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RESEARCH

# The Mathematical Regularity Coefficient for Facade Patterns

Michał Malewczyk[1]

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**Abstract**
This article investigates whether the visual regularity of a building's facade can objectively be expressed by a numerical value. Grounded in universal neurobiological principles and modern digital image processing methods from mathematical and information theories, this study analyzes 245 architectural compositions using an interdisciplinary approach that combines perception, mathematics, programming, and architectural theory. A coefficient was derived with an almost ideal average correlation to regularity and minimal variability. Building on these findings, an open-source computer tool was developed to measure the regularity of flat architectural compositions. These results enhance our understanding of compositional perception and support the development of advanced architectural design tools, including generative algorithms.

**Keywords** Visual regularity · Architectural composition · Digital design tools · Information theory · Perception · Algorithms

## Introduction

This paper investigates the development of a method to measure the consistency of patterns on the exterior of buildings. According to Krier (1992), the correct arrangement of parts is crucial to designing visually appealing and balanced architecture. Combining architectural elements emphasize order, hierarchy, proportion, and rhythm. According to the theory of Birkhoff and Eysenck (Davis 1936; Eysenck 1941), the aesthetics (not only) of architectural works culminates from the right balance between complexity and regularity. This paper discusses the regularity of pat-

✉ Michał Malewczyk
michal.malewczyk@pg.edu.pl

1 Faculty of Architecture, Gdansk University of Technology, Ul. Gabriela Narutowicza 11/12, 80-233 Gdansk, Poland

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terns, focusing on the arrangement of elements such as windows and balconies on façades.

The following research questions guide the work based on the identified research challenges. First, can a mathematical coefficient be defined as one that objectively quantifies the visual regularity of façade patterns? Second, to what extent does this coefficient correlate with the perceived visual regularity observed in different façade pattern types?

This study hypothesizes the existence of a mathematical coefficient that strongly correlates with the visual regularity of façade patterns. This assumption draws on universal biological principles governing the perception of regularity (Amir et al. 2011; Biederman 1987; Kayaert et al. 2005; Kubilius et al. 2014). As a result, visual regularity appears objective and tends to be perceived similarly across human observers (Malewczyk et al. 2022, 2024). Numerous image analysis studies employing entropy-based approaches (Crompton 2012; Grebekina et al. 2018; Güzelci et al. 2021; Güzelci & Alaçam 2019; Güzelci et al. 2020; Redies et al. 2017; Stamps 2004, 2012, 2014; Stanischewski et al. 2020) further support the notion that visual regularity constitutes a quantifiable phenomenon.

If the hypothesis proves valid, a secondary objective involves verifying previous experimental findings, which demonstrated a correlation between the perceived regularity of façade compositions and their respective pattern types (Malewczyk et al. 2022, 2024). The current study adopts a markedly different methodological framework. Despite this divergence, the results align with earlier observations and provide additional support for the previously established correlation.

Assuming the research hypothesis is confirmed, this study proposes the development of an open-source computational tool capable of objectively assessing the regularity coefficient of façade patterns. The scope of the research reaches beyond the field of architecture, drawing from contemporary computational technologies, mathematical statistics, information theory, and architectural theory. By integrating these disciplines, the study addresses fundamental issues related to visual regularity, perceptual principles, and current methodologies for analyzing image structure and façade compositions (Malewczyk et al. 2022, 2024). In the subsequent section, these topics are further elaborated upon.

## Related terms

### Visual Regularity and Its Perception

Kubilius, Wagemans, and Op de Beeck (2014) demonstrated a linear relationship between neural responses in specific brain regions and the regularity of visual stimuli. This linear neural reaction appears consistent across individuals, suggesting that the entire mechanism of visual regularity perception operates universally in humans. Biederman (1987), Kayaert et al. (2005), and Amir et al. (2011) found that regular stimuli activate particular areas of the brain, while irregular stimuli do not. Arnheim's (1977) behavioral research further supports above mentioned conclusion. Consequently, a mathematical indicator of compositional regularity may be an objective measure of perceivable regularity.

Gestalt principles classify a structure as regular when it exhibits a systematic arrangement of objects (Koffka, 1935). According to Wagemans et al. (2005), order emerges from specific governing laws that synergistically influence perception. Arnheim (1977) further conceptualized regularity as a tension-free state of equilibrium, typically underpinned by symmetry in visual arts and architecture, whereas disordered states lack such organizing principles. Pironio (2018) characterizes irregularity as the absence of predictable patterns and positions regularity as its antithesis.

Consistent with these theoretical frameworks, empirical studies in psychology and experimental aesthetics confirm that visual structures perceived as regular are subject to universal laws. Investigations have addressed the impact of environmental context on regularity perception (Sun et al., 2019) and the relationship between bilateral symmetry and aesthetic preference (Pecchinenda 2014; Friedenberg 2021; Bertamini et al. 2013). Furthermore, Katkov et al. (2015) demonstrated that the human brain organizes abstract stimuli according to principles akin to those in statistical mechanics. Observers classify compositions featuring uniform elements and consistent spacing as regular, while they view compositions with variable elements and spacing as more random.

Similar conclusions can be drawn from the experiment by Malewczyk et al. (2024), which proves the relationship between the perceived regularity of the composition and the type of composition pattern (Malewczyk et al. 2022). This typology, which defines six pattern types (labeled A–F), originates from an analysis of 113 housing developments. Figure 1 illustrates the general principles defining each type and reveals their fundamental differences in arranging the compositional elements on the façade's plane. The analysis revealed a strong relationship (68.6%) between regularity and pattern type. An experimental study with human participants (Malewczyk et al. 2024) empirically established the relationship.

## Methods of Image Regularity Analysis

Methodologies for assessing the regularity of visual stimuli and generating images with controlled irregularity are examined. One commonly used technique involves applying random perturbations—jittering—to a regular image, with the level of irregularity scaling with the magnitude of the perturbations (Sun et al. 2019). Researchers have extensively explored the assessment of pattern regularity in the field of image encryption, where they evaluate algorithm performance using measures such as global Shannon entropy, histogram analysis, differential cryptanalysis, and pixel correlation (Li et al. 2017; Xu et al. 2016; Belazi et al. 2016; Benoit et al. 2014). Among these measures, Shannon entropy proves particularly relevant to the present study.

In 19th-century thermodynamics, scientists introduced entropy to quantify a system's disorder. In 1948, Claude Shannon redefined it. Shannon established a link between entropy and information, defining it as the average amount of information per message.

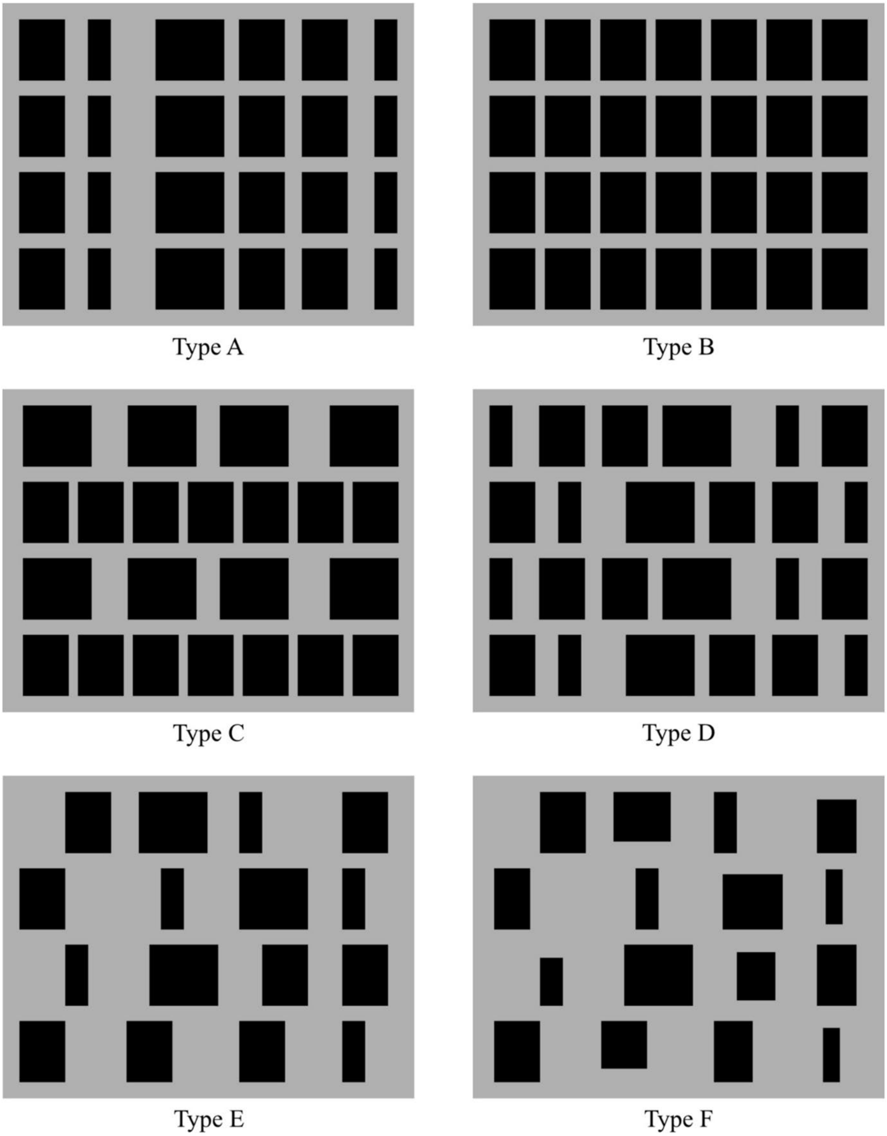


**Fig. 1** Examples of façade patterns (compositions) classified according to the typology. Image: Malewczyk et al. 2022

$$H = -\sum_{i=1}^{n} p_i log_2 p_i$$

Formula 1. Entropy in information theory

According to Formula 1, entropy in information theory (Shannon entropy) depends on the frequency of identical information or values in the whole string. In the 5-element sequence 'AECBD' each of the letters appears once in this 5-element string, so the probability of each of them appearing is 0.2; therefore:

$$H = 5x0.2\log_2 0.2 = 2.32193$$

The invariance of a sequence's entropy with respect 'ABCDE,' 'EDCBA,' and 'ACBDE' yield identical entropy values. Only the distinctiveness of the symbols, not their diversity, influences the measure– hence, sequences such as 'AZMPT,' '12,345,' or '17,290' share the same entropy as 'AECBD'. Moreover, because entropy increases with string length, metric entropy– the entropy (H) divided by the string length– facilitates comparisons across sequences.

Redies et al. (2017) propose that edge orientation entropy differentiates domains of human creativity, noting that Western paintings have lower entropy than East Asian or Islamic art. Stanischewski et al. (2020) found a positive correlation between aesthetic preference for abstract linear patterns and edge orientation entropy, while Grebekina et al. (2018) observed similar patterns in linear designs, abstract textures, album covers, and architectural photographs. Stamps (2004) also demonstrated that perceived diversity in the built environment correlates with entropy.

Although researchers widely apply Shannon's entropy in studies of the built environment (e.g., Güzelci et al. 2021; Güzelci & Alaçam 2019; Güzelci et al. 2020; Crompton 2012; Stamps 2012, 2014), most of these studies rely on the global entropy index, which fails to account for spatial rearrangements of elements—a critical aspect in compositional analysis. In contrast, Wu et al. (2013) introduced a statistical test based on Shannon's local entropy that computes the standard deviation of local entropy values from randomly sampled regions, thereby capturing variations in element positioning.

## Materials and Method

### Materials

All photos used as research material depict residential, multi-family buildings nominated for the 2024 European Union Prize for Contemporary Architecture– Mies van der Rohe Award (EU Mies Award 2024) and classified as 'collective housing.' All photographs were obtained from the award organizer's archive (https://miesarch.com/archive). Randomly selected projects are presented in Fig. 2.

Given that a nomination for this award affirms an object's quality, the study selected buildings based on this criterion, assuming they represent a typical sample of contemporary European multi-family residential architecture.

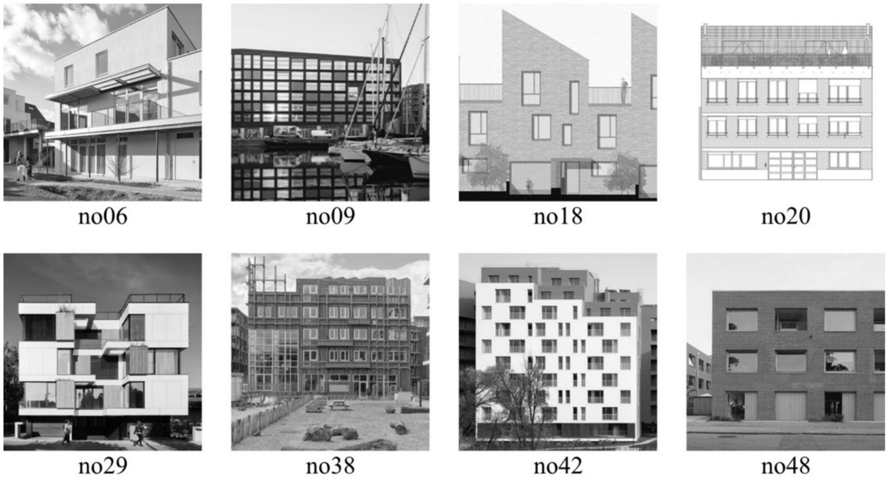


**Fig. 2** Selection of architectural projects from the group of 49 chosen for the survey

## Methods

### *Shannon Entropy and Classical Variability Measures*

Stamps (2004, 2014) calculated entropy from information sequences derived via semantic analysis of architectural façades– a method similarly employed by Crompton (2012) and Güzelci (2019, 2021). Stanischewski et al. (2020) further analyzed the entropy of rotation angles of straight lines in images.

Building on these studies, this research introduces an innovative method that recodes input data from photographs and façade drawings by transforming compositional formulas into sequences of element dimensions and inter-element distances. Subsequent analyses and calculations are performed on these sequences, incorporating classical variability measures such as mean, variance, standard deviation, and coefficient of variation. Figure 3 summarizes the entire procedure of the study.

### *Software*

The study primarily employed Blender version 4.2.1 for MacOS to create compositional objects using its 3D modeling tools and an integrated text editor for running Python 3.12 scripts. The script imports libraries such as bpy, CSV, math, math utils, collections, and datetime. Microsoft Excel version 16.87 for MacOS was used to process the raw data, conduct statistical analyses, and generate the final results.

### *Creating compositions*

The study's first phase involved creating the research compositions. Fifty photographs of buildings nominated for the EU Mies Award 2024 in the 'collective housing' category were imported into Blender, then 49 two-dimensional compositions

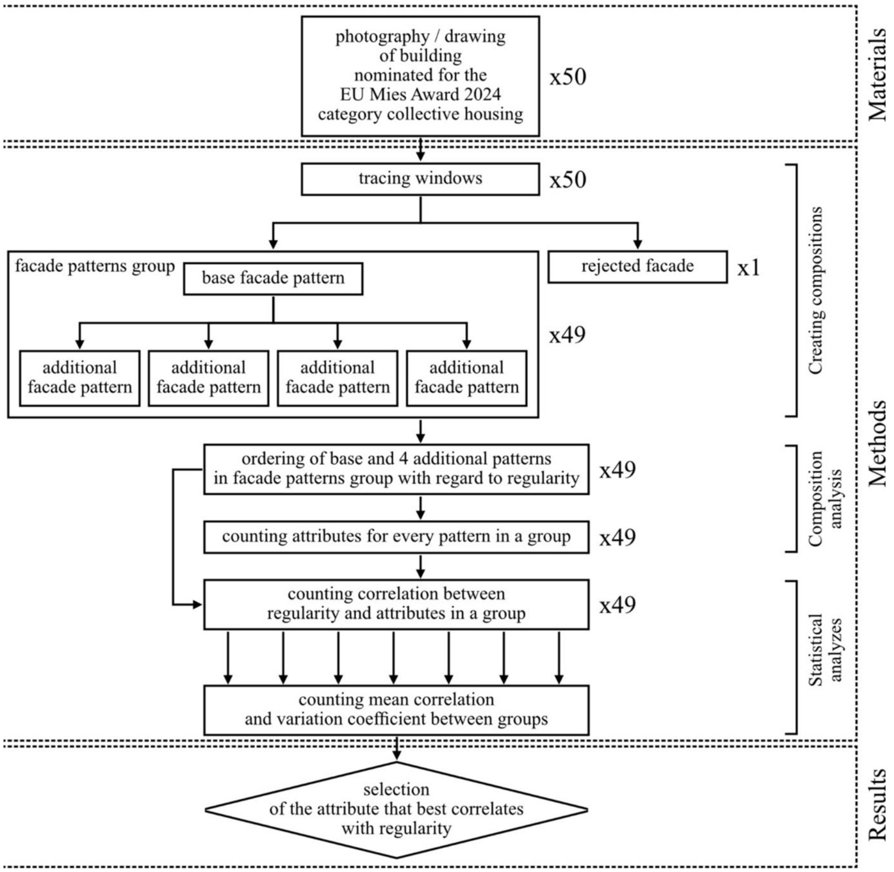


**Fig. 3** Flowchart summarizing the study procedure

were generated. Each composition, representing window arrangements as rectangles on the XY plane, faithfully reproduced the window layout for orthogonal images and their approximation for perspective photos. One building—the 'Kiubo' by Hofrichter-Ritter Architekten, Graz was excluded, as its replication proved challenging.

Subsequently, the study derived four variants from each composition by systematically modifying vertical and horizontal diversification, element displacement, and bilateral symmetry, while also incorporating random perturbations or alignments. Guided by an established typology and prior research (Malewczyk et al. 2022), this process yielded 49 groups of five compositions each, totaling 245, all maintained at consistent proportions to facilitate statistical comparison.

Figure 4 illustrates the process using architectural project no. 12– a residential complex in Ljubljana by Dekleva Gregorič Architects. A photograph from https://miesarch.com/archive extracted window elements rendered as a two-dimensional composition (code no12.001). Two additional variants were generated: composition no12.002 exhibited enhanced regularity with evenly distributed, straight columns, while composition no12.003 showed reduced regularity due to increased offsets and

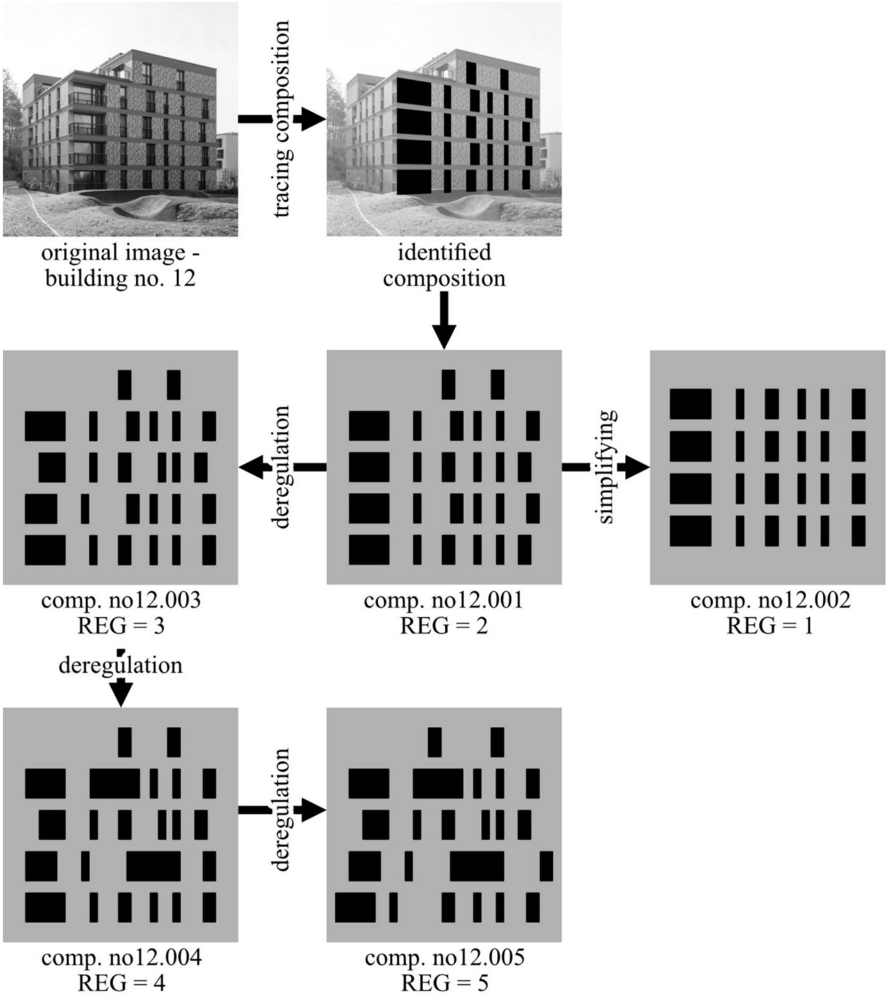


**Fig. 4** Protocol for creating research material on exemplary building no. 12

variability in element widths. By systematically increasing irregularity, two further compositions were produced (no12.004 and no12.005). In this series, the original composition (no12.001) was ranked as second in regularity (REG = 2) and REG values were assigned to the other projects based on their derivative compositions.

#### *Composition analysis*

During composition process, two values were systematically assigned to each composition in advance and recorded them in a spreadsheet:

- REG is a regularity coefficient on a scale from 1 to 5, where 1 means the most regular and five is the least regular. The coefficient was determined for each composition individually, concerning the other four compositions in each group, based on the number of variables—diversification of elements, diversification of shifts of components with each other, symmetry, and others.

- TYPE– a coefficient determining the type of composition, according to the typology of compositions developed by Malewczyk et al. (2022), where one is type'A', 2–'B', 3–'C', 4–'D', 5–'E' and 6–'F'.

A Python 3 script was developed and executed within Blender's integrated interpreter to process the research material. The script performs mathematical analyses using the vertex coordinates of orthogonal compositional elements (FACES) within each MESH object.

First, the essential mathematical functions were defined and used: arithmetic mean (M), standard deviation (SD), variance (var), and coefficient of variation (V). We also defined a function to calculate the metric entropy. Next, all parameters were derived from the information of individual compositional elements (FACES) in each composition (MESH object) (Table 1).

In the next step, the script calculates the coefficients of variation (V), the metric entropy (H), and the products for the individual parameters. Finally, the script determines 30 values (hereinafter referred to as attributes) for each of the compositions (MESH object). These are: $V(x_{coord})$, $H(x_{coord})$, $V(y_{coord})$, $H(y_{coord})$, $V(x_{coord})$ x $V(y_{coord})$, $H(x_{coord})$ x $(H(y_{coord})$, $V(x_{size})$, $H(x_{size})$, $V(y_{size})$, $H(y_{size})$, $V(x_{size})$ x $V(y_{size})$, $H(x_{size})$ x $H(y_{size})$, V(area), H(area), $V(x_{sidedist})$, $H(x_{sidedist})$, $V(y_{sidedist})$, $H(y_{sidedist})$, $V(x_{sidedist})$ x $V(y_{sidedist})$, $H(x_{sidedist})$ x $H(y_{sidedist})$, V(centerdist), H(centerdist), $V(x_{centerdist})$, $H(x_{centerdist})$, $V(y_{centerdist})$, $H(y_{centerdist})$, $V(x_{centerdist})$ x $V(y_{centerdist})$, $H(x_{centerdist})$ x $H(y_{centerdist})$, $V(centerdist_{ang})$, $H(centerdist_{ang})$.

### ***Statistical analyzes***

Statistical analyses were performed in Microsoft Excel by calculating Pearson's correlation coefficient (r) for each of the 49 five-element composition groups. 31 r values for each group were determined– one between the REG coefficient and TYPE and 30 between REG and the other attributes from the Blender script. In addition, the p-value (based on the two-sided Student's T-distribution) was computed for each r, yielding 49 r and p values per attribute and for TYPE.

Subsequently, the arithmetic mean (M) and variation (V) of r for the 30 attributes and the TYPE coefficient was calculated. The number of r values that were highly statistically significant ($p < 0.001$), statistically significant ($p < 0.05$), and insignificant ($p > 0.05$) was also determined.

### ***Research Quality Control and Its Replication***

The script was enhanced to visualize connections for the centerdist, $x_{sidedist}$, and $y_{sidedist}$ parameters, verifying the study's results. Three additional objects were created for each of the three compositions, each containing segments that connect the corresponding points. Figure 5 shows the visualization for composition no21.003.

**Table 1** Proprietary parameters calculated for the composition

| Parameter | Description |
| --- | --- |
| $x_{coord}$ | Information about the coordinates of the vertices of individual rectangles on the horizontal x-axis |
| $y_{coord}$ | Information about the coordinates of the vertices of individual rectangles on the vertical axis– y |
| $x_{size}$ | Width of the individual rectangles |
| $y_{size}$ | Height of the individual rectangles |
| area | Field of individual rectangles |
| centerdist | The absolute distance between the nearest compositional elements, determined in the following steps of the script:<br>Determination of connections between the centers of all elements of a given composition using the greedy method (each with each)<br>Removal of duplicates—overlapping connections<br>Selection of the shortest connection for a given element in a given quadrant rotated by $45^{O}$ (one element can be connected to a maximum of four other closest elements– on the right, left, above, and below– as a result of the nature of architectural objects) |
| $x_{centerdist}$ | Distance between the nearest composing elements, measured following the horizontal axis (x) based on the centerdist parameter |
| $y_{centerdist}$ | Distance between the nearest composing elements, measured by the direction of the vertical axis (y), based on the centerdist parameter |
| $centerdist_{ang}$ | inclination angle of straight lines connecting the nearest compositional elements, determined based on the arcus tangent function |
| $x_{sidedist}$ | horizontal distance between the two closest vertical edges of two equal rectangles, determined in the following steps of the script:<br>Calculation of the center points of the vertical edges, left and right, respectively<br>Creating a list of candidate connections using the greedy method (each with each), connecting each right midpoint with each left midpoint<br>Filtering the list by removing segments where the smaller'x' coordinate of a given connection lies on the left side (removing connections that cross the rectangles) and removing duplicates<br>Selection of the shortest connection for a given center point<br>Final verification that a given center point is used no more than once |
| $y_{sidedist}$ | The script determines the vertical distance between the two closest horizontal edges of two equal rectangles in the same way as the $\mathbf{x_{sidedist}}$ parameter |

Additionally, vertex coordinate data was exported for individual compositional elements from five randomly selected compositions. The parameters computed by the Blender script were recalculated in Microsoft Excel and compared these results to confirm the script's accuracy.

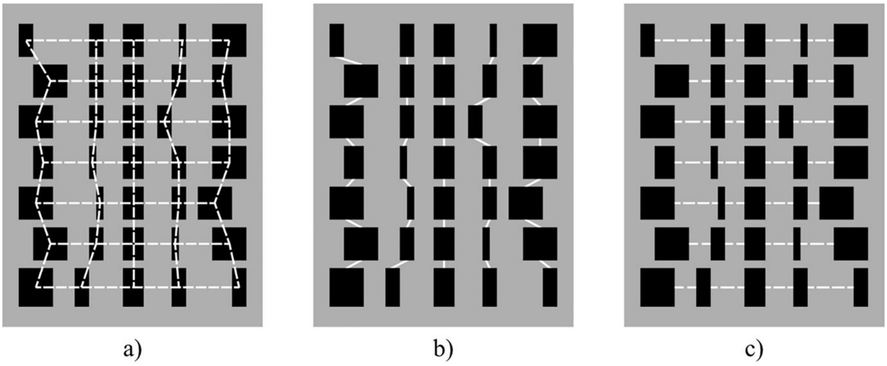

**Fig. 5** Visualization of parameters using composition no. 21.003 as an example: **a** centerdist, **b** $y_{sidedist}$, **c** $x_{sidedist}$

The experiment was repeated three times using identical input parameters and environmental configurations to verify reproducibility. The results fully matched, confirming the method's stability and repeatability. All scripts and configuration files are publicly available (https://doi.org/https://doi.org/10.34808/bemn-rd97), allowing independent replication. The same repository that contains the materials required to replicate the experiment also includes a Blender add-on, developed from the original script used to calculate the 30 attributes analysed in this study.

## Results

### Regularity and Composition of Mathematical Attributes

Under the procedure described in the'Materials and method' section, statistical analyses were carried out based on the collected data for each of the 245 compositions—the values of 30 attributes calculated by the script and the level of regularity—REG. Figure 6 shows the summarization of these results.

For the five attributes, it means $H(y_{sidedist})$, $H(x_{centerdist})$, $H(y_{centerdist})$, $H(x_{centerdist})$ x $H(y_{centerdist})$ and $H(c_{enterdist})$ the average Pearson correlation coefficient (with regularity– REG)– Mr, calculated based on 49 correlation coefficients, corresponding to 49 composition groups, is between 0.9308 and 0.9352 with a variability– Vr– of 7.08 to 9.75%. As mentioned, the results mean a robust correlation with very low volatility. At the same time, in this group of attributes, between 31 and 32 correlations proved to be statistically significant ($p<0.05$) and from 7 to 9– very statistically significant ($p<0.001$). The calculations' average statistical significance level for the r coefficient—Mp—is between 0.0260 and 0.0346.

For the two attributes, it means $H(x_{sidedist})$ x $H(y_{sidedist})$ and $H(centerdist_{ang})$, and the average correlation (with regularity– REG) is also very strong (Mr 0.9063 and 0.9137, respectively). However, their variability is slightly higher– Vr 21.61% and 15.13% respectively. For the attribute $H(x_{sidedist})$ x $H(y_{sidedist})$, the correlation coefficient – r turned out to be statistically significant ($p<0.05$) for 35 and highly statisti-

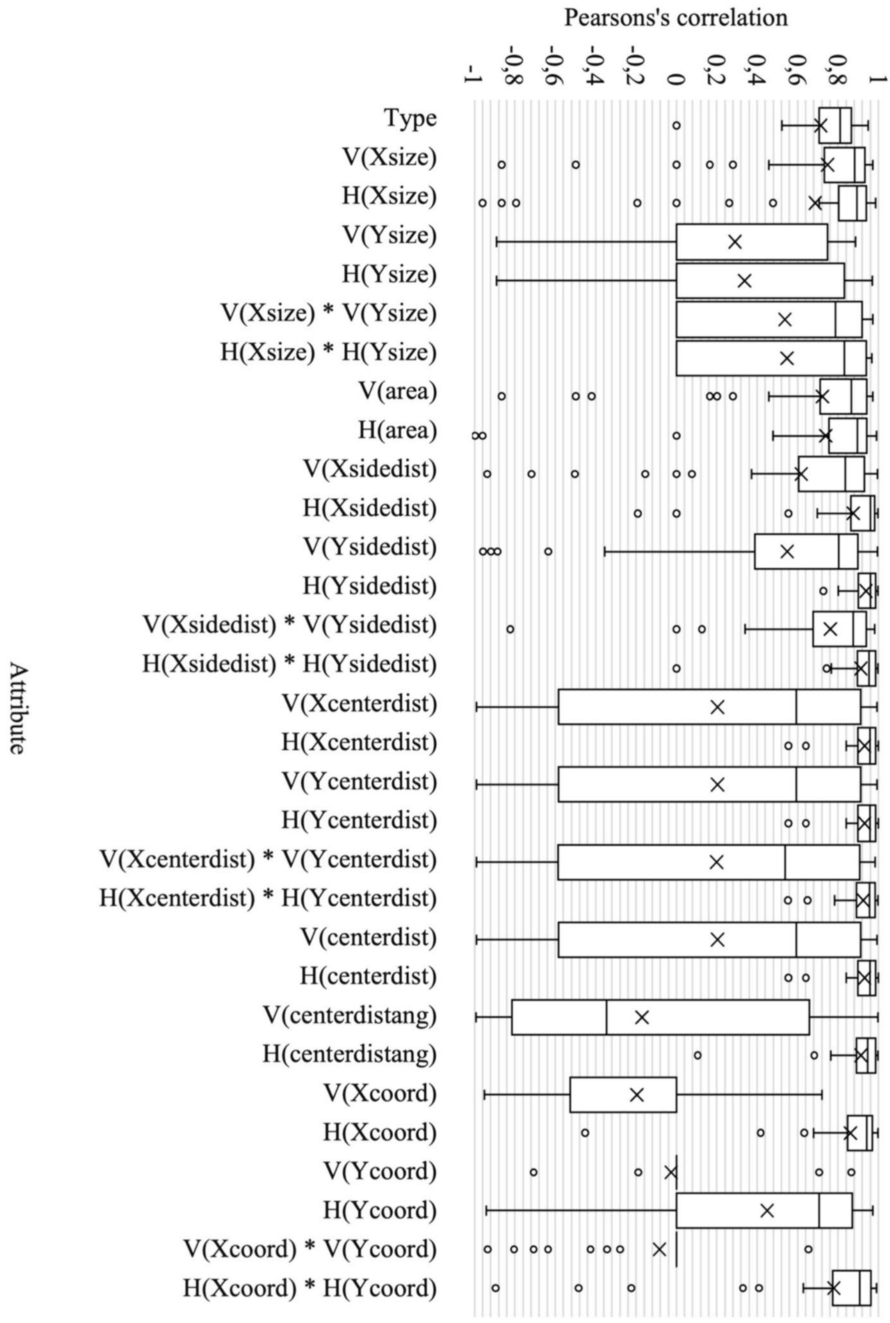


**Fig. 6** Boxplot showing composition attributes by Pearson's correlation r-coefficient with composition regularity

cally significant ($p < 0.001$) for 6, with an average statistical significance level of Mp equal to. 0384. Concerning H($centerdist_{ang}$), $p < 0.05$ for 31 r, $p < 0.001$ for 7 r, while the average statistical significance of the r calculations is $Mp = 0.0464$.

Seven attributes showed a strong average correlation (with regularity—REG)—Mr coefficients ranging from 0.7095 to 0.868. H($x_{sidedist}$) and H($x_{coord}$) exhibit average variability, with Vr values of 29.65% and 27.47%, respectively. The other five attributes in this group of parameters, mean V($x_{size}$), V(area), H(area), V($x_{sidedist}$) x V($y_{sidedist}$), and H($x_{coord}$) x H($y_{coord}$), are characterized by substantial variability. Vr in the range from 48.10% to 63.31%. The average level of statistical significance $Mp > 0.05$– results of calculating the correlation coefficient– r– statistically insignificant on average.

The following five attributes– H($x_{size}$), V($x_{size}$) x V($y_{size}$), H($x_{size}$) x H($y_{size}$), V($\mathbf{x}_{\mathbf{sidedist}}$), and V($\mathbf{y}_{\mathbf{sidedist}}$) show a moderate correlation (with regularity– REG) with Mr at a level of 0.4374 to. 5627. However, the variability of the correlation coefficient for these attributes is substantial—Vr from 78.81% to 102.74%. The correlation coefficient—r—in this group is, on average, statistically insignificant ($Mp > 0.05$).

The average level of correlation of the other 11 attributes with the regularity of composition (REG) is weak (H($y_{coord}$)—$Mr = 0.4498$ and H($y_{size}$)—$Mr = 0.3369$), and there is not even a linear relationship ($Mr < 0.2$). The variability of the correlation coefficient—r in this group of attributes is extreme and oscillates between 127.89% and up to 1290.31%. The average statistical significance for these 11 attributes is $Mp > 0.05$—results are statistically insignificant on average.

## Regularity and Façade Pattern Type Correlation

To verify the relationship between composition type (TYPE), as defined by Malewczyk et al. (2022), and composition regularity (REG), the study calculated correlations for each of the 49 five-element groups. Although the average correlation was strong ($Mr = 0.7145$, $Vr = 36.31\%$), only 6 out of 49 cases achieved significance ($p < 0.05$), and the average p-value ($Mp = 0.2050$) did not indicate statistical significance.

## Blender Add-on to Determine the Regularity of the Composition

Building on the original script– which computed 30 attributes per composition and visualized centerdist and sidedist connections–a Blender add-on was developed. The add-on offers two main functions, written in Python and compatible with Blender 4.0 and above. First: calculates each MESH object's parameter H(centerdist) to measure visual regularity (see Results, point 1, and Discussion, point 1). Second: generates an object that visualizes the centroid connections as segments underlying the H(centerdist) calculation (also referred to as 'Regularity'). Figure 7 shows a screenshot of the add-on.

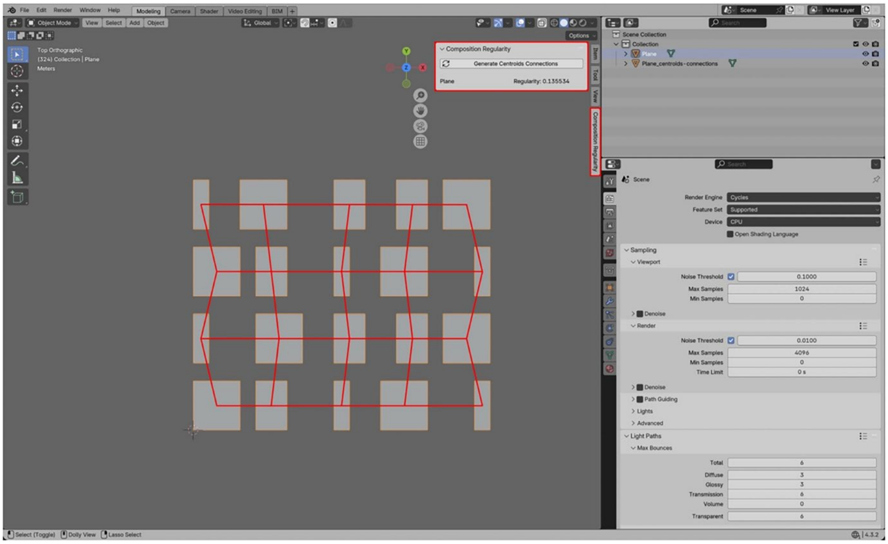


**Fig. 7** Screenshot of the developed Blender add-on

## Findings and Discussion

### Regularity and Composition of Mathematical Attributes

The results indicate that five attributes– $H(y_{sidedist})$, $H(x_{centerdist})$, $H(y_{centerdist})$, $H(x_{centerdist})$ x $H(y_{centerdist})$, and H(centerdist)– form a group that best describes visual regularity. They exhibit near-perfect Pearson correlations (close to 1) with very low variability (<10%), meaning they increase uniformly with irregularity, approaching 0 for ideally regular compositions and theoretically reaching infinity for maximal irregularity. All these attributes derive from Shannon's metric entropy and share a mathematically defined range from 0 to infinity. They align with the concept of visual order: maximum order occurs when elements are evenly distributed, whereas absolute chaos—requiring infinitely unpredictable states—remains physically unattainable.

Despite its strong average results, $H(y_{sidedist})$ is a less reliable indicator compared to the other four ($H(x_{centerdist})$, $H(y_{centerdist})$, $H(x_{centerdist})$ x $H(y_{centerdist})$, and H(centerdist)). In an ideally regular composition (perfectly equal vertical and horizontal intervals), $H(y_{sidedist})$ reached 0 in 26 cases. In contrast, the other parameters reached zero only in additional test compositions with an even square matrix. Moreover, detailed analysis revealed that $H(x_{centerdist})$, $H(y_{centerdist})$, and H(centerdist) produced identical results for each composition, implying that $H(x_{centerdist})$ x $H(y_{centerdist})$ equals $H(centerdist)^2$. Therefore, H(centerdist) is the best reflection of visual regularity, with an average Pearson correlation of $Mr = 0.9308$ and a low variability ($Vr = 9.75\%$). However, its range remains a comparative measure rather than an absolute one.

However, the limitations of the H(centerdist) index must be acknowledged. These constraints arise from the inherent limitations of Shannon entropy, which serves as the mathematical foundation for the parameter. Entropy calculations treat each piece

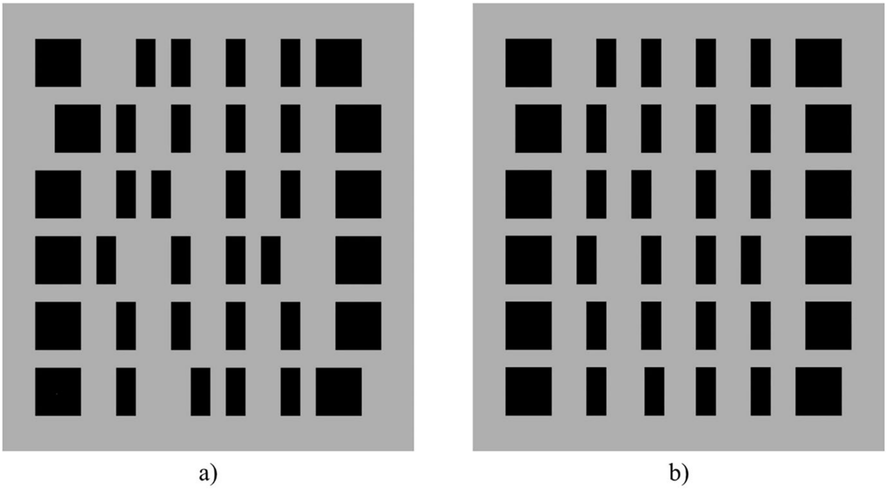


**Fig. 8** Exemplary compositions (**a** and **b**) with the same H(centerdist) value

of information in a sequence as distinct, regardless of its actual degree of diversity. As mentioned, this translates into a lack of sensitivity to subtle regular changes, as shown in Fig. 8. Both compositions (a and b) share the same H(centerdist) value, although composition a) exhibits less regularity than b). In both cases, identical elements shift in the exact directions but by different amounts. While the distances between the centers of the composing elements remain the same, the differences in their values are more pronounced in the left composition.

Nevertheless, the other 25 attributes show a lower average correlation with the composition's regularity, and, importantly, they exhibit more significant variability. Therefore, they will describe the composition's visual regularity as statistically worse.

### Regularity and Façade Pattern Type Correlation

Malewczyk et al. (2024) found a statistically significant, moderate (68.6%) correlation between composition type (as defined in their earlier work, 2022) and compositional regularity, suggesting that type alone does not determine regularity. This study applied a different approach using a larger dataset (245 vs. 12 compositions) derived from existing architectural objects. The average correlation between composition type (TYPE) and regularity (REG) was 71.45%, corroborating previous findings and confirming a strong association between visual regularity and composition type. While a 70% correlation does not prove that a given type inherently possesses regularity, these consistent results show that compositional regularity can be quantified using a mathematical coefficient.

### Blender Add-on to Determine the Regularity of the Composition

The add-on to the Blender program, developed as part of the experiment described in this manuscript, represents its practical outcome. This tool offers two primary functionalities. First, it calculates the numerical value of the parameter H(centerdist), the metric entropy of the lengths of the centroid connections among the nearest compositional elements—regarded as the mathematical quantity that best captures a composition's visual regularity. Second, the program visualizes these centroid connections, reducing the subjectivity inherent in traditional regularity assessments and enabling standardized, repeatable results. Such objectivity plays a critical role in advancing research on architectural composition and its perception.

Another significant aspect involves the capacity to automatically process large volumes of compositional data. Such automation facilitates more robust testing of hypotheses regarding the relationship between composition and regularity. Consequently, integrating this tool with machine learning methods to analyze contemporary and historical architecture– or with generative algorithms that utilize regularity parameters as input– seems promising.

Despite high precision, the tool's performance depends on the quality of the input data. Currently, the script requires compositions to consist of non-touching rectangles on the XY plane, which limits applicability of the add-on. Future developments should analyze actual architectural elements beyond flat representations.

## *Conclusion*

This study presents an innovative approach to analyzing the regularity of façade patterns, integrating digital tools, mathematical methods, and information theory. Drawing on Shannon entropy, the hypothesis formulated the existence of a mathematical coefficient that describes the regularity of architectural element compositions. The proposed parameter correlated with the perception of regularity at a level exceeding 90%, confirming the robustness of the adopted methodology and aligning with earlier findings based on human observation analysis (68.6% vs. 71.4%).

The developed script and a dedicated Blender add-on enable a precise, objective evaluation of façade designs, providing an essential bridge between subjective perception and statistical regularity analysis. This study enriches the theoretical understanding of the phenomenon by demonstrating a relationship between visual regularity and the distribution of distances between elements. It also introduces a practical tool intended for application in architectural design and design education.

Despite the strong results, the study suggests the need for further experiments to improve the coefficient's sensitivity to slight changes in regularity. The study proposes modifying the method of entropy calculation and incorporating alternative measures of variability. These results open new perspectives in the analysis of architectural complexity. They may contribute to a deeper discussion on the regularity of façade patterns in the context of different architectural styles and historical transformations.

In conclusion, the study forms an essential step towards integrating digital technologies with the theoretical analysis of architecture, providing both an innovative research tool and inspiration for further research on the perception of compositional regularity.

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**Availability of Data and Material** On demand.

**Code Availability** Not applicable.

## Declarations

**Conflict of interest** There are no conflict of interests.

**Ethical Approval** Not applicable.

**Consent to Participate** Not applicable.

**Consent for Publication** Sensitive data was not collected.



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**Michał Malewczyk** Assistant at the Faculty of Architecture, Gdańsk University of Technology, and a practicing architect. His research focuses on architectural composition, visual perception—particularly in relation to the built environment—and the design of multi-family residential buildings. He explores the use of computational methods and digital technologies in architectural design processes. He is also involved in teaching descriptive geometry.