final step of the model test: the shear strain map (a), the
 469 original scatter plot (b) and the part of the scatter plot (red dots) corresponding to the area
 470 circled in black in (a), superimposed on the complete scatter plot (c). Similar segmentation to
 471 that shown in Fig.9 for the DEM simulation appears. The right edge of the red area coincides
 472 nicely with one of the lines dividing the original scatter plot into segments, confirming the
 473 qualitative similarity between experimental and numerical data.

474
 475 Based on the preliminary examination presented in this section, we have demonstrated that in
 476 the displacement field behind the retaining wall there exists a structure consisting of the
 477 strongly constrained near-to-rigid motion zones (DZ), weakly constrained large deformation
 478 zones (SZ) and some transition zones (TZ). Material motion in transition zones is quasi-
 479 laminar, with the direction of movement constrained and varying displacements. The simple
 480 procedure described in Section 4.1 can prove only the existence of the structure in the
 481 displacement field. It would be a real challenge to obtain its complete picture, as many time-
 482 consuming hand operations are required in this case.

483 5 *s*-LID analysis

484 In order to extract the shear zones (SZ) and dead zones (DZ) from the displacement fields in
485 the previous section, it was necessary to calculate the shear strains from the given
486 displacements and perform extensive data filtering.

487

488 *s*-LID analysis, as proposed by Tordesillas et al. (2022) and demonstrated on DEM simulation
489 of an ensemble of spherical grains submitted to planar biaxial compression, offers a better
490 way of investigating structured deformation. To demonstrate this, we will compare the results
491 of the procedure presented in the previous sections with the *s*-LID analysis of the same
492 problem.

493

494 *5.1. Terminologies used in the s-LID analysis*

495 - Kinematic clusters:

496 - clusters found in DSS - grains belong to the same kinematic cluster if they
497 share very similar displacements,

498 - Kinematic outliers:

499 - grains that are moving significantly differently from the others,

500 - *s*-LID value, ranging from 0 to $+\infty$, indicates the outlying-ness of a grain's
501 motion - the higher the value, the more outlying the grain's motion is.

502 The only independent parameter in the *s*-LID analysis is the size of the examined
503 neighbourhood *s*, expressed by the number of belonging measuring points.

504

505 *5.2 s-LID estimator*



506 The local intrinsic dimensionality LID (Houle, 2017) of the grain motion data in a
507 neighbourhood of the displacement-state space (DSS), centred at the focal grain's
508 displacement, is measured by the s -LID estimator, defined by Eq (3):

$$509 \quad s\text{-LID}(\mathbf{v}) = -\left(\frac{1}{s} \sum_{i=1}^s \log \frac{d_i(\mathbf{v})}{d_s(\mathbf{v})}\right)^{-1}, \quad (3)$$

510

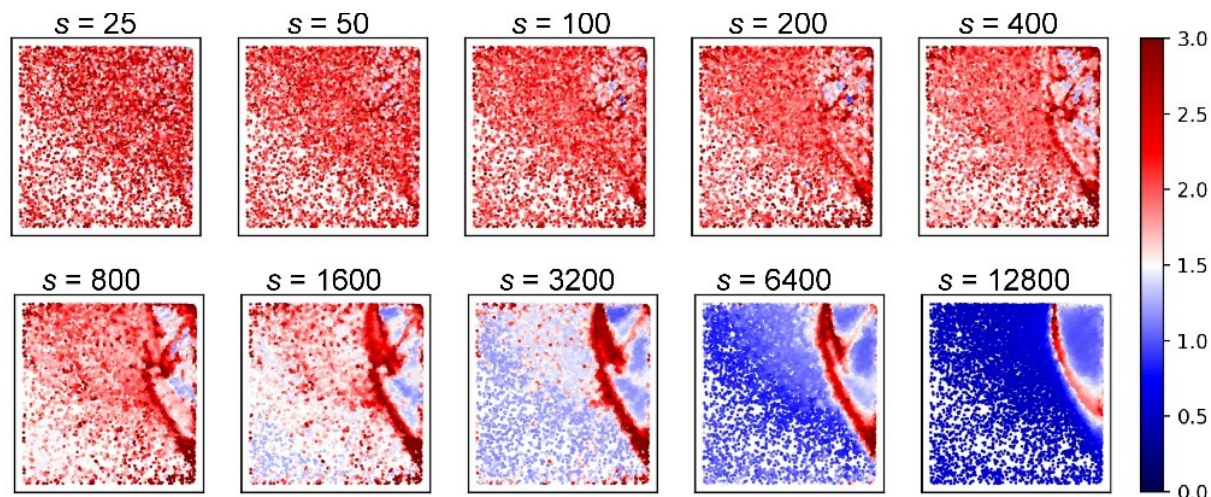
511 where \mathbf{v} is the displacement of a focal grain, a 2-dimensional vector representing its
512 translation in the horizontal and vertical directions, $d_i(\mathbf{v})$ is the Euclidean distance between the
513 focal grain displacement and its i -th nearest neighbour in DSS; $d_s(\mathbf{v})$ is the maximum of the
514 neighbour distances in DSS; and $s \geq 2$ is a parameter to control the size of the neighbourhood
515 to investigate (i.e., the number of nearest neighbours to use in the estimation procedure). As a
516 result, the s -LID is discriminative of the complexity of collective grain motions (Zhou et al.,
517 2021). The results of s -LID calculations can be transferred back to the coupled physical space
518 (PSS) to indicate the actual geometric position of areas distinguished by similar s -LID values
519 in DSS. Increasing s allows the identification of outliers for a larger neighbourhood of DSS,
520 thereby enabling the identification of patterns with more complex kinematics.

521

522 *5.3 Results of the s -LID analysis for the retaining wall model*

523 Usually, s -LID results are given in PSS and DSS space using some colour code. In this paper,
524 the s -LID score increases from blue to red - the bluer the colour, the lower the s -LID value,
525 indicating no significant difference in the specific grain compared to its nearest neighbours in
526 terms of the motion. Searching for a structure using the s -LID estimator requires adopting the
527 neighbourhood size s , which may correspond to the searched structure elements (kinematic
528 clusters). The choice of the value depends on the number and density of the measurement
529 points - $s = 25$, for example, corresponds to the area occupied by 5 grains in the case of the

530 experimental displacement field and 2.5 grains for the DEM simulation. The study of an a
 531 priori unknown deformation structure requires selecting the appropriate neighbourhood size
 532 and thus applying some range of s values by trial and error.



533
 534 **Fig.13.** Sensitivity of s-LID analysis results on s parameter in PSS space.

535 Fig.13 gives the results of such a parametric study in PSS space for the selected experimental
 536 displacement field and s between 25 and 12800. They depend on s : up to $s = 1600$, the s-LID
 537 values uncover a majority of areas with uncoordinated material movement (high s-LID score),
 538 corresponding mainly to small displacements to the left of the motion wedge but also present
 539 inside it.

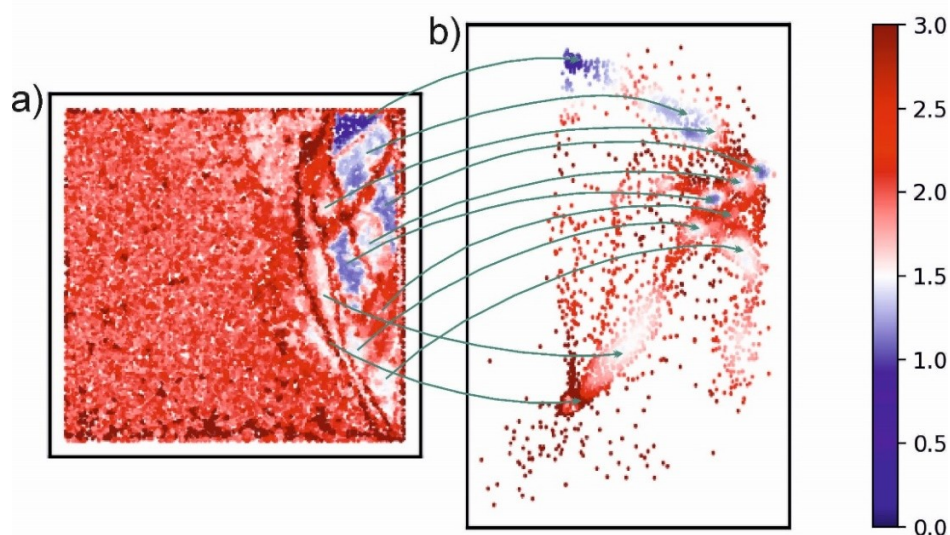
540
 541 The explanation of the high s-LID score in the small displacement area (the left side of the s-
 542 LID maps) is the scale of the selected neighbourhood. From the perspective of 5 grains ($s =$
 543 25), their small local movements around the equilibrium positions cannot be correlated.

544
 545 This lack of correlation can result from small, independent local grain movements or a
 546 relative error of displacement measurement, which is usually the biggest in the areas of
 547 negligible deformation. Despite the relatively high values of s-LID in this area, their
 548 distribution is random, with no visible structure. Only when the neighbourhood size starts to

549 compare with some structure elements do the larger scale correlations emerge, as Fig.13
550 illustrates. It is clear from the figure that there is an optimal value of s for the examined
551 displacement field, which reveals the deformation structure most clearly.
552 Precise determination of this optimal value is quite challenging: the first signs of ordering on
553 the s -LID map appear already for $s = 50$. However, they do not create a continuous structure
554 with dimensions comparable to the model's height. Such a structure appears around $s = 400$
555 and develops with s increasing to around 3200. However, the further increase of s causes the
556 structure details to be lost – the tool to look for the structure becomes too big. Instead, the
557 hierarchical order of the various elements can be revealed, e.g. dominant shear band is best
558 isolated at $s = 12800$ (about 2500 grains).

559

560 In the next part of the paper, the following values of s were selected for further analysis: 400,
561 600 and 3200 for the DEM simulation and 1600, 3200, 6400 and 12800 for the experimental
562 test. The selected examples of the s -LID results for the final step 38 of the DEM simulation
563 are shown in Figs 14, 15b, 16, 17, 21a and 22a. Figs 13, 18, 19 and 20 relate to the
564 experiment. The number of ‘measurement points’ in the original displacement fields had to be
565 down-sampled for s -LID analysis to fulfil the software requirements. Due to it the scatter
566 plots presenting s -LID results (e.g. Fig.14b) contain less detail than the original ones (e.g.
567 Fig.5f) but preserve enough information to be further studied.

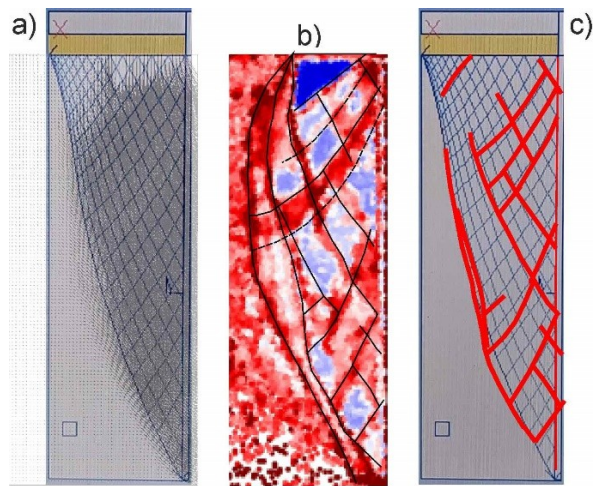


568
 569 **Fig.14.** The example of s-LID results for DEM simulation of a retaining wall problem (step 38, $s = 400$). Dark
 570 blue is attributed to near-rigid motion, dark red - extensive deformation or lack of coordinated motion: (a) PSS
 571 view, (b) corresponding scatter plot (DSS).

572 This section analyses the *slid_uniform_c* data set, consisting of uniformly down-sampled
 573 cumulative displacements. The right side of the map in Fig. 14a shows a network built of
 574 intersecting, fine bands with a high s-LID score (dark red), suggesting weak bonds between
 575 the grains. The areas closed between the bands in the lattice meshes have lower s-LID values,
 576 that is, stronger inter-grain bonds, and are marked in Fig.14a with white, pink and shades of
 577 blue. The lattice is so well defined that a fairly detailed sketch of it can be made, such as in
 578 Figs 15b or 17. Fig.15b shows the sketch of the right part of the network from Fig.14a. It is
 579 the region where the corresponding boundary value problem solution using the rigid-plastic
 580 approach and the method of characteristics exists.

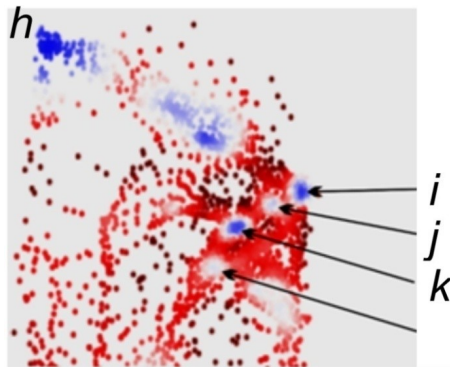
581
 582 The typical solution consists of the stress distribution on the top of a retaining wall,
 583 determining its bearing capacity and the stress characteristics, forming a so-called 'slip line'
 584 field. The slip line field obtained for the retaining wall model used in this paper (both
 585 experimental and simulated by DEM) was calculated using the RES program (Leśniewska,
 586 1993). The input data were the geometry of the problem and strength parameters of the
 587 granular material: internal friction angle $\hat{\varphi} = 30^\circ$ and soil unit weight $\gamma = 16.7\text{kN/m}^3$.





588

589 **Fig.15.** Comparison between the *s*-LID network and the solution of the corresponding boundary value problem
 590 using the rigid-plastic approach: **a)** stress characteristics superimposed on the displacement field from Fig.5b, **b)**
 591 sketch of the grid of lines dividing the moving wedge area according to the value of the *s*-LID parameter, **c)** the
 592 selected dividing lines with directions close to stress characteristics.



593

594 **Fig.16.** Close-up of Fig.14b – the scatter plot for DEM simulation of a retaining wall problem (step 38, $s = 400$)
 595 – objects *h* and *i-k* recognized in Fig.9c and here.

596 Fig.15a shows this 'slip line' network superimposed on the displacement field from Fig.5b.
 597 The extent of the 'slip line' field coincides nicely with the area of large displacements but not
 598 with the region outside it. On the contrary, *s*-LID analysis finds elements of a discrete
 599 network also within the area of negligible displacements to the left of the motion area (e.g.
 600 Fig.17a).

601

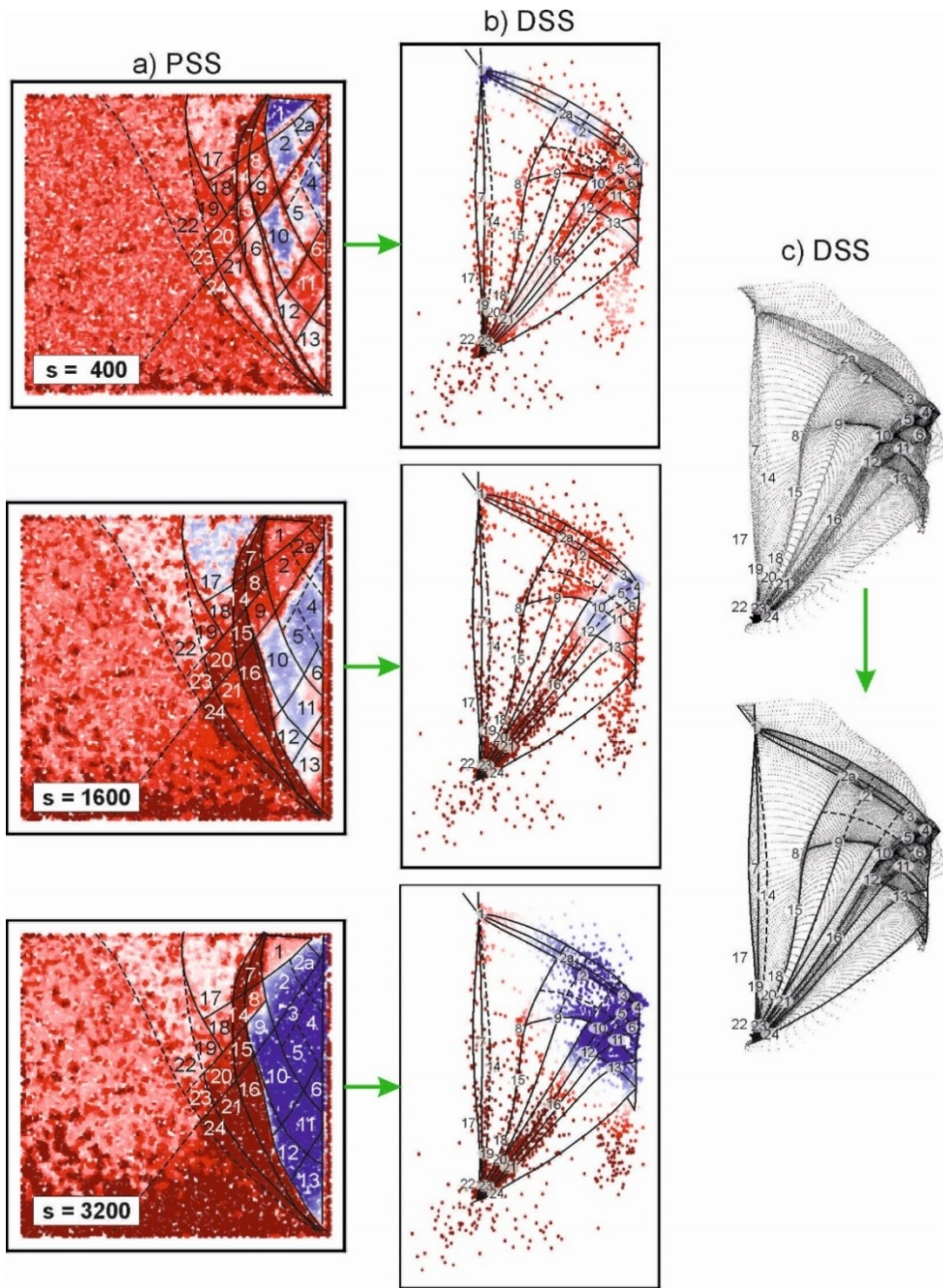
602 The sketch of the network consisting of black lines in Fig.15b resembles the solution of a
 603 bearing capacity problem for a retaining wall. However, the stress or velocity characteristics
 604 represent an un-deformed state, so they do not describe motion and should therefore be

88
 89
 90

605 interpreted as indicating its potential and not actual directions. Nevertheless, Fig.15c shows
606 that quite a significant part of the s-LID network (red lines) is close to some directions of the
607 slip line field, which can be the result of the small deformation regime adopted in the
608 experimental program.

609

610 The value of s (neighbourhood dimension) determines the scale of objects we can extract in
611 the DSS space based on the value of s -LID. For example, $s = 400$ finds point-like objects such
612 as those indicated by some green arrows in Fig.14b. The indicated objects have a physical
613 meaning, evidenced by the comparison of their location with the position of the clusters of
614 yellow points in Fig.9c.



615
616 **Fig.17.** DEM simulation – the study of the influence of the s parameter on s -LID results.

617 The same objects can be recognized in both figures, as shown in Fig.16, where the areas
618 marked in Fig.9c as h, i, j and k are also present. Fig.17 illustrates the effect of the selected
619 values of s . Column a) of this figure shows a sketch of an extended (compared to Fig.15b)
620 grid, delineated on the $s = 400$ map by the high s -LID values (black lines) and superimposed
621 on $s = 1600$ and 3200 maps. Each area lying in the meshes of this grid and marked with
622 uniform colour was assigned a number facilitating further analysis.

623 These areas are approximately diamond-shaped (e.g. 2, 5, 9, 10), triangular (e.g. 1, 4, 6), or
624 take the form of elongated bands (e.g. 3 or 16). They differ significantly in the values of the s -
625 LID parameter, which is colour coded according to the scale in Fig.14. For $s = 400$, the colour
626 of individual meshes differs; within one mesh, it is relatively uniform, suggesting similar
627 kinematics (kinematic cluster).

628

629 The dimensions of individual diamond-shaped and triangular clusters are several times
630 smaller than the sample (the height of the model retaining wall). As a result, they can be
631 classified as mesoscale objects at most, except for the bands 7, 14, 15, and 16, whose range
632 covers the entire height of the model, thus classifying them as macro-scale objects. Column b)
633 in Fig.17 shows the results of the s -LID analysis of column a) as scatter plots. As these scatter
634 plots include a reduced number of points in the displacement field, for ease of interpretation, a
635 sketch of the division grid of the complete scatter plot is superimposed, marked with solid
636 lines in the lower figure of column c).

637

638 Taking advantage of the fact that the area moving as a rigid body in the PSS space
639 corresponds to one point in the DSS space, some clusters in Fig.17 ($s = 400$) classify as DZ.
640 The point-like objects in DSS (circular objects of relatively small radius) were already
641 pointed out in Fig.16. They are numbered 1, 4, 5 and 10 in Fig.17.

642

643 Apart from them, also clusters 2, 2a, 12 and 13 show a tendency to converge around a point in
644 the DSS space. Among the above-mentioned, cluster 1 best meets both conditions for close-
645 to-rigid motion – the lowest s -LID score (less than 0.5) and the concentration of the
646 displacement values around one point on the scatter plot. Clusters 4, 5, 10 and 12 show



647 equally good convergence to a point on the scatter plot, but a higher s -LID value, ranging
648 from 0.5 to 1.5.

649 Changing the size of the neighbourhood from $s = 400$ to $s = 1600$, which is analogous to the
650 change of the magnification in a photograph, i.e. the transition from detailed observation to
651 more coarse but capable of detecting less defined and larger objects, results in merging of
652 some clusters into bigger ones and a new value of their s -LID score. The merged clusters still
653 fit into the initial division grid of $s = 400$, in which essential lines do not change, while only
654 some weaker internal ones have disappeared. An example of such a merge is the blue triangle
655 in PSS space, which includes clusters 4-6 and 10-13, forming an object of dimensions in the
656 range of the model's height and uniformly distributed s -LID score of about 1.5.

657

658 In the DSS space, however, this triangle does not correspond to a circular area concentrated
659 around any point. Instead, it forms a stretched area, filling the characteristic triangle on the
660 right of the scatter plot, the side of which is labelled BB' in Fig.6. For this reason, the large
661 blue triangle in the PSS cannot be considered quasi-rigid. However, since it has been clearly
662 distinguished by the s -LID analysis for $s = 1600$, the motion of the points it covers must have
663 some common constraint, revealed in Section 5.4.

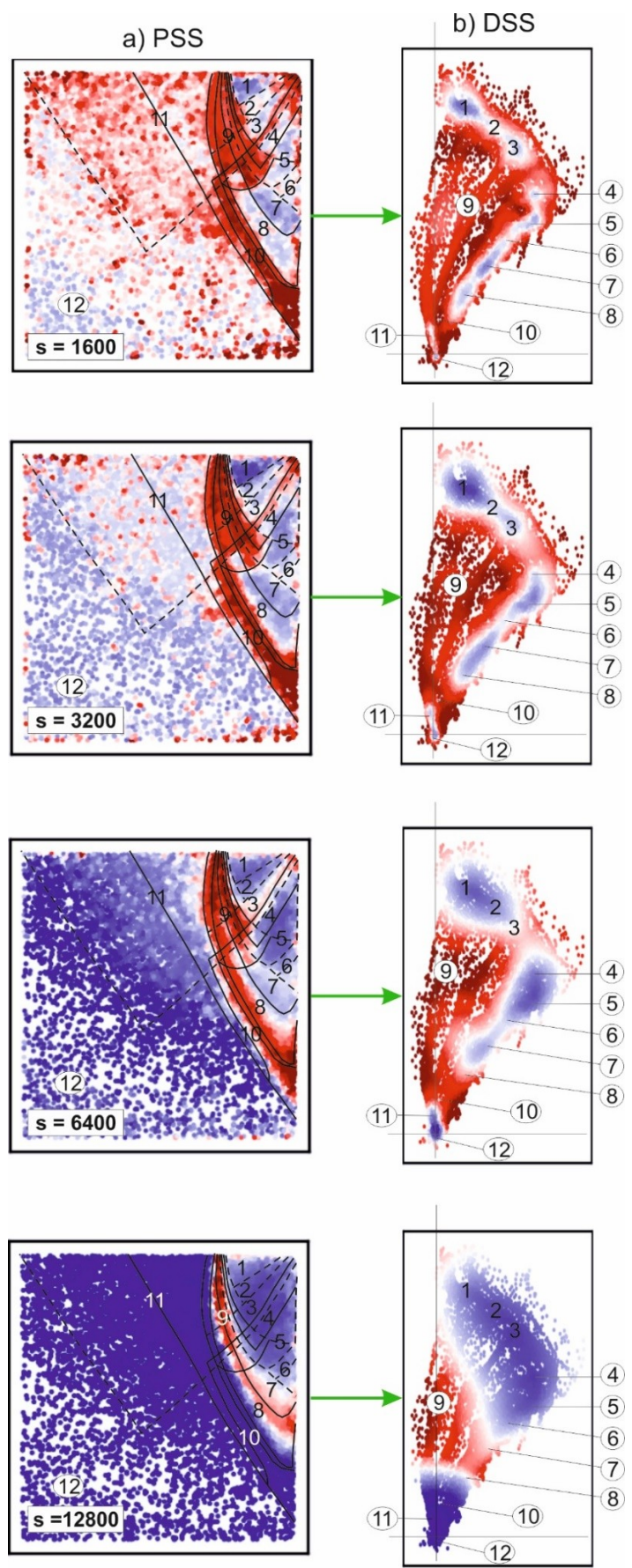
664

665 Another example of clusters merged in the PSS at $s = 1600$ are 1, 2 and 2a - they gained a
666 similar s -LID value around 2.25, and in the DSS space, their displacement points form a part
667 of a well-defined stretched band, limiting the highest displacement values. Due to it, the new
668 cluster cannot classify as DZ. Increasing the value of s to 3200 causes a further loss of detail
669 in the PSS and DSS. At the same time, the triangle built in the PSS from areas 3-6 and 10-13
670 corresponds in the DSS to the above-mentioned well-defined triangle on the right side of the

100
101
102



671 scatter plot, showing a regular internal division, which proves that the component clusters
672 retain a certain kinematic distinctiveness.
673



674
675
106
107
108

Fig.18. The experimental model test – the study of the influence of the s parameter on s-LID analysis results.

676 Clusters 7 and 14-16 belong to the same band, regardless of the size of the s . They do not
677 connect with others; the only change is some differences in the values of s -LID, ranging from
678 1.5 to 3.0 ($s = 400$) and 3.0 or higher ($s = 1600$ and 3200). Similar observations are made in
679 the analysis of experimental data. Fig.18 shows the example for s equal to 1600, 3200, 6400
680 and 12800.

681

682 In the case of $s = 1600$, cluster 4 is indicated as a quasi-rigid area because its s -LID value is
683 relatively low (between 1.0 and 1.5), and its image in the DSS space is a regular circular area
684 of a small radius. The remaining blue clusters (1-3, 5-8) do not satisfy the last condition -
685 their representations in DSS space are quite concentrated but far from a circular shape,
686 suggesting they are kinematically complex. Separating them into components is possible for
687 smaller values of s , as evidenced by Fig.19, corresponding to $s = 800$. A series of circular blue
688 areas of small radii in Fig.19b proves that the small blue clusters visible in Fig.19a classify as
689 DZ.

690

691 Further increase of parameter s leads to the isolation of the primary shear band, with the
692 highest values of s -LID (SZ), which in Fig.18a occurred for $s = 12800$. It agrees with the
693 previous research on the biaxial test (Tordesillas et al., 2022), where it was found that any
694 persistent shear band is exposed at higher s values.

695

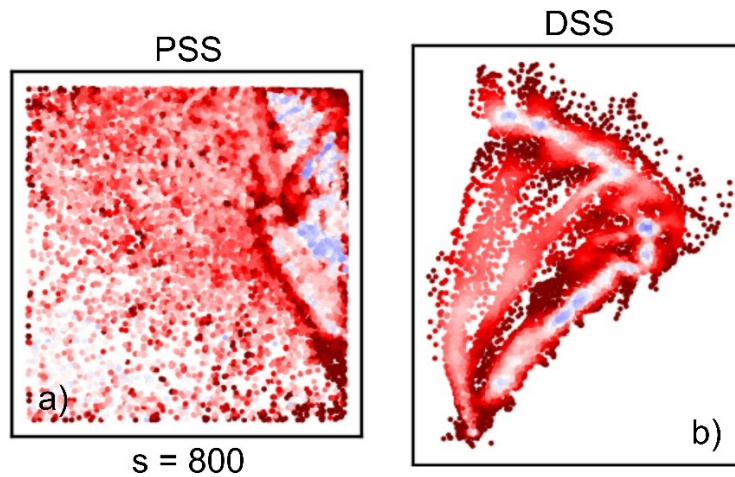
696 The separation of the primary shear band in the PSS allows its location in the DSS space to be
697 determined. Fig.18b ($s = 12800$) shows that the localised band of the highest s -LID score
698 covers the extended area in the very centre of the scatter plot. A similar location of the
699 primary shear band is also confirmed for the experimental displacement field using the
700 procedure introduced in section 4.1.

109

110

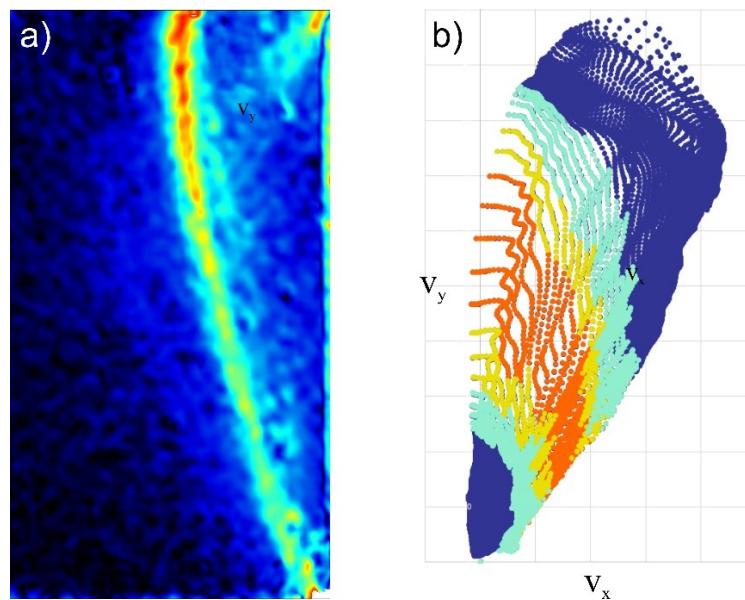
111





701
702

Fig.19. The experimental model test – the result of s-LID analysis for $s = 800$.



703

Fig.20. The primary shear band in the experimental model test: PSS (a) and DSS space (b).

704

705 Fig.20 shows the result - Fig.20a illustrates the shear band in PSS and Fig.20b its
706 transformation on the scatter plot, keeping the colour code from Fig.20a. As can be seen, both
707 figures (18 and 20) locate the primary shear band in the same area of the scatter plot.

708

709 It was necessary to calculate shear strains from the displacements to prepare Fig.20. The
710 apparent qualitative agreement between Fig.18 ($s = 12800$) and Fig.20 shows that the s-LID
711 analysis is the appropriate independent tool for studying granular materials from a new
712 perspective of local intrinsic dimensionality.

112

113

114

713 The undoubted advantage of this method, presented in this Section, is its ability not only to
 714 reproduce strain localisation from individual grain movements but also to give an insight into
 715 the detailed kinematic structure of granular material deformation, inaccessible for the
 716 continuum approach.
 717

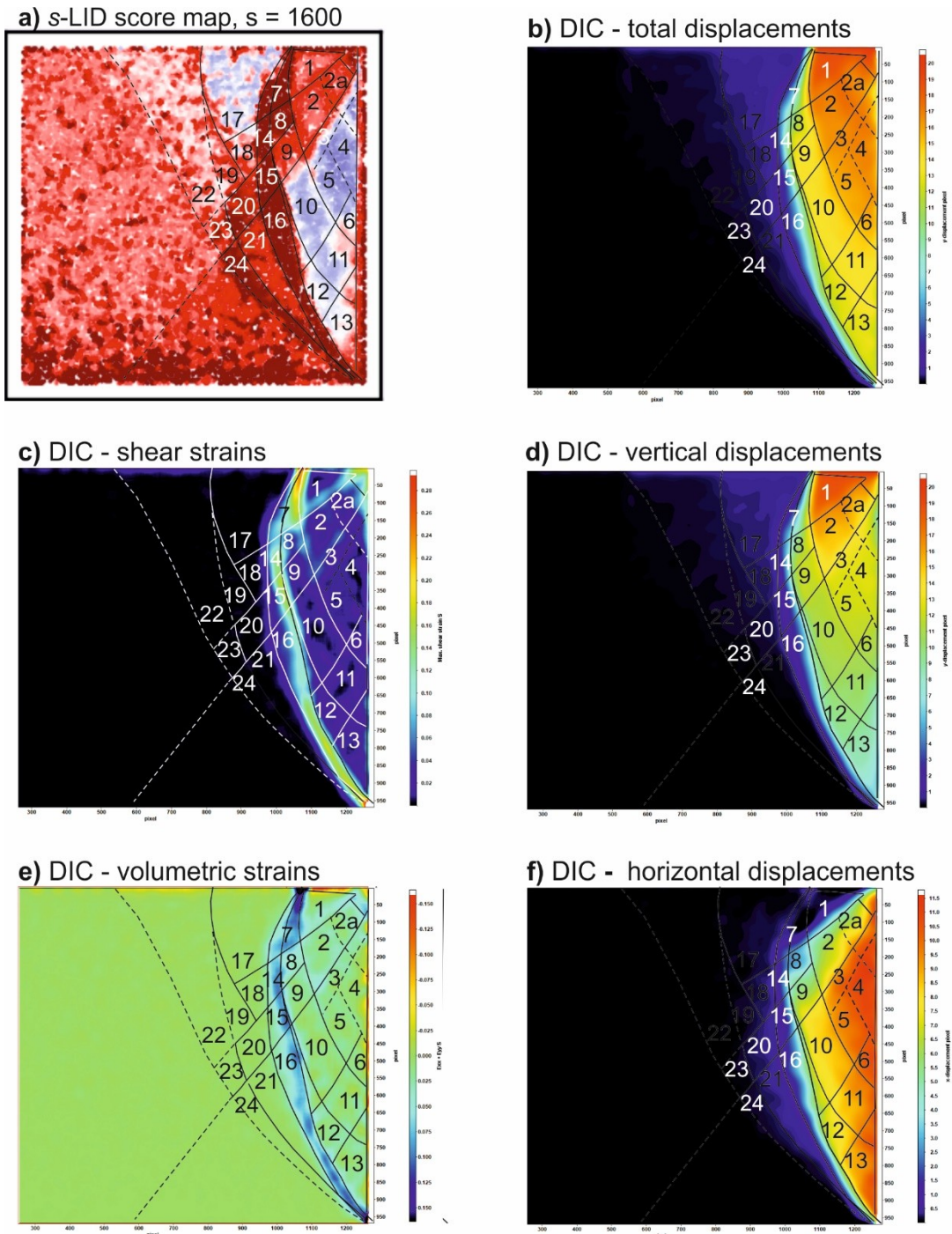


Fig.21. The s-LID score map (a) versus DIC analysis results (b)-(f).

718
719

115
116
117

720

721 *5.4. s-LID and DIC: physical reasons for clusters' forming*

722 To verify s-LID results for a retaining wall, the typical results of DIC analysis (maps of
723 displacements and strains), performed for step 38 of the DEM simulation, are compared in
724 Fig.21 with the s-LID score map for $s = 1600$. The same division grid as in Fig.17 is
725 superimposed on the DIC maps.

726

727 The total displacements map (Fig.21b) supplemented by this grid shows the role of the grid's
728 radial lines: they separate the areas of different, uniformly distributed displacement values.
729 The line between clusters 1-7, 2-8, 5-10 and 6-11 coincides with the well-defined border
730 between red (the highest) and yellow (lower) displacement zones. The divisions by the polar
731 lines present in the s-LID grid (Fig.21a) are not visible in total displacements but exist in the
732 vertical and horizontal displacement maps (Fig.21d, f).

733

734 The largest vertical displacements concentrate almost exactly in cluster 1, having a relatively
735 high and uniformly distributed s-LID score. Clusters 3-6 and 10-13 coincide with the area of
736 most significant horizontal displacements, so it is the displacement direction which makes the
737 common feature connecting them into one entity in Fig.21a (the triangle 3-6, 10-13) and
738 Fig.17 ($s = 1600$).

739

740 Clusters 1, 2, and 2a move primarily down in a vertical direction, while clusters 2 and 2a, in
741 addition to the predominant downward movement, experience some horizontal displacements,
742 thus constituting the transition zone between the small (cluster 1) and the big (blue) triangle.
743 It follows that the 'polar' lines are associated with a change in the direction of grains'
744 movement.

118

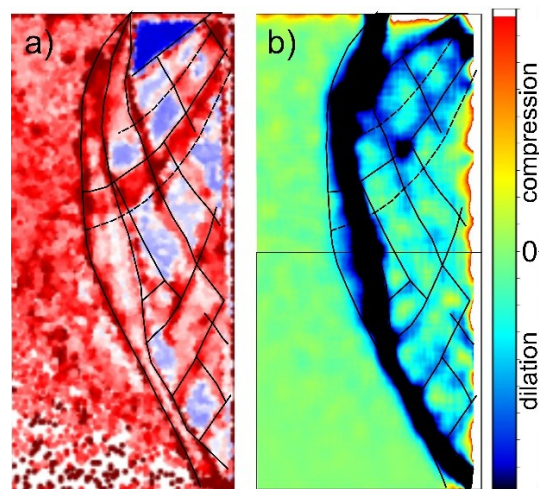
119

120

745 The clusters above 16 are practically not visible on displacement maps when the full scale of
746 displacements is used, like in Fig.21b, d and f. However, they are visible in Fig.21a, which
747 shows greater sensitivity of the s-LID value in the search for kinematic clusters in small
748 displacements region. Such sensitivity gives another essential advantage of the s-LID method
749 - access to areas of small deformation, immeasurable with the DIC method. It may be of great
750 practical importance, e.g., in assessing the risk of landslide movements, which can be very
751 slow in the long initial development period (Zhou et al., 2022).

752

753 The maps of shear and volumetric strains (Fig.21c and e) confirm that the highest s-LID
754 score, dark red clusters, organized in elongated bands, like 7, 14-16, coincide very well with
755 shear bands and experience significant extension, as expected. Several faint micro shear bands
756 are also present in Fig.21c, inside the high displacement region. They coincide with some of
757 the 'radial' and 'polar' lines detected by s-LID analysis in this area. Similar observations are
758 valid for the other steps of the DEM simulation, not shown in this paper. If we narrow the
759 scale in Fig.21e in such a way as to emphasize the volume changes in the area of the wedge
760 between the main shear band and the retaining wall (Fig.22b), numerous clusters subject to
761 slight compression appear. Their location fits well with the s-LID partition grid (Fig.22a).



762

763

Fig.22. s-LID cluster division grid (a) superimposed on volumetric strain map, calculated by DIC (b).

121

122

123

764 At the same time, the lines of this grid appear to follow the bands of slight extension between
765 the clusters (micro shear bands). It may explain the occurrence of small but stable DZ-s in the
766 area of global deformation behind the retaining wall, the structure of which is determined by
767 the micro shear bands mesh.

768

769 **6 Summary and conclusions**

770 A recently introduced metric called s-LID has been applied to study the classical geotechnical
771 problem of a retaining wall from the perspective of collective motion. A rigid wall translating
772 horizontally out of a granular sample (in active mode) with a constant displacement rate was a
773 specific object of the study.

774

775 Granular material movement is usually presented in physical space (PSS). The general
776 advantage of analysing the deformation of granular media in the DSS displacement space is
777 an insight into the processes that are difficult to capture in the PSS, such as forming kinematic
778 clusters with different motion characteristics. Combined with the s-LID method, built on local
779 intrinsic dimensionality, it allows a quantitative assessment of the deformation structure,
780 unavailable with the continuous approach. Several detailed observations come from the study:

- 781 • The s-LID method could successfully find shear bands in granular materials without
782 using the concept of continuum strain, thereby making a new independent and
783 powerful tool to study the deformation of granular materials.
- 784 • Local intrinsic dimensionality was entirely consistent with the DIC analysis in the
785 areas of significant displacement and expanded the knowledge of the deformation
786 structure in the areas of small displacements, where the DIC method loses its
787 accuracy. This feature may help assess the risk of slow landslide movements.

124
125
126



- 788 • The co-existing DZ and SZ constitute the deformation structure behind a retaining
789 wall. They differ in scale: SZ have prevailing dimensions in the range of the model
790 height and DZ are more numerous but much smaller, having two comparable
791 dimensions determined by the meshes of the s-LID dividing grid, where they occur.
- 792 • It is easy to recognize near-to-rigid zones (DZ) in DSS, even in the area of high
793 displacements, by their point-like representations and low s-LID score.
- 794 • SZ can be kinematically complex – the primary shear band, in the case of the studied
795 retaining wall problem, exhibited internal structure and consisted of several segments
796 of different types of motion,
- 797 • results of the s-LID analysis are s sensitive - the study of an a priori unknown
798 deformation structure requires selecting the appropriate neighbourhood size and thus
799 applying some range of s values by trial and error.

800

801

802 **Acknowledgements**

803 This research was funded in part by National Science Centre, Poland, grant no.
804 2020/39/B/ST8/01685. For the purpose of Open Access, the authors have applied a CC-BY
805 public copyright licence to any Author Accepted Manuscript (AAM) version arising from this
806 submission.

807

808

809

127
128
129



810 **Bibliography**

- 811 Coulomb, C.A. (1776) Essai sur une application des regles de maximis et minimis a quelques problemas de
812 stratique relatifs a l'architecture. Memoires de mathematique et de physique. Presentes a l'Academie Royale des
813 Sciences, pp. 343–382.
- 814 Chèvremont W., Bodiguel H., Chareyre B. (2020) Lubricated contact model for numerical simulations of
815 suspensions. *Powder Technology* (372), 600 - 610. DOI: doi.org/10.1016/j.powtec.2020.06.001
- 816 Cui, D., Wu, W., Xiang, W., Doanh, T., Chen, Q., Wang, S., Liu, Q., Wang, J. (2017) Stick–slip behaviours of
817 dry glass beads in triaxial compression. *Granul. Matter* 19(1), 111.
- 818 Fathipour, H., Payan, M., Chenari, RJ, Senetakis, K. (2021) Lower bound analysis of modified pseudo-dynamic
819 lateral earth pressures for retaining wall-backfill system with depth-varying damping using FEM-Second order
820 cone programming, *Int J Numer Anal Methods Geomech.* 45:2371–2387.
- 821 Franks, D. M., Stringer, M., Torres-Cruz L.A., Baker E., Valenta R., Thygesen, K., Matthews, A., Howchin J. &
822 Barrie, S. (2021) Tailings facility disclosures reveal stability risks, *Nature Scientific Reports* 11:5353,
823 <https://doi.org/10.1038/s41598-021-84897-0>
- 824 Frost J.D., Dejong J.T., Recalde M. (2002). Shear failure behavior of granular-continuum interfaces. *Eng Fract*
825 *Mech* 69(17):2029–2048
- 826 Greco, V. (2013). Active Thrust on Retaining walls of Narrow Backfill Width. *Comput. Geotechnics* 50, 66–78.
827 doi:10.1016/j.compgeo.2012.12.007
- 828 Gu X., Chen Y., Huang M. (2017) Critical state shear behavior of the soil-structure interface determined by
829 discrete element modeling. *Particuology* 35:68–77
- 830 Guan, X., Madabhushi, G.S.P. (2022), Dynamic response of a retaining wall with a structure on the dry backfill,
831 *Soil Dynamics and Earthquake Engineering* 157, 107259, //doi.org/10.1016/j.soildyn.2022.107259
- 832 Hall, S.A., Muir Wood, D., Ibraim, E., Viggiani, G.(2010) Localised deformation patterning in 2D granular
833 materials revealed by digital image correlation, *Granular Matter* 12:1–14 doi: 10.1007/s10035-009-0155-1.
- 834 Hartmann P., Thoeni K., Rojek J. (2022) A generalised multi-scale peridynamics – DEM framework and its
835 application to rigid–soft particle mixtures. *Computational Mechanics*, DOI: 10.1007/s00466-022-02227-1
- 836 Hegde, A., Murthy, T.G. (2022) Experimental studies on deformation of granular materials during orthogonal
837 cutting. *Granular Matter* 24, 70.



838 Hossein, K. M., Hassan, S., Thirapong, P., Araz, S. (2022) Active Earth Pressure Distribution inside Narrow
839 Backfill Considering Soil-Archiving Effect, International Journal of Geomechanics, Vol 22, No 7, doi:
840 10.1061/(ASCE)GM.1943-5622.0002434.

841 Houle, M.E.(2017) Local intrinsic dimensionality i: an extreme-value-theoretic foundation for similarity
842 applications. In: SISAP, pp. 64–79.

843 Houle, M.E. (2017) Local intrinsic dimensionality ii: multivariate analysis and distributional support. In: SISAP,
844 pp. 80–95.

845 Hu, W., Zhu, X., Zeng, Y. et al. (2022) Active earth pressure against flexible retaining wall for finite soils under
846 the drum deformation mode. Sci Rep 12, 497, <https://doi.org/10.1038/s41598-021-04411-4>

847 James, R.G. (1965) Stress and strain fields in sand, PhD Thesis, University of Cambridge.

848 James, R. G. and Bransby, P. L. (1971) A velocity field for some passive earth pressure problems.
849 Geotechnique21, 61–83.

850 Ke, W., Luo, W., Fang, T., Chen, Q., Xu, Ch., Yan, J. (2020) A simple closed-form solution for kinematic
851 responses of retaining wall incorporating the effects of shear stiffness of soils, Soil Dynamics and Earthquake
852 Engineering 134, 106163, [//doi.org/10.1016/j.soildyn.2020.106163](https://doi.org/10.1016/j.soildyn.2020.106163)

853 Khosravi, MH, Pipatpongsa, T., Takemura, J. (2016) Theoretical analysis of earth pressure against rigid
854 retaining walls under translation mode, Soils and Foundations 56(4):664–675,
855 [//dx.doi.org/10.1016/j.sandf.2016.07.007](https://dx.doi.org/10.1016/j.sandf.2016.07.007)

856 Khosravi, MH, Sarfaraz, H., Pipatpongsa, T. and Sharifdeljuyi, A., (2022) Active Earth Pressure Distribution
857 inside Narrow Backfill Considering Soil-Archiving Effect, [https://doi.org/10.1061/\(ASCE\)GM.1943-](https://doi.org/10.1061/(ASCE)GM.1943-5622.0002434)
858 5622.0002434

859 Kozicki, J., Donze, F.V. (2008) A new open-source software developer for numerical simulations using discrete
860 modelling methods. Comput. Methods Appl. Mech. Eng. 197, 4429–4443.

861 Kozicki J., Gladky A., Thoeni K. (2022) Implementation of high-precision computation capabilities into the
862 open-source dynamic simulation framework YADE. Computer Physics Communications, DOI:
863 10.1016/j.cpc.2021.108167

864 Kozicki, J., Niedostatkiewicz, M., Tejchman, J., Mühlhaus, H.-B. (2013) Discrete modelling results of a direct
865 shear test for granular materials versus FE results. Granul. Matter 15(5), 607–627.

866 Lai F., Yang D., Liu S., Zhang H., Cheng Y.(2022) Towards an improved analytical framework to estimate
867 active earth pressure in narrow $c - \phi$ soils behind rotating walls about the base, Computers and Geotechnics,
868 141.

869 Leśniewska, D (1993), RES — A numerical program for reinforced-soil slopes based on the rigid-plastic
870 theoretical model, Geotextiles and Geomembranes, 12, 5, 435-439, DOI: 10.1016/0266-1144(93)90018-J

871 Leśniewska, D. & Muir Wood, D. (2009). Observations of stresses and strains in a granular material. J Engng
872 Mech., ASCE 135, No. 9, 1038–1054, doi: 10.1061/_ASCE_EM.1943-7889.0000015.

873 Leśniewska, D. (2000) Shear Band Pattern Formation in Soil. Monograph, Institute of Hydro-Engineering,
874 Polish Academy of Science, Gdańsk, Poland, doi:10.13140/2.1.3653.0244.

875 Leśniewska, D. & Mróz, Z. (2000). Geotechnique 50, No. 5, 521-536,

876 Leśniewska, D. and Muir Wood, D. (2010) Photoelastic and photographic study of a granular material.
877 Geotechnique 60, 903–911, doi: 10.1680/geot.2010.60.00.1.

878 Leśniewska, D., Nitka, M., Tejchman, J., Pietrzak, M., 2020, Contact force network evolution in active earth
879 pressure state of granular materials: photo-elastic tests and DEM, Granular Matter (2020) 22:71,
880 <https://doi.org/10.1007/s10035-020-01033-x>.

881 Loukidis D., Salgado R. (2013) Active earth thrust on walls supporting granular soils: effect of wall movement,
882 Proceedings of the 18th International Conference on Soil Mechanics and Geotechnical Engineering, Paris 2013,
883 2043-2046

884 Luding S. (2004) Micro-macro transition for anisotropic, frictional granular packings. Int. J. Solids Struct.,
885 41:5821-5836

886 McGovern, D.J., Allsop, W., Rossetto, T., Chandler, I. (2023) Large-scale experiments on tsunami inundation
887 and overtopping forces at vertical sea walls, Coastal Engineering 179, doi.org/10.1016/j.coastaleng.2022.104222

888 Milligan, G.W.E. (1974a). The Behaviour of Rigid and Flexible Retaining Walls in Sand, PhD Thesis,
889 University of Cambridge .

890 Milligan, G. W. E. (1974b). The behaviour of rigid and flexible retaining walls in sand. Geotechnique 26, 473–
891 494.

892 Milligan, G. W. E. (1983) Soil deformations near anchored sheet-pile walls. Geotechnique 33, 41–55.

893 Maurin R. (2018) YADE 1D vertical VANS fluid resolution : validations. Yade Technical Archive.

894 Muir Wood, D. and Leśniewska, D. (2011) Stresses in granular materials. Granular Matter 13, 395–415, doi:
895 10.1007/s10035-010-0237-0.

136
137
138

- 896 Nitka M., Grabowski A. (2021) Shear band evolution phenomena in direct shear test modelled with DEM.
 897 Powder Technology, 391:369-384
- 898 Rechenmacher, A. L. and Finno, R. J. (2004) Digital image correlation to evaluate shear banding in dilative
 899 sands. Geotechnical Testing Journal, 27, 13-22
- 900 Ren, F., Huang, Q., Chen, J. (2022) Centrifuge modelling of geosynthetic-reinforced soil retaining walls
 901 subjected to the combined effect of earthquakes and rainfall, Geotextiles and Geomembranes 50, 470–479
- 902 Sailer, E., Taborda, DMG, Zdravkovi´c, L., Potts, DM, Cui, W. (2021) Thermo-hydro-mechanical interactions in
 903 porous media: Implications on thermo-active retaining walls, Computers and Geotechnics 135, 104121,
 904 <https://doi.org/10.1016/j.compgeo.2021.104121>
- 905 Salazar A., Sáez E., Pardo G. (2015) Modeling the direct shear test of a coarse sand using the 3D discrete
 906 element method with a rolling friction model. Comput. Geotech., 67:83-93
- 907 Schmüdderich, C., Tschuchnigg, F. & Schweiger, H.F. (2022) Significance of flow rule for the passive earth
 908 pressure problem. ActaGeotech. 17, 81–92.
- 909 Schmüdderich, C., Tschuchnigg, F., Wichtmann T. (2020) Rigorous lower and upper bounds for the 3D passive
 910 earth pressure problem, Géotechnique Letters, Volume 10 Issue 2, pp. 100-105, June.
- 911 Slominski, C. C., Niedostatkiewicz, M. and Tejchman, J. (2007) Application of particle image velocimetry (PIV)
 912 for deformation measurement during granular silo flow. Powder Technology, 173, 1-18
- 913 Stanier, S., Dijkstra, J., Lesniewska, D., Hambleton, J., White, D., Muir Wood, D., (2016) Vermiculate artefacts
 914 in image analysis of granular materials. Computers and Geotechnics 72, 100-113, DOI:
 915 10.1016/j.compgeo.2015.11.013
- 916 Schweiger H. F., Tschuchnigg F. (2021) A numerical study on undrained passive earth pressure, Computers and
 917 Geotechnics, 140.
- 918 Thoeni, K., Giacomini, A., Lambert, C., Sloan, S.W., Carter, J.P. (2014) A 3D discrete element modelling
 919 approach for rockfall analysis with drapery systems. International Journal of Rock Mechanics and Mining
 920 Sciences 68, 107119, DOI:10.1016/j.ijrmms.2014.02.008.
- 921 Thoeni K. (2021) A novel and robust contact detection algorithm for arbitrarily shaped blocks. Computers and
 922 Geotechnics, DOI: 10.1016/j.compgeo.2021.104430
- 923 Tordesillas, A., Zhou, S., Bailey, J. and Bondell, H. (2022) A representation learning framework for detection
 924 and characterization of dead versus strain localization zones from pre- to post- failure, Granular Matter.

- 925 Wang K, Liu G, Zhang Y and Lin J (2022) Quantification of the Active Lateral Earth Pressure Changes on
926 Retaining Walls at the Leading Edge of Steep Slopes. *Front. Earth Sci.* 10:796232.
- 927 White, D. J., Take, W. A. and Bolton, M.D. (2003) Soil deformation measurements using particle image
928 velocimetry (PIV) and photogrammetry. *Geotechnique* 53, 619-631.
- 929 White, D. J., Randolph, M. and Thompson, B. (2005) An image based deformation measurement system for the
930 geotechnical centrifuge., *Int. Jnl Phys. Model. Geotech.* 3, 1-12
- 931 Widulinski, L., Tejchman, J., Kozicki, J., Leśniewska, D. (2011) Discrete simulations of shear zone patterning in
932 sand in earth pressure problems of a retaining wall. *Int. J. Solids Struct.* 48(7–8), 1191–1209.
- 933 Xu, L., Xu Y., Wang, C. and Feng, K. (2019) Data-Driven Deformation Reliability of Retaining Structures in
934 Deep Excavations Considering Measurement Error *Appl. Sci.* 2019, 9, 5466; doi:10.3390/app9245466.
- 935 Xua, P., Hatamib, K., Baoc, JJ., Lid T. (2020) Bearing capacity and failure mechanisms of two-tiered reinforced
936 soil retaining walls under footing load, *Computers and Geotechnics* 128, 103833
- 937 Yang, M. and Deng, B. (2019). Simplified Method for Calculating the Active Earth Pressure on Retaining Walls
938 of Narrow Backfill Width Based on DEM Analysis. *Adv. Civil Eng.* 2019, 1–12. doi:10.1155/2019/1507825
- 939 Yang, Z., Liu, J., Nie J., Li X. (2022) Efficient estimation of cumulative distribution functions of multiple failure
940 modes using advanced generalized subset simulation, *Int J Numer Anal Methods Geomech.* 2022;46:1093–1108.
- 941 Zhao S., Evans T.M., Zhou X. (2018) Shear-induced anisotropy of granular materials with rolling resistance and
942 particle shape effects. *Int. J. Solids Struct.*, 150 (1):268-281
- 943 Zhou, S., Tordesillas, A., Pouragha, M., Bailey, J., Bondell, H. (2021) On local intrinsic dimensionality of
944 deformation in complex materials. *Sci. Rep.* 11(1), 1
- 945 Zhou, S., Tordesillas, A., Intrieri, E., Di Traglia, F., Qian, G., & Catani, F. (2022). Pinpointing early signs of
946 impending slope failures from space. *Journal of Geophysical Research: Solid Earth*, 127, e2021JB022957.
947 <https://doi.org/10.1029/2021JB022957>



948 List of symbols and abbreviations

- PSS - the physical state space, where the measurement points (e.g. grains) are described in 2D (x, y) space by their x and y coordinates
- v_x, v_y - the horizontal and vertical displacements
- v_z - the out-of-plane displacement
- DSS - the displacement state space, where the measurement points (e.g. grains) are described in 2D (v_x, v_y) space by horizontal and vertical displacements
- LID - the local intrinsic dimensionality
- s - a number of the nearest neighbours in DSS (the size of the investigated neighbourhood)
- sNN - s nearest neighbours (grains)
- s-LID - the measure quantifying how outlying a grain's motion is relative to its s nearest neighbours in DSS
- DEM - discrete element method
- DIC - discrete image correlation
- P-3D test - pseudo three-dimensional test
- E_c - modulus of elasticity of grain contact
- ν_c - Poisson ratio of the grain contact
- μ - inter-particle friction angle
- coefficient of rolling stiffness
- rolling coefficient
- R - particle (grain) radius
- ρ - particle density
- a - numerical damping parameter
- DZ - 'dead' zone – quasi rigid cluster
- SZ - shear zone
- TZ - transition zone

- D_{50} - median particle diameter
- d_k - total displacement of a focal measuring point after k time steps
- d_{N-av} - mean displacement of the neighbourhood consisting of N measuring points
- $\circ \mathbf{v}$ - the difference between the selected focal point displacement and the average displacement of its neighbourhood
- \mathbf{v} - the focal grain displacement vector
- $d_i(\mathbf{v})$ - the Euclidean distance between the focal grain displacement and its i -th nearest neighbour in DSS
- $d_s(\mathbf{v})$ - maximum of the neighbour distances in DSS

949

950 Tables

951

952 **Table 1:** Material constants used in DEM simulations

E_c [GPa]	ν_c [-]	μ [°]	β [-]	η [-]	R [m]	ρ [kg/m ³]	α [-]
2.4	0.3	20	0.3	0.005	0.001	2550	0.08

953

954 **Figures' captions:**

955

956 **Fig.1.** Radiographs made at Cambridge University, showing examples of soil failure patterns (*structured*
957 *deformation*) behind flexible (a) and rigid (b) model retaining walls. White dots on the radiographs are images of
958 lead shot markers.

959

960 **Fig.2.** The out-of-plane grain displacement v_z in 3D DEM simulation of a P-3D test. a) schematic geometry of a
961 sample in a P-3D test, b), c), d) – v_z as a function of z , y and x .

962

963 **Fig.3.** Idealized case - a rigid wedge sliding on an inclined plane: a) discretized displacement field, b) scatter
964 plot, c) evolution of the wedge displacement as a function of the number of wall displacement increments (time
965 steps).

966 **Fig.4.** The layout of the retaining wall model test and its boundary conditions: the incremental horizontal
967 translation of the right boundary and constant external loading of the top boundary (through five adjusting rigid
968 blocks, Leśniewska et al., 2020).

969

970 **Fig.5.** Displacement data, representing the initial stage of the experiment (left column) and the final stage of the
971 DEM simulation (right column): **(a), (b)** - displacement vectors, **(c), (d)** - the corresponding shear strain maps
972 (PSS), **(e), (f)** - displacement fields (a) and (b) as scatter plots (DSS).

973

974 **Fig.6.** Approximate division of the scatter plots, representing: **a)** the initial stage of the experiment (Fig.5e), **b)**
975 the final stage of the DEM simulation (Fig.5f).

976

977 **Fig.7.** Scatter plots of displacements after steps 01, 08, 18, 28 and 38 of the DEM simulation.

978

979 **Fig.8.** Areas of different shear strain values selected for data interpretation in DSS: **a)** – the approximate location
980 on the shear strain map, **b)** – extension and colour code.

981

982 **Fig.9.** Scatter plots with the reduced number of points showing locations corresponding to the areas from Fig.8b
983 in DSS space.

984

985 **Fig.10.** Displacement value evolution for the selected focal points compared to their closest neighbourhood's
986 average displacement in the areas: **a)** *a*, *e* and *c*, **b)** *b* and *d*, **c)** *h*, *i* and *n* (Fig.8b).

987

988 **Fig.11.** The statistical trend of the difference between the selected focal points' displacements and the average
989 displacement of their neighbourhoods, in areas **a**, **d** and **h** in Fig.8b.

990

991 **Fig.12.** The example of the structure in the experimental data (the final stage of the retaining wall model test: **a)**
992 shear strain map, calculated by DIC, **b)** corresponding scatter plot of the displacement field, **c)** location on the
993 scatter plot of the area circled in black in (a).

994

995 **Fig.13.** Sensitivity of s-LID analysis results on s parameter in PSS space.

996

997 **Fig.14.** The example of s-LID results for DEM simulation of a retaining wall problem (step 38, $s = 400$). Dark
998 blue is attributed to near-rigid motion, dark red - extensive deformation or lack of coordinated motion: **(a)** PSS
999 view, **(b)** corresponding scatter plot (DSS).

1000

1001 **Fig.15.** Comparison between the s-LID network and the solution of the corresponding boundary value problem
.002 using the rigid-plastic approach: **a)** stress characteristics superimposed on the displacement field from Fig.5b, **b)**
.003 sketch of the grid of lines dividing the moving wedge area according to the value of the s-LID parameter, **c)** the
.004 selected dividing lines with directions close to stress characteristics.

.005

.006 **Fig.16.** Close-up of Fig.14b – the scatter plot for DEM simulation of a retaining wall problem (step 38, $s = 400$)
.007 – objects **h** and **i-k** recognized in Fig.9c and here.

.008

.009 **Fig.17.** DEM simulation – the study of the influence of the s parameter on s-LID results.

154

155

156

- 1010
1011 **Fig.18.** The experimental model test – the study of the influence of the s parameter on s -LID analysis results.
1012
1013 **Fig.19.** The experimental model test – the result of s -LID analysis for $s = 800$.
1014
1015 **Fig.20.** The primary shear band in the experimental model test: PSS **(a)** and DSS space **(b)**.
1016
1017 **Fig.21.** The s -LID score map **(a)** versus DIC analysis results **(b)-(f)**.
1018
1019 **Fig.22.** s -LID cluster division grid **(a)** superimposed on volumetric strain map, calculated by DIC **(b)**.