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Thermal self-action effects of acoustic beam in a vibrationally relaxing gas

Anna Perelomova *

Gdansk University of Technology, Faculty of Applied Physics and Mathematics, ul. Narutowicza 11/12, 80-233 Gdansk, Poland

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ABSTRACT

Thermal self-action of acoustic beam in a molecular gas with excited internal degrees of molecules' freedom, is studied. This kind of thermal self-action differs from that in a Newtonian fluid. Heating or cooling of a medium takes place due to transfer of internal vibrational energy. Equilibrium and non-equilibrium gases, which may be acoustically active, are considered. A beam in an acoustically active gas is self-focusing unlike a beam in a standard viscous gas. The self-action effects relating to wave beams containing shock fronts, are discussed. Stationary and non-stationary kinds of self-action are considered.

1. Introduction

In a Newtonian fluid, where the sound velocity c enlarges with increasing temperature T, an acoustic beam is defocused, while in a fluid with negative thermal coefficient $\delta = (\partial c/\partial T)_p/c_0 < 0$, it is focused ($c$ denotes the infinitely small-signal sound speed in a fluid, and c_0 is its value at the unperturbed pressure and temperature, p_0 and T_0). The first theoretical results were reviewed in Ref. [1], and the first experiments confirming the theory, were described in Ref. [2,3]. Considerable attention was paid to the thermal self-action of quasi-harmonic sound waves because many interesting results obtained in nonlinear optics have their counterparts in acoustics [4,5]. The comprehensive review by Rudenko and Sapozhnikov [6] concentrates on the self-action of beams containing shock fronts in media with quadratic and cubic nonlinearities. As usual, thermal self-action is caused by variation of the background temperature of a fluid due to Newtonian absorption of the wave energy. The scale of thermal inhomogeneities is much larger than the acoustic wavelength, and they form slowly, with characteristic time of formation much larger than the wave period.

This study is devoted to the non-Newtonian kind of self-action of sound beams in a gas where internal degrees of freedom are excited. In contrast to Newtonian fluids, acoustic heating in the thermodynamically excited gases occurs due to transfer of acoustic energy into that of internal degrees of molecule's freedom. In the non-equilibrium gas, sound may enhance under some conditions. The nonlinear effects of sound also may reveal anomalous features. Apart from acoustic cooling, the mean flow induced in the field of sound, is directed oppositely to the direction of beam propagation. An anomalous behavior of sound and relative nonlinear phenomena are specific not only in vibrationally excited gases, but in all media, where thermodynamic equilibrium is disturbed, such as non-isothermal plasma, chemically active fluids, suspensions of microparticles in a gas, the interstellar gas and upper atmosphere [7–16].

* Tel.: +48 583472078.
E-mail address: anpe@mif.pg.gda.pl

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A number of problems relating to the nonlinear effects and rate processes in gases with internal relaxation, have been studied previously. The attention was also paid to acoustic waves with discontinuities [17–19]. It was discovered in Ref. [20], that the nonlinear exchange of energy between sound and the thermal mode may lead to cooling instead of heating in the non-equilibrium gas, if the standard attenuation is small. The possibility of non-stationary self-focusing of sound beams in a vibrationally excited molecular gas was discovered by Molevich in Ref. [21], but effects associated with propagation of the shock wave over it were not considered there.

The method worked out by the author is fruitful in derivation of instantaneous dynamic equations for sound and non-wave modes accounting for their interaction. It has been applied in studies of streaming and heating in Newtonian fluids, as well as in studies of nonlinear phenomena of sound in non-equilibrium media like chemically reacting and relaxing gases [22–24]. In this study, we consider propagation of an axially symmetric sound beam over a relaxing gas in the geometrical approach. That allows to neglect diffraction. The statement of problem actually consists of two parts, one to describe the sound field itself, and second to account for variations of temperature in the field of sound and their influence on a sound beam. The simplified system of equations includes the analogue of the Khokhlov–Zabolotskaya equation supplemented by the term responsible for the vibrational relaxation, and an equation which describes slow dynamics of the entropy mode. The last equation was derived by the author in [25]. The mathematical content of further solution is very close to that one which has been developed by Rudenko et al. in studies of self-action of sound beams in a Newtonian fluid [6].

We consider a gas whose steady but non-equilibrium state is maintained by pumping energy into the vibrational degrees of freedom by power I (I refers to a unit mass). The relaxation equation for the vibrational energy E per unit mass has the form:

$$\frac{dE}{dt} = -\frac{E - E_{eq}(T)}{\tau_R} + I. \tag{1}$$

The equilibrium value of the vibrational energy at given temperature T is denoted by $E_{eq}(T)$, and $\tau_R(\rho, T)$ marks the vibrational relaxation time. The quantity $E_{eq}(T)$ equals in the case of a system of harmonic oscillators:

$$E_{eq}(T) = \frac{\hbar\Omega}{m(exp(\hbar\Omega/k_BT) - 1)}, \tag{2}$$

where m is the mass of a molecule, $\hbar\Omega$ is the magnitude of the vibrational quantum, k_B is the Boltzmann constant. Eq. (2) is valid over the temperatures, where one can neglect anharmonic effects, i.e., below the characteristic temperatures, which are fairly high for most molecules [8,10].

2. The governing equations and starting points

The system of equations describing thermal self-action in an axially symmetric flow of a vibrationally relaxing gas, take the form

$$\frac{\partial}{\partial\tau}\left(\frac{\partial p}{\partial x} - \frac{\delta T}{c_0}\frac{\partial p}{\partial\tau} - \frac{\varepsilon}{c_0^3\rho_0}p\frac{\partial p}{\partial\tau} - Bp\right) = \frac{c_0}{2}\Delta_\perp p. \tag{3}$$

$$\frac{\partial T}{\partial t} - \frac{\chi}{\rho_0 C_p}\Delta_\perp T = -\frac{BT_0(\gamma-1)}{c_0^3\rho_0^2}\langle p^2\rangle. \tag{4}$$

Here, x and r are cylindrical coordinates, the x axis coincides with the axis of a beam, $c_0 = \sqrt{\frac{\gamma R T_0}{\mu}} = \sqrt{\frac{\gamma p_0}{\rho_0}}$ denotes the infinitely small-signal sound speed in a perfect uniform gas (γ is the adiabatic index for high-frequency processes with frequency ω much larger than $1/\tau_R$), $\tau = t - x/c_0$ is the retarded time in the reference frame which moves with the sound speed c_0, p is acoustic pressure, $\Delta_\perp$ is the Laplacian with respect to the radial coordinate, $\varepsilon = (\gamma+1)/2$ is the parameter of nonlinearity, the angular brackets denote averaging over fast acoustic oscillations. The quantity B was derived in Ref. [20]:

$$B = -\frac{(\gamma-1)^2 T_0}{2c_0^3}\left(\frac{C_v}{\tau_R} + \frac{E - E_{eq}}{\tau_R^2}\left(\frac{\partial\tau_R}{\partial T} + \frac{\rho}{(\gamma-1)T}\frac{\partial\tau_R}{\partial\rho}\right)\right)_0. \tag{5}$$

It is evaluated at unperturbed p_0, T_0 and $C_v = dE_{eq}/dT$ [26]. Eq. (3) describes an acoustic pressure in a beam which propagates in the positive direction of axis x. In contrast to the well-known Khokhlov–Zabolotskaya–Kuznetsov [KZK] equation [27], Eq. (3) describes modulation of the wave velocity due to variation in temperature T; it includes also the term responsible for damping (or amplification) of sound different form that in the Newtonian fluids. This term is proportional to B which may take negative or positive values (if a non-equilibrium medium is acoustically active). Eq. (3) is derived under the same conditions, as the KZK equation. It is the leading-order equation for acoustic pressure which is imposed be a function of μx, $\sqrt{\mu}y$, $\sqrt{\mu}z$, τ, where μ is a small parameter responsible for a beam's divergence.

An acoustic source of the thermal mode in Eq. (4) follows from the general expression which was derived by the author in [25] in the case of periodic sound after averaging it over the sound period. Both Eqs. (3) and (4) are valid for the high-frequency sound, $\omega\tau_R \gg 1$, and its slow attenuation (or amplification) during a period, $|B|c_0/\omega \ll 1$.

The non-equilibrium excitation is possible in principle due to negative $\frac{\partial \tau_R}{\partial T}+\frac{\rho}{(\gamma-1)T}\frac{\partial \tau_R}{\partial \rho}$. The relaxation time in the most important cases may be thought as a function of temperature accordingly to Landau and Teller with some positive constants $\tilde{A}$ and $\tilde{B}$, $\tau_R(T)=\tilde{A}\exp(\tilde{B}T^{-1/3})$ [8,28,29]. There exists the threshold quantity of pumping magnitude I starting from which the excitation is non-equilibrium, since $E-E_{eq}\approx I\tau_R$.

The coupled system of Eqs. (3) and (4) allows to describe both harmonic waves and strongly distorted ones with a broad spectrum. In the cases where the acoustic nonlinearity is important, an acoustic pressure may be found in the form which follows from the theory of geometrical acoustics [6],

$$p=p(x,r,\theta=\tau-\psi(x,r)/c_0). \tag{6}$$

This leads to equations for unknown eikonal ψ and p,

$$\frac{\partial p}{\partial x}-\frac{\varepsilon}{c_0^3\rho_0}p\frac{\partial p}{\partial \theta}-Bp+\frac{\partial \psi}{\partial r}\frac{\partial p}{\partial r}+\frac{\Delta_\perp \psi}{2}p=0, \tag{7}$$

$$\frac{\partial \psi}{\partial x}+\frac{1}{2}\left(\frac{\partial \psi}{\partial r}\right)^2+\delta T=0. \tag{8}$$

Eqs. (7) and (8) in the new variables $P=\exp(-Bx)p$, $\Psi=\exp(-Bx)\psi/B$, $X=\exp(Bx)-1$, may be readily rearranged into the set

$$\frac{\partial P}{\partial X}-\frac{\varepsilon}{Bc_0^3\rho_0}P\frac{\partial P}{\partial \theta}+\frac{\partial \Psi}{\partial r}\frac{\partial P}{\partial r}+\frac{\Delta_\perp \Psi}{2}P=0, \tag{9}$$

$$B(X+1)\frac{\partial}{\partial X}(B(X+1)\Psi)+\frac{1}{2}\left(\frac{\partial \Psi}{\partial r}\right)^2 B^2(X+1)^2+\delta T=0. \tag{10}$$

Eq. (9) is analogous to the purely nonlinear equation, it differs from it by the last two terms responsible for variation in the cross-section of the ray tubes. Note that in spite of that B may take positive or negative values, the nonlinear distortions occurs similarly in the both cases in view of that the product XB is always positive. The second equation from the set, Eq. (10) determines distortion of the rays due to variations in temperature. For the validity of approximation of geometrical acoustics, the diffraction should be insignificant over the length of self-focusing. That is true for enough powerful beams. The characteristic length of diffraction is $x_d=\pi a_0^2/\lambda$, where a_0 and λ are characteristic initial transversal dimension of a beam and its wavelength. Assuming, that in the saw-tooth wave

$$P(X,r)=A(X,r)\cdot\begin{cases}-\frac{\omega\theta}{\pi}-1, & -\pi<\theta\omega<0,\\ -\frac{\omega\theta}{\pi}+1, & 0<\theta\omega<\pi\end{cases}, \tag{11}$$

Eqs. (4) and (9) transform into

$$\frac{\partial A}{\partial X}+\frac{\varepsilon\omega}{B\pi c_0^3\rho_0}A^2+\frac{\partial \Psi}{\partial r}\frac{\partial A}{\partial r}+\frac{\Delta_\perp \Psi}{2}A=0, \tag{12}$$

$$\frac{\partial T}{\partial t}-\frac{\chi}{\rho_0 C_P}\Delta_\perp T=-\frac{BT_0(\gamma-1)}{c_0^3\rho_0^2}\langle P^2\rangle(X+1)^2=-\frac{BT_0(\gamma-1)}{3c_0^3\rho_0^2}A^2(X+1)^2. \tag{13}$$

Eq. (12) can be solved by assuming the parabolic wave front,

$$\Psi(X,r,t)=\Psi_0(X,t)+\frac{r^2}{2}\frac{\partial}{\partial X}\ln F(X,t). \tag{14}$$

With account for (14), the exact solution of nonlinear Eq. (12) is [6]

$$A=\frac{P_0}{F}\Phi\left(\frac{r}{aF}\right)\left[1+\frac{1}{X_s}\Phi\left(\frac{r}{aF}\right)\int_0^X\frac{dX'}{F(X',t)}\right]^{-1}, \tag{15}$$

where P_0 is the initial amplitude on the beam axis, and function Φ describes the initial transverse distribution, $A(X=0,r)=P_0\Phi\left(\frac{r}{a_0}\right)$,

$$X_s=\frac{\rho_0c_0^3B\pi}{P_0\varepsilon\omega}=B\tilde{x}_s, \tag{16}$$

where $\tilde{x}_s$ is the distance of shock formation in a planar wave which propagates in an equilibrium gas with $B=0$. It is remarkable that X_s may take positive ($B>0$) or negative values ($B<0$), but $x_s=\frac{1}{B}\ln(1+X_s)$ (if exists in real numbers; this quantity is derived in [30]) should be always positive for a beam progressing in the positive direction of axis Ox. The discontinuity does not form at all if $X_s<-1$. That may happen for enough large negative B correspondent to strong attenuation. In the acoustically active media, when B is positive, the discontinuity always forms. In accordance to Eq. (10), evolution of eikonal Ψ is described by equation

$$\frac{1}{F}\left(\frac{\partial^2 F}{\partial X^2}+\frac{1}{X+1}\frac{\partial F}{\partial X}\right)=\frac{\delta T_2}{B^2(X+1)^2}, \tag{17}$$

where $T_2(X,t)$ is the coefficient in the transverse-coordinate expansion of the temperature,

$$T=T_0-\frac{r^2}{2}T_2+\ldots \tag{18}$$

Eqs. (13) and (17) with amplitude in the form (15) describe evolution of F. The paraxial approximation, which allows to consider temperature as series in powers of r (Eq. (18)), will considerably simplify solution of the system.

3. Thermal self-action of a sound beam

3.1. Stationary regime

If t is much larger than t_0, where

$$t_0=\frac{\rho_0 C_P a_0^2}{12\chi} \tag{19}$$

is the characteristic time of temperature establishment, the temporal derivative in the heat transport Eq. (13) may be put zero. We should establish the initial distribution of pressure across the beam. In the most practical applications, a beam is Gaussian with $\Phi(\xi)=\exp(-\xi^2)$. Expanding A in series in powers of r, one can derive from Eqs. (13) and (17) the equation for unknown function of one variable, $F(X)$,

$$F\left(\frac{d^2F}{dX^2}+\frac{1}{X+1}\frac{dF}{dX}\right)\left(1+\frac{1}{X_s}\int_0^X\frac{dX'}{F(X')}\right)^2=-\frac{T_0(\gamma-1)C_P\delta P_0^2}{6\chi c_0^3\rho_0 B}. \tag{20}$$

The boundary conditions are

$$F|_{X=0}=1,\quad \left.\frac{dF}{dX}\right|_{X=0}=\frac{1}{BR}, \tag{21}$$

where R is the wavefront curvature at the boundary $X=0$. In this study, we consider an ideal in equilibrium gas with positive $\delta,\ \delta=1/2T_0$. By use of dimensionless variable $z=\frac{x}{x_0}$, where

$$x_0=\frac{12\chi c_0^3\rho_0}{(\gamma-1)C_P P_0^2} \tag{22}$$

is some characteristic length depending on thermodynamical properties of the gas and initial magnitude of the saw-tooth wave, Eq. (20) readily rearranges into

$$F\frac{d^2F}{dz^2}\left(1+\frac{\Pi}{X_s}\int_0^z\frac{\exp(\Pi z')dz'}{F}\right)^2=-\Pi\exp(2\Pi z), \tag{23}$$

where

$$\Pi=Bx_0=\frac{12B\chi c_0^3\rho_0}{(\gamma-1)C_P P_0^2}. \tag{24}$$

Note that Π and X_s are both positive (in the case of acoustically active gas) or negative (otherwise). The normalized amplitude at the axis of a beam, $r=0$, equals

$$\frac{p_A}{P_0}=\frac{A(z)\exp(\Pi z)}{P_0}=\frac{\exp(\Pi z)}{F}\left(1+\frac{\Pi}{X_s}\int_0^z\frac{\exp(\Pi z')dz'}{F}\right)^{-1}. \tag{25}$$

The width of a beam may be evaluated as a distance from axis where the amplitude becomes e times smaller than that at the axis, $A(r=a,z)=A(r=0,z)/e$. It follows from Eq. (15), that

$$\frac{a}{a_0}=F\left(\ln e+(e-1)\frac{\Pi}{X_s}\int_0^z\frac{\exp(\Pi z')dz'}{F}\right)^{\frac{1}{2}}. \tag{26}$$

Eq. (23) was solved numerically. Thermodynamically equilibrium gas is always defocusing, and non-equilibrium gas, if it is acoustically active ($B>0$), it becomes focusing. In the domain of strong nonlinear absorption, where the distortion of rays takes place, the distance from the rays to the axis of beam propagation varies insignificantly. The so-called approximation of "thin lens", and F may be approximately equated to 1. Eq. (23) takes the form

$$\frac{\partial^2 F}{\partial z^2}\left(1+\frac{\exp(\Pi z)-1}{X_s}\right)^2 = -\Pi \exp(2\Pi z). \tag{27}$$

The ratio of Π and X_s,

$$\frac{\Pi}{X_s} = \frac{12\chi\varepsilon\omega}{\pi(\gamma-1)C_P P_0}, \tag{28}$$

is determined by thermodynamical properties of a gas, initial magnitude of pressure in a saw-tooth wave, and frequency of sound. We may estimate this ratio for a typical laser mixture $CO_2 : N_2 : He = 1:2:3$ at normal conditions $p_0 = 1$ atm $= 101325$ Pa, $T = 300$ K. The density of this mixture correspondent to its molar mass, $\mu = 0,019\ \mathrm{kg\cdot mol^{-1}}$, is $\rho_0 = 0,76\ \mathrm{kg\cdot m^{-3}}$, $\chi = 0,07\ \mathrm{W\cdot m^{-1}\cdot K^{-1}}$[20], $c_0 = 422\ \mathrm{m\cdot s^{-1}}$, $C_P = 1780\ \mathrm{J\cdot kg^{-1}\ K^{-1}}$, $\gamma = 1,33$, $\varepsilon = 1,17$. The dependence of the relaxation time τ_R on temperature and density is as follows,

$$\tau_R = 10^{-7}\frac{\mu}{\rho}\left(0,22\exp(-62,75T^{-1/3}) + 0,99\exp(-75,46T^{-1/3}) + 0,55\cdot 10^{-2}\sqrt{T}\exp(-58,82T^{-1/3})\right)^{-1}, \tag{29}$$

where τ_R is measured in seconds, μ in $\mathrm{kg\cdot mol^{-1}}$,T in Kelvins, ρ in $\mathrm{kg\cdot m^{-3}}$ [29]. That gives approximately $\tau_R = 5\cdot 10^{-5}$ s, $\frac{T}{\tau_R}\frac{\partial \tau_R}{\partial T} = -3,4$, $\frac{\rho}{(\gamma-1)\tau_R}\frac{\partial\tau_R}{\partial\rho} = -3$ and $\frac{\Pi}{X_s}\approx 5\cdot 10^{-9}\ \mathrm{s}\cdot\omega/M$, where $M = P_0/p_0$ is the effective acoustic number at the entrance in the medium, $x = 0$. The value of B depends on the pumping intensity I in accordance to Eq.(5); the threshold quantity is $I_{th}\cdot\rho_0 = 1,5\cdot 10^6\ \mathrm{W\cdot m^{-3}}$ [18]. For $I\cdot\rho_0 = 5,3\cdot 10^8\ \mathrm{W\cdot m^{-3}}$, $B = 3,3\ \mathrm{m^{-1}}$.Along with $\omega = 10^6$ Hz, $M = 10^{-3}$ that gives $X_s = 5$ and $\Pi = 26$.The diffraction length at this frequency and ten centimeter transducer, x_d, equals 12 m, and $x_0 = 8$ m. That provides large x_d as compared with the focal length, $x_f \approx 2$ m, and hence validity of approximation of the geometrical acoustics. If $I\cdot\rho_0 = 1,1\cdot 10^8 W\cdot m^{-3}$, $B = 0,66\ \mathrm{m^{-1}}$. In this case, for$\omega = 10^6$ Hz and $M = 10^{-2}$, $X_s = 0,1$, $\Pi = 0,05$, $x_0 = 0,08$ m, and $x_d = 12$ m is also greater then the focal length $x_f \approx 1$ m, as it is clear from Fig. 1. Fig. 1 shows the characteristic dimensionless width and amplitude of acoustic pressure in a beam with initial planar front, $1/R = 0$ for some quantities of Π and X_s during propagation over the equilibrium and non-equilibrium gas.

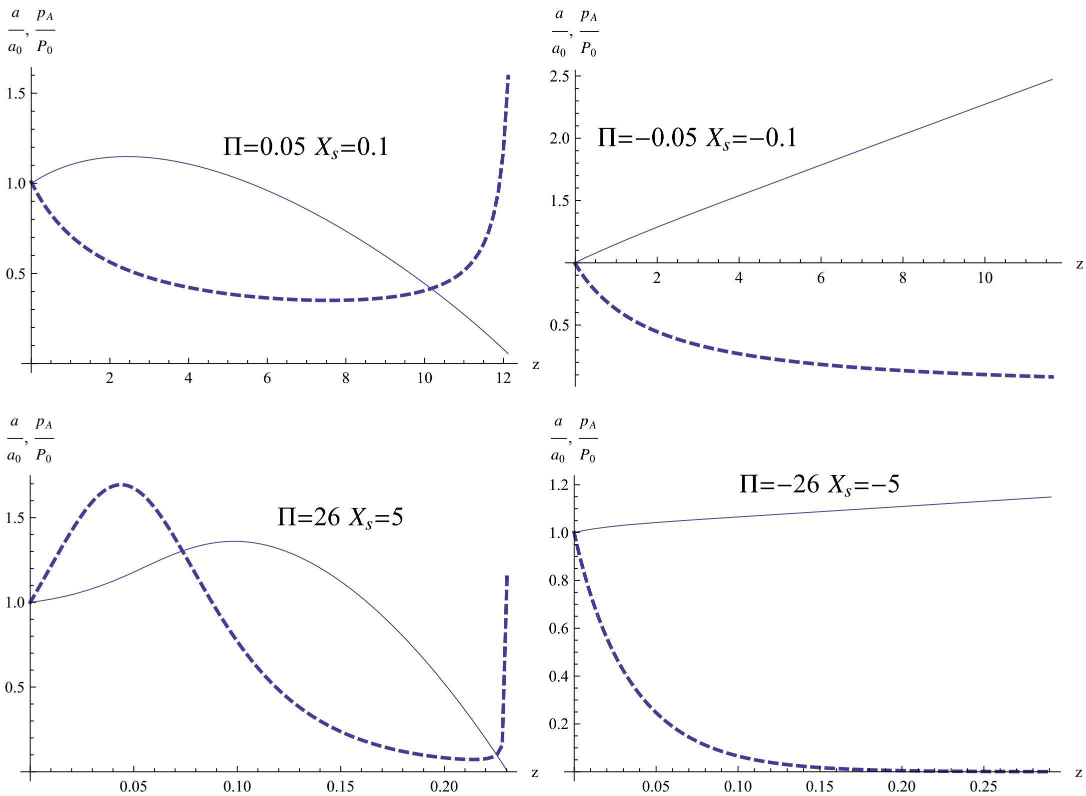

Fig. 1. The characteristic width of a beam (solid lines) and amplitude of acoustic pressure (dotted lines) at the axis of a beam with initially planar front in equilibrium or acoustic active gas.

The most simple estimations may be done if $X_s = 1$, and $F \approx 1$. In this case, Eq. (24) with account for initial conditions (21) is readily integrated (we consider initially planar beam with $1/R = 0$):

$$F = 1 - \Pi \frac{z^2}{2}. \tag{30}$$

That allows to evaluate approximately the dimensionless focal distance z_f, $x_0 \cdot z_f = \sqrt{2\Pi}/B$. If $\Pi < 0$, the characteristic distance of beam's broadening equals $x_0 \cdot z_f = -\sqrt{-2\Pi}/B$. For the mixture $CO_2 : N_2 : He = 1 : 2 : 3$ at normal conditions, the condition $X_s = 1$ yields $B/(M\omega) \approx 7 \cdot 10^{-4}$ s/m and may be satisfied, for example, by quantities $\omega = 10^6\ \mathrm{s}^{-1}$, $M = 10^{-3}$, $B = 0,7\ \mathrm{m}^{-1}$. Since the ratio $x_0 \cdot z_f / x_d$ should be less than unit, it means

$$\frac{96 c_0 \gamma^2 \chi}{a_0^4 B(\gamma - 1) C_P \rho_0 M^2 \omega^2} < 1. \tag{31}$$

The listed above quantities of ω, M and B result in the initial size of transducer $a_0 > 0,05$ m. The nonlinear length of focusing, $x_0 \cdot z_f$, equals 4,7 m. The condition $X_s = 1$ may be also satisfied, for example, by quantities $\omega = 5 \cdot 10^6\ \mathrm{s}^{-1}$, $M = 10^{-2}$, $B = 0,33\ \mathrm{m}^{-1}$, and in this case the nonlinear length of focusing equals 0,7 m. It is useful to compare results with the focal length in the case of the wave without discontinuities, which was obtained by Molevich in the limit $BR \ll 1 (B > 0)$ in Ref. [21]:

$$X_f = \ln(1 + BR)/B. \tag{32}$$

It gives $R = 0,79$ m in the case of $B = 0,33\ \mathrm{m}^{-1}$ and $X_f = 0,7$ m. The considered focusing of shock waves corresponds to the initially planar beam with $1/R = 0$, so that, it is much more effective than that of waves without discontinuities. To be focused at the same distance, a beam without discontinuities requires additional focusing at the transducer.

3.2. Non-stationary self-focusing

We now consider the case of non-stationary self-focusing where heat conductivity is small and the diffusion term in the Eq. (13) can be neglected. That takes place at initial stage, $t \lesssim t_0$. In this case,

$$\frac{\partial T}{\partial t} = -\frac{B T_0 (\gamma - 1)}{3 c_0^3 \rho_0^2} A^2 (X + 1)^2. \tag{33}$$

Using Eqs. (17) and (18) performing the expansion of A in the transverse coordinate in the vicinity of a beam axis, one gets the equation for $F(X, t)$:

$$\frac{\partial}{\partial t}\left(F^{-1}\left(\frac{\partial^2 F}{\partial X^2} + \frac{1}{X+1}\frac{\partial F}{\partial X}\right)\right) = -\frac{2(\gamma - 1)P_0^2}{3 B c_0^3 \rho_0^2 a_0^2 F^4 \left(1 + \frac{1}{X_s}\int_0^X \frac{dX'}{F(X')}\right)^2}, \tag{34}$$

which in dimensionless variables takes the form

$$\frac{\partial}{\partial \theta}\left(F^{-1}\frac{\partial^2 F}{\partial \tilde{z}^2}\right) = -\frac{\tilde{\Pi}\exp(2\tilde{\Pi}\tilde{z})}{F^4\left(1 + \frac{\tilde{\Pi}}{X_s}\int_0^{\tilde{z}} \frac{\exp(\tilde{\Pi}\tilde{z}')}{F(z')} d\tilde{z}'\right)^2}, \tag{35}$$

where $\theta = t/t_0$,

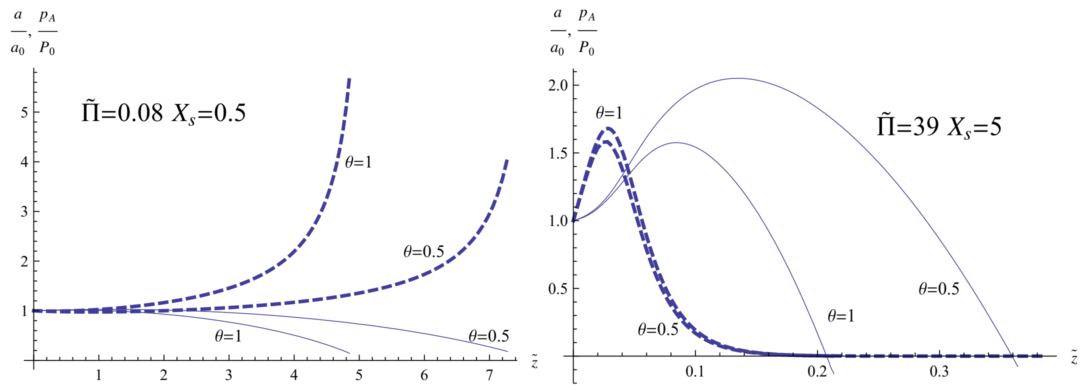

Fig. 2. The characteristic width of a beam (solid lines) and amplitude of acoustic pressure (dotted lines) at the axis of a beam with initially planar front at different times θ.

$$\tilde{\Pi} = B\tilde{x}_0 = \frac{18B\chi c_0^3\rho_0}{(\gamma-1)C_pP_0^2} = \frac{3}{2}\Pi, \quad \tilde{z} = \frac{x}{\tilde{x}_0}. \tag{36}$$

Eq. (35) is solved numerically under conditions

$$F(\tilde{z}=0,\theta) = F(\tilde{z},\theta=0) = 1, \quad \frac{\partial F}{\partial \tilde{z}}(\tilde{z}=0,\theta) = \frac{\tilde{x}_0}{R}. \tag{37}$$

If $B = 0,66\ \mathrm{m}^{-1}$, $\omega = 2\cdot 10^5\ \mathrm{s}^{-1}$, $M = 10^{-2}$, $\tilde{\Pi} = 0,08$, $X_s = 0,5$, $\tilde{x}_0 = 0,12$ m, and $x_d = 2,4$ m for ten centimeter transducer, $a_0 = 0,1$ m. The second series represent $B = 3,3\ \mathrm{m}^{-1}$, $\omega = 10^6\ \mathrm{s}^{-1}$, $M = 10^{-3}$, $\tilde{\Pi} = 39$, $X_s = 5$, $\tilde{x}_0 = 12$ m, and $x_d = 12$ m, $a_0 = 0,1$ m. For the considered typical laser mixture and ten centimeter transducer, $t_0 = 16$ s.

4. Concluding remarks

Similar inertial self-action can occur by means of formation of hydrodynamic streams in a medium ("acoustic streaming") due to the radiation pressure of an intense shock wave. This mechanism in a Newtonian fluid always leads to additional divergence because the drift caused by streaming makes the wave velocity increase in the central part of a beam, where the ultrasound intensity is higher and hence streaming is stronger. So that, the sound beam is divergent in a Newtonian gas due to both non-acoustic motions, the entropy mode, which forms a thermal lens, and the vortex mode, which is responsible for a bulk motion of a gas. In the media with unusual thermodynamic properties, like a gas with excited internal degrees of molecules freedom, streamlines may be directed oppositely as compared with a Newtonian fluid. This may enhance unusual focusing properties of a gas. In this study, we assumed that self-action occurs in a static medium. The effects associated with the occurrence of flows in saw-tooth wave fields and the hydrodynamic convection nonlinearity in Newtonian fluids were discussed in Ref. [31].

Figs. 1 and 2 reveal some important features of sound beams propagating over acoustically active gas. The width of a beam always decreases (in some cases somewhat increasing in the beginning), but amplitude of acoustic pressure may increase or decrease along the axis of a beam. That reflects two contrary mechanisms, one increase in the magnitude of sound, and second, its nonlinear attenuation, which becomes stronger with increase in magnitude of perturbations. Nonlinearity competes with the self-focusing of the wave front, and acoustic pressure decreases. The nonlinear broadening of a beam can be explained by flattening of the transverse beam profile due to stronger absorption near the axis (the so-called isotropization of the directional distribution). In the non-stationary regime, the thermal lens becomes stronger with time and the focal point moves towards the transducer. Near the nonlinear focus x_f, the width of a beam vanishes and amplitude infinitely grows. In this region, the description becomes inadequate because it does not take into account the diffraction divergence. These features of sound beams with discontinuities are very similar to those in majority of Newtonian liquids, which are defocusing due to positive δ and absorption of the sound energy [6]. Account for pumping I in the zero-order hydrodynamic equations would lead to dependence of the background density and pressure on spatial coordinates. Eqs. (3) and (4) are derived in the case, where the gradients of the background parameters are weak and they depend exclusively on the transversal coordinate r [32]. For large intensity of pumping, nor definitions of modes, nor Eqs. (3) and (4) are longer valid. The mathematical content also becomes fairly difficult [33]. Some features of wave propagation, like size of the domain stability of waves, look different, if spatial heterogeneity were taken into account [34]. The standard Newtonian attenuation of a gas is not considered in this study.

The conclusions of this study are readily applied to media with similar relaxation which may have different physical reasons. Among them, we may list fluids with Maxwell relaxation at high frequencies. At low frequencies, they behave as Newtonian. The system of initial equations which describes acoustic pressure of high-frequency sound in these media, looks similar to Eqs. (3) and (4) with some different coefficient standing by acoustic source.

Whereas the mathematical method used in this study, originates from that which has been used by Rudenko and co-authors, it is useful to underline the difference of the initial equations and further evaluations. Rudenko and co-authors investigated thermal self-action of the shock waves in Newtonian fluids, the equations which they use relate to pure attenuation of the shock wave when $\frac{b\omega}{\rho_0 c_0^2}$ tends to zero (b is the total Newtonian attenuation). Eq. (3) includes attenuation different from Newtonian, and Eq. (4) includes the acoustic source different from Newtonian: it is proportional to the mean square acoustic pressure, not to its mean squared temporal derivative. The results depend on B of the essence, in contrast to the pure nonlinear attenuation of shock waves in Newtonian fluids. Molevich starts from the system of conservation equations and seek solutions in the form of series of perturbations. That is valid in the case of weak nonlinearity and strong dispersion which takes place in the majority of problems relating to optic waves and may be applied in studies of beams without discontinuities. If the waveform is strongly nonlinearly distorted, the higher harmonics are effectively generated and do interact, and the whole waveform should be considered. In acoustically active media, nonlinearity is strong due to growing magnitude of acoustic pressure in the course of propagation, and discontinuities rapidly form in a weakly diffracting beam.

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