THE CALCULATION LONG-TERM PHYSICAL FIELD CREATED BY WATER FLOW AROUND THE SHIP

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The paper presents a description of the problems with fluid flow around a ship. Using the described solution of the problem were performed numerical calculations using the boundary element method. Were also presented preliminary results of the calculation of pressure fields at the bottom of the ship, taking into account the impact of the bottom.

INTRODUCTION

Pressure field around a ship moving at a constant speed on calm water, have a static character in the moving coordinate system associated with him (SM - system mobile). The stationary system associated with the land (SS - an immovable property), this field is characterized by dynamic changes of a different time period. Elapsed for the checkpoint, she observed an initial increase in pulse pressure with a relatively short duration, then generally long-term decline in pressure for vessels with an average length (period and amplitude of this phenomenon we may call respectively the primary period of the pressure on ship - *PPPS* and amplitude of the pressure drop ship - *APDS*), then again increases the pressure pulse when passing the stern. Time passing by a point in the SS, she is equal to the quotient of the length between perpendiculars Lpp and ship speed Vs as $T0 = Lpp / Vs$, ($PPPS \approx T0$).

For larger ships moving with considerable speed may not occur aft pressure pulse (followed by flow separation at the stern and the base period is increasing, $PPPS >$ T0) or may receive an extra boost pressure during the mid-beam, (with the creation of k pulse pressure increase base period may be as high as $PPPS \approx T0 / (k-1)$). However, the amplitude of the pressure drop *APDS* is a fraction of stagnation pressure $q = (\rho V_s^2)/2$ and equal to the $APDS = f(L,B,T,Cb,r)*q$. This fraction is a function $f\ (L,\ B,\ T,\ Cb,\ r)$ depends on the ship's main dimensions and their relations that is the shape of the ship hull and the distance r from the checkpoint of the ship. "At a given shape of the ship (eg, resulting from the good properties of resistance) function $f(L,B,T,Cb,r) = const \approx C_1/r^4$ i $APDS = C_2 * V_s^2$, where the constant $C_2(r) \approx (C_1/r^4)*\rho/2$, ie, the amplitude of the pressure drop becomes dependent only on the ship's speed and distance of the checkpoint. *APDS* will decrease so will automatically reduce the operating speed of the ship to the speed limit Vs:

$V_{limit}=(APDS / C_2)^{1/2}$.

When moving the ship on a stormy sea, overlapping systems of waves of the ship and wind waves significantly shorten *PPPS*.

Possible changes in the value of *f(L,B,T,Cb,r)* ,and thus the value of a constant C_2, is much greater at the stage of ship design. This allows to change the size *APDS* without changing the operating speed V_s . Much cheaper than changing the way *APDS* are changes in speed of the ship, at a distance r equals the change in period *PPPS*.

For the purposes of a preliminary numerical analysis based upon the preliminary program of the described method for calculation of flow around the ship and hydrodynamic fields. Singularities in the form of a single layer spread over the surface of moistened simplified shape of the vessel and at the bottom. For the same shape and the small value of the Froude number hydrodynamic field results are correct. For higher speeds it is necessary to extend a single layer on the free surface area surrounding the ship.

The results obtained using numerical calculation methods of fluid mechanics (CFD) and the results of the experiments will help to develop an algorithm allowing the required accuracy without having to use labor intensive methods to determine the parameters of the ship pressure. These algorithm will also adjust the operating parameters on the maximum safe in relation to risk of execution of the ship mission under special conditions.

1. THE MATHEMATICAL MODEL OF THE WATER FLOW AROUND THE SHIP

The ship runs at a speed of v_0 = const. Determination of the pressure and velocity fields in fluid flow around the hull of the ship will be made by solving linearized problem formulated by the following equation of continuity of flow (Laplace equation for the velocity potential φ) and the boundary conditions at the bottom and the free surface and away from the ship's vertical surfaces defining the closed area around the vessel.

The equations were formulated in the coordinate systems Oxyz, Oξηζ , the first of which inertial coordinate system, the x-axis located on the free surface and directed towards the ship's velocity vector v_0 and z-axis directed downward vertical, coinciding with the second only at the beginning of the movement. The second movable coordinate system rigidly linked to the hull of the ship. A mathematical model of fluid motion surrounded the ship describe the equations :

$$\Delta\varphi = 0 \tag{2.1}$$

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{g}{v_0^2}\frac{\partial\varphi}{\partial z} - \frac{\mu}{v_0}\frac{\partial\varphi}{\partial x} = 0 \quad for\ z = 0 \tag{2.2}$$

$$\frac{\partial\varphi}{\partial z}(z = h) = 0 \tag{2.3}$$

$$\frac{\partial\varphi}{\partial y}\left(y = \pm\frac{b}{2}\right) = 0 \tag{2.4}$$

$$\frac{\partial\varphi}{\partial u}(S) = v_0 n \tag{2.5}$$

$$\zeta = -\frac{v_0}{g}\frac{\partial\varphi}{\partial x} \quad for\ z = 0 \tag{2.6}$$

where: h - water depth, b - width of a multiple of the ship, S - wetted surface ship moving at a speed v_0 progressive, Ω - ship wetted surface property, n - vector normal to the surface in equation (2.5) to the surface ship hull, g - gravitational acceleration vector.

The velocity potential φ is determined by the method of Kirchhoff-splitting layer sources on the surface S, so. single layer of the first type. To determine the potential φ must know the potential sources moving uniformly with velocity v_0 at the free surface of water, called the Havelock source function as well as Green's function for the source of the following form:

$$\varphi(\vec{r}) = -\frac{Q}{4\pi} G(x, y, z, \xi, \eta, \zeta) \tag{2.7}$$

where: $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ - radius vector.

Green's function has the form:

$$G(x,y,z,\xi,\eta,\zeta) = G(M,N) = \frac{1}{r} - \frac{1}{r_1} + G(M,N) \tag{2.8}$$

where the points M and N are the coordinates of, respectively: M(x, y, z), N (ξ, η, ζ).

The potential φ at the point M from the source unit at the point of N has the form:

$$\varphi(M(x,y,z)) = -\frac{1}{4\pi} \frac{1}{\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}} \tag{2.9}$$

This potential, as a solution of the Poisson equation as follows,

$$\Lambda\varphi = q(\xi,\eta,\zeta) \tag{2.10}$$

has the integral form:

$$\varphi(x,y,z) = -\frac{1}{4\pi} \iiint_V \frac{q(\xi,\eta,\zeta)d\xi d\eta d\zeta}{r} \tag{2.11}$$

when: $q(\xi,\eta,\zeta) = \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta)$.

After applying the Fourier transform and use Residue Theorem and Jordan's lemma, by [3,4], Green's function for a source moving in the unrestricted area take the form of :

$$G(x,y,z,\xi,\eta,\zeta) = \frac{1}{r} = \frac{1}{2\pi} \iint_{-\infty}^{\infty} \frac{1}{k} e^{i[u(x-\xi)+v(y-\eta)]} e^{-k|z-\zeta|} du dv \tag{2.12}$$

Potential and the Green's functions for the source of a moving near free surface in deep and shallow water is obtained by following the same, by [4].

Consider the first case of deep water, when the source of constant expense Q is at point N(ξ,η,ζ) and moves at a speed of v_0 in the direction of the axis Ox in the deep water.

The velocity potential φ is determined by formulas (2.7-8) and satisfies the Laplace equation with boundary conditions (2.2-3). Source of N (ξ, η, ζ) is accompanied by a point source N1 (ξ, η,-ζ) moving with the first. Substituting the expression for φ in the form :

$$\varphi = -\frac{1}{4\pi}\left(\frac{1}{r} - \frac{1}{r_1} + G_1\right) \tag{2.13}$$

where $r = |\overline{MN}|$, $r_1 = |\overline{MN_1}|$,

insert into Laplace's equation :

$$\Delta G_1 = 0 \tag{2.14}$$

and the boundary condition at the bottom of sea:

$$\frac{\partial G_1}{\partial z}(z \to \infty) = 0 \tag{2.15}$$

Substituting relation (2.7) to the boundary condition (2.2) for z = 0 were obtained:

$$\frac{\partial^2 G_1}{\partial x^2} - \frac{g}{v_0^2}\frac{\partial G_1}{\partial z} - \frac{\mu}{v_0}\frac{\partial G_1}{\partial x} = \frac{g}{v_0^2}\left[\frac{\partial}{\partial z}\left(\frac{1}{r}\right) - \frac{\partial}{\partial z}\left(\frac{1}{r_1}\right)\right] \quad for\, z = 0 \tag{2.16}$$

1/r1 dependence by (2.12) obtains the form :

$$\frac{1}{r_1} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\Theta \int_0^{\infty} e^{ikw} e^{-k(z+\zeta)} dk \tag{2.17}$$

However, formula (2.12) in pursuit of the free surface of the z→ 0 converges to the equation:

$$\frac{1}{r} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\Theta \int_0^{\infty} e^{ikw} e^{k(z-\zeta)} dk \tag{2.18}$$

where: w= (x-ξ)cosθ+(y-η)sinθ.
As for the z=0

$$\frac{\partial}{\partial z}\left(\frac{1}{r}\right) = -\frac{\partial}{\partial z}\left(\frac{1}{r_1}\right) \tag{2.19}$$

and condition (2.16) take the form of:

$$\frac{\partial^2 G_1}{\partial x^2} - \frac{g}{v_0^2}\frac{\partial G_1}{\partial z} - \frac{\mu}{v_0}\frac{\partial G_1}{\partial x} = -2\frac{g}{v_0^2}\frac{\partial}{\partial z}\left(\frac{1}{r_1}\right) \tag{2.20}$$

When you differentiate (2.17) and substituting into (2.20) becomes a boundary condition:

$$\frac{\partial^2 G_1}{\partial x^2} - \frac{\mu}{v_0}\frac{\partial G_1}{\partial x} - \frac{g}{v_0^2}\frac{\partial G_1}{\partial z} = \frac{g}{\pi v_0^2}\iint_{-\infty}^{\infty} e^{-k\zeta}\; e^{i[u(x-\xi)+v(y-\eta)]} du dv \tag{2.21}$$

After re-applying the method of integral Fourier transformation according to [4] obtained the Green's function G1 form below:

$$G_1(x,y,z,\xi,\eta,\zeta) = \frac{g}{\pi v_0^2}\int_{-\pi}^{\pi} d\Theta \int_0^{\infty} \frac{e^{ikw} e^{-k(z+\zeta)} dk}{\frac{g}{v_0^2} - k\cos^2\Theta - \frac{\mu}{v_0}\cos\Theta} \tag{2.22}$$

And the potential φ has a solution:

$$\varphi(x,y,z) = -\frac{Q}{4\pi}\left(\frac{1}{r} - \frac{1}{r_1} + G_1\right) \tag{2.23}$$

Using the above relation and relation (2.6) obtained the formula for the wave profile :

$$\zeta(x,y) = -\frac{v_0}{g}\frac{\partial \varphi}{\partial x}(z=0) = \frac{Q v_0}{4\pi g}\frac{\partial G_1}{\partial x}(z=0) \tag{2.24}$$

and, after the differentiation and substituting (2.22) :

$$\zeta(x,y) = \frac{Qi}{4\pi^2 v_0}\int_{-\pi}^{\pi} d\Theta \int_0^{\infty} \frac{e^{ikw} e^{-k\zeta}}{\frac{g}{v_0^2} - k\cos^2\Theta - i\frac{\mu}{v_0}\cos\Theta}\, k\, dk \tag{2.25}$$

Consider the second case for the shallow water, when the source of constant expense Q is at point N (ξ, η, ζ) and moving at a speed v_0 in the direction of the axis Ox in water depth h. Coordinate systems Oxyz, Oξηζ are associated with a source. Source of N (ξ, η, ζ) is accompanied by a source at the point N1 (ξ, η, ζ-2h), moving with the first. By introducing some simplifications write, $K_0=\frac{g}{v_0^2}$, $\mu_1 = \frac{\mu}{v_0}$, equations describing the problem take the form:"

$$\Delta\varphi = 0 \tag{2.26}$$

$$\frac{\partial^2\varphi}{\partial x^2} - K_0\frac{\partial\varphi}{\partial z} - \mu_1\frac{\partial\varphi}{\partial x} = 0 \; for\; z = 0 \tag{2.27}$$

$$\frac{\partial\varphi}{\partial z}(z = h) = 0 \tag{2.28}$$

and the boundary condition at the bottom :

$$\frac{\partial G_1}{\partial z}(z = h) = 0 \tag{2.29}$$

and φ:

$$\varphi(x, y, z) = -\frac{Q}{4\pi}\left(\frac{1}{r} + \frac{1}{r_1} + G_1\right) \tag{2.30}$$

Where r and r_1 see (2.17-18) taking into account the different position of the N1(ξ,η,ζ-2h). Proceeding similarly as in the case of deep water was obtained the following relationship for the Green's function for the shallow water and free surface:

$$G_1(M,N) - \frac{1}{2\pi}\int_{-\pi}^{\pi} d\Theta \int_0^{\infty} \frac{\cosh[k(h-z)]}{\cosh(kh)} \frac{(K_n + k\cos^2\Theta)e^{ikw}(e^{-k\zeta} + (e^{k(\zeta-2h)})dk}{K_0 tgh(kh) - k\cos^2\Theta - i\mu_1\cos\Theta} \tag{2.31}$$

and potential φ from equation (2.30).

2. THE CALCULATION OF THE WATER FLOW AROUND THE SHIP. MODELLING OF THE SHIP WATER FLOW WITH THE SINGLE LAYER AND/OR DOUBLE LAYER

Solution of the problem will be through the designation of potential in terms of linear speed. The function G (M, N) will be determined taking into account the linear boundary condition on the free surface. Surface on which the singularities are spreading S_s is equal to the hull surface S and at the same time it is wetted surface area Ω ship property, ie, $S_s = S = \Omega$. The free surface S_F within the hull, so. S_{F0}, is the area of the waterline S_{WL}. The limit of the volume displacement of the ship and a closing surface area of this volume is marked as: $\partial V = \Omega + S_{WL}$.

The relationship describing the velocity potential φ takes the form:

$$\varphi(M) = -\frac{1}{4\pi}\int_{\Xi=S_s}\left(q_s G + \sigma\frac{\partial G}{\partial n}\right)dS - \frac{1}{4\pi}\int_{S_{WL}=S_{F0}}\left[G(M,N)\left(\frac{\partial\varphi_e}{\partial n} - \frac{\partial\varphi_i}{\partial n}\right) + (\varphi_i - \varphi_e)\frac{\partial G}{\partial n}\right]dS \tag{3.1}$$

If we assume that on the surface of a moist hull Ω are spread only source q_s (N), on the waterline S_{WL} only dipoles σ (N) and on the surface of the hull there is not the potential jump, then formula (3.1) the potential φ take the form :

$$\varphi(M) = -\frac{1}{4\pi}\int_{\Xi=S_s}(q_s(N)G(M,N))dS - \frac{1}{4\pi}\int_{S_{WL}=S_{F0}}\left[\sigma(N)\frac{\partial G}{\partial n}(\zeta = 0)\right]dS \tag{3.2}$$

Then, taking into account that there is an equation on S_{WL} :

$$\frac{\partial G(M,N)}{\partial n}(\zeta = 0) = -\frac{\partial G(M,N)}{\partial \zeta}(\zeta = 0) = G^*(M,N) \tag{3.3}$$

what will save the equation (3.2) in the form as follows:.

$$\varphi(M) = -\frac{1}{4\pi}\int_{\Xi=S_s}(q_s G)\, dS - \frac{1}{4\pi}\int_{S_{WL}=S_{Fo}}(\sigma G^*)\, dS \tag{3.4}$$

Boundary condition on the surface of S_s leads to the following equation describing the distribution of q_s (N) at $S_s = \Omega$, and σ (N) for S_{WL}:

$$q_s(M) = 2v_0 n_x(M) + \frac{1}{2\pi}\int_{\Omega} q_s(N)\frac{\partial G(M,N)}{\partial n_M}\, dS + \frac{1}{2\pi}\int_{S_{WL}} \sigma(N)\frac{\partial G^*(M,N)}{\partial n_M}\, dS \tag{3.5}$$

where the differentiation in the normal direction at the point N (ξ, η, ζ) - (location of singularities), we describe as $\frac{\partial}{\partial n}$, while differentiation at the point M (x, y, z) as $\frac{\partial}{\partial n_M}$.

As is apparent from the above formulas, timetables sources qs (N) and distributions of dipoles σ(N) are not dependent on each other. The question is whether, for example part of the surface ∂V, such as the ship's waterline S_{WL}, it can be reset? The answer to this and similar questions can give a numerical study compared with the results of experiments. In case of confirmation of such a possibility these patterns to simplify the form as below:

potential φ:

$$\varphi(M) = -\frac{1}{4\pi}\int_{\Xi=S_s}(q_s G)\, dS \tag{3.6}$$

distributions of sources in the form :

$$q_s(M) = 2v_0 n_x(M) + \frac{1}{2\pi}\int_{\Omega} q_s(N)\frac{\partial G(M,N)}{\partial n_M} dS \tag{3.7}$$

Numerical solution of this equation is obtained using the boundary element method. This solution describes the distribution of singularities on the surface of the ship hull S_s and other strongly interacting at the flow of surfaces, such as S_d bottom surface (especially in the shallow water flow).

Using a single layer and method for allocating the surface ∂V of the panels, the solution of the integral equation (3.6) we obtain from the condition of impervious surface. Solution to the problem boils down to solving the system of linear equations in the following form :

$$\sum_{j=1}^{N} A_{ij} q_j = \overrightarrow{v_\infty}\cdot\overrightarrow{n_i} = v_{ni} \quad i = 1, \dots, N \tag{3.8}$$

where the elements of the matrix effects A_{ij} are in the form:

$$A_{ij} = \frac{1}{4\pi}\frac{\overrightarrow{n_i}\cdot\overrightarrow{r_{ji}}}{r_{ji}^3}\Delta S_j \quad dla\ i \neq j \tag{3.9}$$

$$A_{ij} = \frac{1}{2} \quad dla\ i = j \tag{3.10}$$

where:

A_{ij} - projection of velocity induced by the singularity of a panel of the unit j on the panel i, and the direction normal to the i-th panel $\overrightarrow{n_i}$,

$r_{ji} = \left|\overrightarrow{r_{ji}}\right|$ – length of the vector from the j-th to the i-th panel",

ΔS_j - surface area of the j-panel.

Using a double layer, in turn, obtained the system of equations will have the form :

$$\sum_{j=1}^{N} A_{ij}\sigma_j = \overrightarrow{v_\infty}\cdot\overrightarrow{n_i} = v_{ni} \quad i = 1, \dots, N \tag{3.11}$$

where the elements of the matrix effects A_{ij} are in the form:

$$A_{ij} = \frac{1}{4\pi}\left[-\overrightarrow{n_i}\cdot\overrightarrow{n_j} + 3(\overrightarrow{n_i}\cdot\overrightarrow{e_{rji}})\overrightarrow{e_{rji}}\cdot\overrightarrow{n_i}\right]\frac{\Delta S_j}{r_{ji}^3} \quad dla\ i \neq j \tag{3.12}$$

$$A_{ij} = 0 \quad for\ i = j \tag{3.13}$$

where the sign above, see (3.10).

3. THE NUMERICAL SOLUTION OF THE PROBLEM

Based on the above address theoretical issues has been developed a model for the preliminary numerical tests. Single layer has been deployed on the surface of the hull of the ship and at the bottom, $\partial V = S_s + \Omega$.

For the numerical studies adopted a simplified pre-shape of the hull. Surface of the hull was divided into square panels, in which the center of gravity placed checkpoints C. It was assumed that the total expenditure on the panel is concentrated in point C. Used of a single layer.

4. EXAMPLE RESULTS OF CALCULATIONS

Sample results of calculations show the distribution of pressure at the bottom of the ship with simplified elliptical shapes. The center of the ship is in place to coordinate x = 0 .

The first chart shows the distribution of pressure at the bottom, and the second distribution of hydrodynamic pressure.

Dimensions of the ship and its speed have the following values:

- length of the ship, L = 40m, - width of the ship, B = 8m, - draft of the ship, T = 6m, - velocity, v = 3 knots, - atmospheric pressure, p_a = 10.1325 kPa, - water density, ρ = 998 kg/m_3, - bottom depth, h = 10m.

5. CONCLUSIONS

Preliminary assessment of the results of calculations can be defined as the correct values of pressure amplitude APDS. But it is not possible to define the basic pressure field PPPS period. This is due to the lack of pressure impulse from the waves of ship's bow and ship's stern . To describe this phenomenon is necessary to include in the algorithm and the mathematical model of the phenomenon of free surface deformation. This will affect the distribution of a single layer on the surface of free water around the ship S_F and met there the boundary condition in the form of equation (2.6).

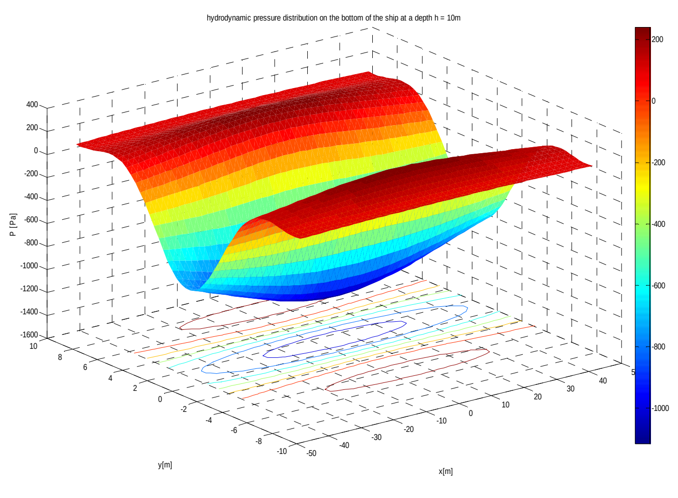

Fig.1. Distribution of hydrodynamic pressure at the bottom of the ship

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