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The Use of the Language of Mathematics as an Inspiration for Contemporary Architectural Design

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Abstract

The purpose of the article is to present the evolution of the use of mathematical language as an inspiration for creating spatial, three-dimensional forms in art and architecture. The article focuses on the possibilities for art and architectural design ideas gained by contemporary mathematics, algorithms and computational parametric approach. The analysis of various examples represents the relationships between the composition of spatial forms and the rules of mathematics. It is evident in different time frames, different styles and different approaches to thinking about and creating art and architecture. The starting point for this analysis is the symbolic Vitruvian golden ratio and its impact on the principles of spatial forms composition. Next, using the study of the geometric art of folding and cutting paper to create three-dimensional spaces, elements of applied art or even furniture or clothing, the article reaches to the mathematical issue of fractals, as the most accurate illustration of the aims of the parameterization in contemporary architecture. Parametric architecture, as a way of thinking about building as a set of numerically coded aspects, demonstrates the possibilities of using mathematical resources to improve functioning of the building by using optimization algorithms. The article shows possibilities of such uses of mathematics in generating spatial forms. Further, this analysis asks about the possible risks and disadvantages of such an approach, wondering if the correct definition of architecture is possible to achieve simply by using language of mathematics without all the other immeasurable aspects.

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1. Introduction

Starting the creative process in the visual arts, consciously or unconsciously we surrender the rigid, logical and measurable means of the composition. In architecture, the primary means of composition are: geometric characteristics of form, divisions (arrangements), weight ratios, rhythm, symmetry, symmetry of axes, accents, hierarchies, proportions, and finally colour and texture. Creating a work of art or architecture is about combining those elements. Already at this stage of the creative process we have to deal with such parameters that create functional dependencies by their mere interrelatedness. The eminent architect and theoretician Le Corbusier defined architecture as: "*(Architecture is)... the art above all others which achieves a state of platonic grandeur, mathematical order, speculation, the perception of the harmony which lies in emotional relationships. This is the AIM of architecture*" [1].

But is the creative, design process something more? Le Corbusier also admitted that "*Architecture is the masterly, correct and magnificent play of masses brought together in light. Our eyes are made to see forms in light; light and shade reveal these forms; cubes, cones, spheres, cylinders or pyramids are the great primary forms which light reveals to advantage; the image of these is distance and tangible within us and without ambiguity. It is for that reason that these are beautiful forms, the most beautiful forms. […] It is of the very nature of the plastic arts*" and "*...a thing of art, a phenomenon of the emotions, lying outside questions of construction and beyond them*" [1].

With development of the possibilities of modern mathematics and algorithms, many questions arise nowadays, such as: Are we able to represent by a mathematical function every aspect influencing the architectural project? Is it possible to describe the mathematical relationships existing between all indicators? Are there measurable, non-parametric aspects of correct architecture? The biggest problem in the definition of Le Corbusier seems to be the logical and mathematical notation of emotional relations, which architecture should provide. Are we (or will we ever be) able to define architecture using only the language of mathematics? Following the evolution of the use of mathematical language as an inspiration for creating spatial, three-dimensional forms in art and architecture we can answer this question. At present possibilities for art and architectural design ideas gained by contemporary mathematics, algorithms and computational parametric approaches are opening up a new way of thinking about the relation between mathematics and art.

2. Evolution of the influence of mathematics in the creation of 3D spatial forms

Analysis of various examples where mathematical achievements inspire art and architecture shows the relationships between the composition of spatial forms and the rules of mathematics. The evolution of this influence is evident in different time frames, and among different styles and approaches in architecture.

2.1. The golden ratio as a mathematical record of proportion (The golden ratio in applied arts and architecture)

The symbolic starting point for this analysis is the Vitruvian *Golden Ratio*[1] and its impact on the principles of spatial form composition since the Vitruvian formula is still present in contemporary graphic design. The value of the Golden Ratio (golden number) (1), which has influenced different fields of spatial composition and been noticeable for more than 2 400 years, is as follows:

[1] The Golden Ratio (Lat. *Sectio aurea*) – or in other words, the harmonious division or divine proportion, is defined as a division of a segment into two parts so that the ratio of the length of the longer one to the shorter is the same as the entire section to the longer part. In the language of mathematics: the length of the longer portion is to be the geometric average of the shorter part and the whole section according to the following formula: $(a+b) / a = a / b = \varphi$. The ratio defined by the above formula is called the golden number and is represented by the Greek letter φ.[2]

$$\varphi = \frac{1+\sqrt{5}}{2} = 1{,}6180339887 \tag{1}$$

It is not exactly known why this ratio has been declared a canon of beauty, but it can be said that ahead of its time, it may be regarded as the symbolic start of parametric thinking in art and architecture design. The sense of parametric architecture is to determine the aspects that affect the form by using numerical values and to create relationships between them using mathematical functions. From a mathematical point of view, the golden ratio is a very simple record of the relationship between the dimensions of the building's elements.

We can observe the use of the divine proportion in modern graphic elements, such as logos of companies known all around the world (Fig. 1a). The golden ratio is noticeable also in a geometrical figure, a regular asteroid shaped polygon, known as the Pythagorean star, or a pentagram (Fig. 1b).

a) b) c) d)

Fig. 1. (a) Golden Ratio in contemporary logo design, source:[3]; (b) Golden Ratio in pentagram, source:[4]; (c) Golden ratio in a Doric order, source:[5]; (d) Golden ratio in the inclination angle of the wall of a pyramid (Cross-section of a pyramid), source:[4].

In architecture, an ancient demonstration of the influence of the golden ratio is the Parthenon, whose construction was begun in 447 BC. It is considered to be the most perfect example of Doric order, and the perfect example of a composition according to the rules of the golden ratio. The front facade, as well as its modules, and even the details of the frieze, cornice and architrave were composed according to the divine proportion (Fig. 1c) [6].

Another important example of the creation of form according to the principle of the golden ratio is the Pyramid of Cheops. The shape of the Pyramid of Cheops is extremely close to the "*golden pyramid.*"[2] The angle of inclination (51°52') is very close to the inclination of the "golden" pyramid, which equals 51°50', and to the inclination based on "*π pyramid*"[3] which equals 51°51'. The cross-section of an equilateral pyramid with a square base is described by its central triangle whose sides are: hypotenuse of the pyramid, half of the base (a / 2), and height (h) (Fig. 1d). In the "golden" pyramid, the ratio of the hypotenuse to the base equals the golden number [8]. There is no actual evidence that the constructors of these pyramids were aware of the idea of golden ratio, so it may just be a coincidence. There are many other famous proportions which can be found during the history.

Vitruvius shows in his ground-breaking work "Ten books about architecture" that the intended proportions in architecture are based on the relationships between different parts of the human body. His book about architecture includes the message that the dimensions in human nature are composed so that the handspan is equal to four fingers, the foot is equal to four hands, the elbow is equal to six hands and a man is equal to 24 hands [9]. The above dimensions and proportions are noticeable in his buildings. By analogy, the Swiss architect Le Corbusier also based his design philosophy for harmony by (this time significantly modified) proportions of the human body. His fascination with mathematical order inspired by the Fibonacci sequence reflected his scale and proportion system called "*Modulor*"[4].

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[2] A pyramid based on Golden Triangle of sides √Φ, 1 and Φ. (where height to base ratio of the section triangle is golden ratio). In reference to cross-section in Fig.2 in a Golden Pyramid: h=√Φ, and a/2=1). [7]

[3] It is a pyramid in which the radius of a circle with the same circumference as the base of the pyramid has approximately the same value, as the height of the pyramid. [7]

[4] The"*Modulor*" according to Le Corbusier is a continuation of the "*Vitruvian Man*" by Leonardo da Vinci and is the evolution of the work of

Le Corbusier bravely linked the golden ratio with the human body. He divided his model height of a man in two parts according to the golden ratio at the height of the navel, then divided the obtained segments also in this ratio at the knees and neck [10]. Villa Stein in Garches, which was designed by Le Corbusier in 1927, is a significant example of the Modulor system. The rectangular plan of the villa, elevation and internal structure are a good approximation of golden rectangles.

2.2. Models of paper geometry: Origami and Kirigami in applied arts and architecture

The influence of mathematics in designing 3D spatial forms in applied arts and architecture can also be seen in the oriental art of folding paper - Japanese Origami[5] and its derivative Kirigami. Origami, as an example of algorithmic paper geometry, can be a strong base point in the experimentation with a new form while starting the design process. Creating small, easy to make models (e.g. paper miniatures) in order to search for the intentional form is an essential aspect of the artist’s or architect's work on the conceptual level. This raises the question: If the spatial form can be created out of paper, can it also be formed as a building?

Inspirations of both Origami and Kirigami can be found in the applied arts, such as fashion, furniture, accessories and even in architecture itself. An amusing use of Origami technique was found in a Christmas card designed by Volkswagen, where by unfolding the paper logo of the company, a snowflake appeared (Fig. 2a). The influence of Origami can also be noticed in many other fields of art, even on fashion runways, as for example in the collection called Sculpting Mind by Yuki Hagino from 2013 (Fig. 2b).

Fig. 2. (a), Volkswagen Christmas card, source:[12]; (b), Sculpting Mind collection by Yuki Hagino, source:[13]; (c), Origami inspired architecture, source:[14]; (d), Origami inspired architecture, source:[15].

Also in contemporary architecture, examples inspired by the art of Origami can be discerned in designs such as *Ice Cube* by McBrige Charles Ryan in Melbourne, (Fig. 2c) or *Embedded Project* by HDD FUN in Shanghai (Fig.2d). However the above observations raise some doubts: Can any geometry be represented in the form of an algorithm or by using the language of mathematics? Does designing a 3D spatial form require something more than just geometry?

2.3. Fractals in nature, art and architecture

The next step of investigation is the issue of fractals[6] a symbolic link in the evolution of mathematical inspirations in architectural design, as it announces a deeper level of complexity in algorithmic record of geometry and makes a

Leon Battista Alberti and other artists who used the proportions of the human body to search for the perfect appearance and functionality in architecture.

[5] Origami is considered traditional Japanese art. Rules of Origami are simple: The starting point must be only a square sheet of paper, which must not be cut or glued, but in order to create a 3D form it can only be folded. The process of folding the paper is written using an algorithm of procedure. Kirigami is a variation of Origami that allows small incisions in the paper [11].

[6] The word fractal (lat. *Fractus*) means exactly broken, partial, fractional. Commonly it refers to a self-similar object (the fragments of which are similar to the whole object) or "infinitely subtle" (showing the subtle details even in multiple enlargements). Due to a wide variety of examples, mathematicians avoid forming rigid definitions and suggest to determine fractals as sets that have all of the following characteristics, or at least the majority of them: A fractal in every scale has a non-trivial structure, which cannot be easily described in traditional Euclidean geometry. It is self-similar, if not in the exact sense, then approximately. It has a relatively simple recursive definition and its Hausdorff dimension is greater than its topological dimension. Colloquially, we describe fractals as having a natural, ragged, whirling appearance [16,17].

strong indication towards the idea of parametrization. Fractals are a part of nature, can be represented by mathematical algorithms and serve as inspiration for nontrivial geometry in interesting spatial forms. Through the development of computational capabilities of modern technologies, fractals can be considered as inspired by nature algorithmic notation of a complicated language of mathematics. This gives a new insight for artists and architects to gain inspiration from the surrounding nature, as nowadays they can be inspired by "computerized" nature. Architecture has come full circle, back to nature, but in a completely new meaning, which is adapted to the cybernetic world in which we live today. What is more, fractals, by its simple recursive definition and its self-similarity direct to the idea of optimization and – what may come with it – the issue of sustainable development. The issue of fractals creates a link in mathematical evolution between simple record of geometry and contemporary parametric design.

Fractals are geometric forms originally found in nature. An example of a fractal can be a cauliflower, a lightning, a snowflake or a fern (Fig. 3 a, b)

a)  b)  c)  d) 

Fig.3. (a), Fractal in nature, source:[4], Fig. by R. Bartz; (b), Fractal in nature, source:[18]; (c), Fractal ornament, source:[4]; (d), Geometric fractal of metabolism, source:[19]

Searching for fractals in art and architecture, we can find examples in oriental culture and art that often occur in the form of ornamentation, for example on vaults (Fig.3c). We can also find fractals in various forms. In Tokyo, an example, inspired by a simple geometric fractal is Kisho Kurokawa's design for the *Nakagin Capsule Tower* – office and residential building, which represents the architectural syle of metabolism (Fig.3d).

3. Parametric design as a modern stage for evolution of the use of mathematics

Reaching the present day, we should take into account current technological possibilities, the computing power of modern computers and the strong correlations between mathematics and algorithms. This way, by observing contemporary design evolutionarily, the presence of mathematics in the creation of spatial forms must be sought in the idea of parameterization [20]. It can even be said that modern architecture increasingly derives from technological capabilities. As a result, the way of thinking about mathematical approaches to architectural design has now changed. Parametric architecture provides such a way of thinking about the creation of a building, which under the given restrictions ensures the largest number of parameters that the designer cares about. Computational design comes down to finding functional dependencies between individual elements both in space and compositions, as well as (what is most important in the parametric approach) the functioning of the building [21]. If aspects affecting the realization of a project can be measured using numeric values as performance indicators, then the object may be parametrically designed and mathematics is the tool that will enable its optimization [22]. Parametric architecture is therefore a crucial leap in the use of mathematical notation, because from that moment, besides influencing the form, we are able to intervene primarily in the functioning of the building.

4. Parametric design ideas

The idea of parametric design, by its functional use goes far beyond art and architecture. One example is the project of a parametric plaster by Jake Eville from Victoria University in Wellington. The 3D-printers that Eville designed have produced, possibly the lightest, flexible plaster, which stiffens the broken bone as effectively as a traditional plaster. In architecture, thanks to the use of advanced technology, we can find a variety of forms generated parametrically. One example is *Absolute Towers*, a project by Mad Architects. Figure below depicts the algorithm for a twisted ellipsoidal tower with the use of the computer technology tool (Fig 4a). Another example is *Swallow's Nest For Taichung*, a new cultural center by Vincent Cellebaut. The other figure depicts an attempt to reproduce the algorithm of its creation (Fig.4b).

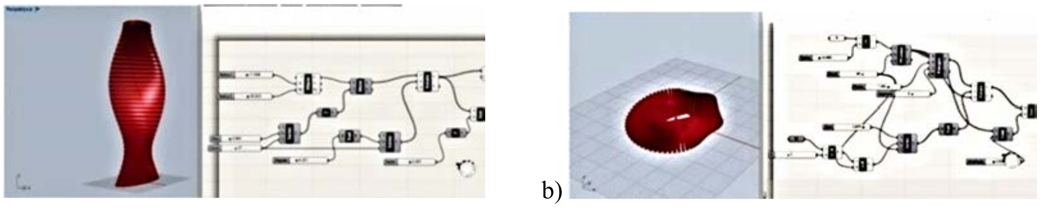
a) b)

Fig.4. (a), Parametric tower, Grasshopper algorithm; figure by author: A.Czech; (b), Parametric “nest” Grasshopper algorithm; figure by author:A.Czech

4. Conclusions

The direction in which we now seek ways to combine applied mathematics with art, architecture and other forms of creation has many advantages and gives us a broad spectrum of possibilities. Such possibilities involve: the effort to achieve (energetically) optimal forms, the "extinction" of unfavorable (non-functional) forms, the full use of technological advances, and innovative approaches to spatial forms while opening the mind to a new way of designing.

Considering the definition of architecture quoted by Le Corbusier in the introduction of this paper, we still do not have the certainty that parametric thinking is able to provide all aspects affecting good design. This forms a question about the possibility of the human element getting lost in the design process, and resulting in the loss of the "soul" of the building. Since the process begins to roll itself out after the inception of the algorithm, the creator seems to be significant only at the beginning while targets are being specified. Such an abandonment of standards and habits generates a transformation of civilization and is contrary to the archetype of the house/building. In this context, we have to keep in mind that architecture has always been and must remain more than a set of measurable parameters.

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