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Transient Response of Oscillated Carbon Nanotubes with an Internal and External Damping

Abstract

The present work aims at modeling a viscoelastic nanobeam with simple boundary conditions at the two ends with the introduction of the Kelvin-Voigt viscoelasticity in a nonlocal strain gradient theory. The nanobeam lies on the visco-Pasternak matrix in which three characteristic parameters have a prominent role. A refined Timoshenko beam theory is here applied, which is only based on one unknown variable, in accordance with the Euler-Bernoulli theory, whereas the Hamilton's principle is applied to derive the equations of motion. These are, in turn, solved for a carbon nanotube with some fixed material properties. An analytical method has been used to discretize the equations in the displacement field and time, while computing the time-response of the system. For validation purposes, the results based on the proposed formulation are successfully compared to several references. A final parametric investigation focuses on the sensitivity of the time-response of a nanotube under simple boundary conditions, to different parameters such as the length scale, the viscoelasticity coefficients or the nonlocal parameter.

Keywords: Nonlocal strain gradient theory; Refined Timoshenko beam theory; Simple boundary conditions; Viscoelastic nanobeam

1. Introduction and literature review

Carbon nanotubes are one of the most widely used carbon structures recently discovered, with unique properties and features, namely, the excellent flexibility and twistability, in addition to their high strength. The most important property of the nanotubes, however, is related to their electrical conductivity, which depends on the order of the atoms. They are cylindrical carbon molecules with open or closed ends. The structure of the nanotubes is rolled like a graphite sheet. Consider a graphite layer to better understand the structure of the nanotube. To place a pipe from a plane, it would be enough to place a point on the other. A nanotube is like a graphite sheet that is in the shape of a pipe. Depending on how the two sections of the graphite sheet are connected to each other, we will have different types of nanotubes. For the application of nanotubes and their properties, we can refer to the use of these compounds as "continuous fibers" in composites. The most important applications of such materials include airplanes, helicopter bodies, tennis rackets, etc. [1].

Therefore, given these remarkable applications, high mechanical properties have been increasingly required for many practical purposes, in the last decades. Arghavan and Singh [2] investigated the natural frequencies of single-walled carbon nanotubes (SWCNTs) based on a molecular mechanics method for clamped-free and clamped-clamped boundary conditions. A further application of the molecular mechanics is proposed by Ansari et al. [3] for a numerical study about the natural frequencies of nanotubes. Zhen et al. [4] examined the free vibrations of double-walled carbon nanotubes (DWCNTs), by applying the Euler-Bernoulli beam model and the nonlocal continuum theory of Eringen. In addition, Brischetto [5] analyzed the free vibrations of a SWCNT based on three-dimensional elasticity using a nonlocal continuum theory. In another work, Wang and Li [6] studied the free vibrations of nanotubes based on the nonlocal elasticity theory of Eringen placed in a viscous matrix. Torkaman-Asadi et al. [7] studied natural frequencies of carbon nanotubes on the basis of the nonlocal theory of Eringen

embedded on a Winkler foundation. El-Borgi et al. [8] formulated analytically the nonlinear equations for small-scale free longitudinal vibrations of SWCNTs placed on the Winkler-Pasternak foundation under several boundary conditions. Malikan et al. [9] studied the damping excited vibrations of SWCNTs bridged on a visco-Pasternak substrate while considering possible thermal effects. They used a higher-order nonlocal strain gradient theory in conjunction with a novel one variable beam model. Ren et al. [10] modeled the nanotubes as a porous material and studied the free vibrations of such a model using a refined beam theory in conjunction with the nonlocal strain gradient size-dependent theory. Shamani and Aghdam [11] analyzed the nonlinear buckling and post buckling of a lipid supramolecular micro- and nanotubes. They used the nonlocal theory of strain gradient to capture the small scale influences. Also the third-order theory of shear deformation was employed to derive constitutive equations. In a further research, Shamani and Aghdam [12] considered some nanoporous biomaterials under a static transverse loading. A modified truncated cube cell was also taken into account. The small scale equations were formulated by a nonlocal strain gradient model and an explicit analytical model was proposed to solve the differential relations governing the problem. Mohammadi et al. [13] calibrated a nonlocal theory of strain gradient for a shell model of nanotubes which conveyed a viscous fluid. The work was solved by means of the molecular dynamics and an amplitude was introduced to tune the nonlocal parameter. Wang et al. [14] examined the natural frequencies of a nanobeam moved axially, while deriving the differential equations of the problem according to the nonlocal theory of strain gradient, and the Euler-Bernoulli theory. Arefi and Zenkour [15] in a complete research, analyzed wave propagation and natural frequencies and also tensioning of a composite micro/nano rod in under electric field. Pasternak medium was embedded as a pedestal for the system. The model of Love's rod was employed to derive the Hamilton principle. They used strain gradient theory to investigate micro

and nanoscale influences. Arefi [16] showed the small scale effects by considering surface impacts and nonlocal elasticity theory for vibrational analysis of a functionally graded nano rod with considering piezoelectricity impacts. Arefi and Zenkour [17] presented a deep study on the bending and vibrations of a piezomagnetic laminated nanoscale beam. They considered both electric and magnetic fields whilst the model was placed on an elastic matrix. The Timoshenko beam theory was combined with the nonlocal theory of elasticity to derive the equations. Arefi [18] modeled vibrational of a functionally graded nano rod by using nonlocal theory of elasticity when the effects of magnetoelectric were taken into account. Arefi and Zenkour [19] applied couple stress and surface elasticity models for investigating wave propagation of a functionally graded nanoscale rod with taking piezoelectricity influences.

Further relevant works on the topic can be found in [20-54], where different nanostructures have been considered for varying external conditions.

Based on the available literature, in this work we focus on the transient response of oscillating nanotubes resting on a viscoelastic support, where both the internal and external damping has been taken into account. Thus, structural damping (hysteresis) is examined, and a new beam theory is applied to solve the vibrational problem, that has only one unknown variable, as in the classical model. In detail, the behavior of the nanomaterial on a small-scale is here studied according to the theory of nonlocal strain gradient, and the Navier's solution is proposed for the numerical examples, instead of an high-costly numerical solution. A large parametric investigation is finally presented to check for the sensitivity of the history response of oscillations for varying input parameters, i.e. lengths scale, viscoelasticity coefficients, nonlocal parameters.

2. Mathematical relations

Fig. 1 shows a single-walled carbon nanotube (SWCNT) resting on a visco-Pasternak foundation, as analyzed in what follows. The model has length L, outer diameter d and thickness h in the rectangular coordinate system.

A new refined beam theory (OVFSDT) derived in the past research by the authors [55-56] is here adopted to approach the problem, as in the following

$$\begin{Bmatrix} U(x,z,t) \\ V(x,z,t) \\ W(x,z,t) \end{Bmatrix} = \begin{Bmatrix} u(x,t) + z\varphi(x,t) \\ 0 \\ w(x,t) \end{Bmatrix} \tag{1}$$

where u and w are the displacement components of the neutral axis in the x- and z-directions, and φ refers to the twisting angle around the y-axis, at time t.

Based on a simple FSDT, the following expression for the deflection w is assumed

$$w(x,t) = w_b(x,t) + w_s(x,t) \tag{2}$$

$w_b(x,t)$ and $w_s(x,t)$ being its bending and shear components, respectively. On the other hand, the rotation φ is defined as

$$\varphi(x,t) = -\frac{\partial w_b(x,t)}{\partial x} \tag{3}$$

By replacing Eqs. (2), (3) in Eq. (1), the displacement field of the simple FSDT can be rewritten as

$$\begin{Bmatrix} U(x,z,t) \\ W(x,z,t) \end{Bmatrix} = \begin{Bmatrix} u(x,t) - z\dfrac{\partial w_b(x,t)}{\partial x} \\ w_b(x,t) + w_s(x,t) \end{Bmatrix} \tag{4}$$

The adoption of Eq. (2) might not be cognizable. Hence, the simple FSDT is refined in the following

$$\begin{Bmatrix} U(x,z,t) \\ W(x,z,t) \end{Bmatrix} = \begin{Bmatrix} u(x,t) - z\dfrac{\partial w_b(x,t)}{\partial x} \\ w_b(x,t) + W' \end{Bmatrix} \tag{5}$$

where W' denotes an indirect effect for $w_s(x,t)$ that has to be determined.

According to the constitutive equations of FSDT, we can compute $w_s(x,t)$ as a functions of $w_b(x,t)$ after manipulating the following relations

$$\frac{dM_x}{dx} - Q_x = 0 \tag{6}$$

combined with the stress resultants expression of the simple FSDT, i.e.

$$EI_c \frac{\partial^3 w_b(x,t)}{\partial x^3} - AG\frac{\partial w_s(x,t)}{\partial x} = 0 \tag{7}$$

where E is the elastic modulus, G is the shear modulus, and $\left(I_c = \int_A z^2 dA\right)$ is the moment of area of the cross-section A of the nanotube.

integrating Eq. (7) regarding x, and neglecting the integral constant terms, after a mathematical manipulation, the W' can be defined as

$$w_s(x,t) = W'(x,t) = B\frac{\partial^2 w_b(x,t)}{\partial x^2} \tag{8}$$

where B is defined as

$$B = \frac{EI_c}{AG} \tag{9}$$

Therefore, according to the OVFSDT, the displacement field becomes as follows

$$w_b = w_0 \;\; ; \;\; \begin{Bmatrix} U(x,z,t) \\ W(x,z,t) \end{Bmatrix} = \begin{Bmatrix} u(x,t) - z\dfrac{\partial w_0(x,t)}{\partial x} \\ w_0(x,t) + B\dfrac{\partial^2 w_0(x,t)}{\partial x^2} \end{Bmatrix} \tag{10}$$

Based on the Hamiltonian rule, the potential energy of the domain V is employed in the variational case, as below [57-63]

$$\delta V = \delta \int_0^t (S + \Omega - T) dt = 0 \tag{11}$$

Here, the virtual strain energy δS, the virtual kinetic energy δT, and the work of external forces $\delta \Omega$ are used to derive the governing equations in the following steps.

First, the virtual strain energy is obtained as below

$$\delta S = \iiint_v \sigma_{ij} \delta \varepsilon_{ij} dV = 0 \tag{12}$$

where the strain tensor is expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} - z \dfrac{\partial^2 w_0}{\partial x^2} + \dfrac{1}{2}\left(B \dfrac{\partial^3 w_0}{\partial x^3} + \dfrac{\partial w_0}{\partial x} \right)^2 \\ B \dfrac{\partial^3 w_0}{\partial x^3} \end{Bmatrix} \tag{13}$$

On the other hand, the kinetic energy is computed as follows [57]

$$T = \frac{1}{2} \rho \int \int_A \left(\left(\frac{\partial U}{\partial t} \right)^2 + \left(\frac{\partial W}{\partial t} \right)^2 \right) dA dx = 0 \tag{14}$$

If the virtual kinetic energy is expanded, it is

$$\delta T = \rho \int \int_A \left(-z^2 \frac{\partial^4 w_0}{\partial x^2 \partial t^2} - \frac{\partial^2 w_0}{\partial t^2} - B^2 \frac{\partial^6 w_0}{\partial x^4 \partial t^2} - 2B \frac{\partial^4 w_0}{\partial x^2 \partial t^2} \right) \delta w_0 dA dx = 0 \tag{15}$$

where $I_m \left(I_m = \rho I_c \right)$, $m_0 \left(m_0 = \rho \int_A dA \right)$ and ρ, are the mass moment of inertia, volumetric mass density and sectional density, respectively.

The visco-Pasternak foundation was applied as an external force which is defined as follows [64]:

$$\delta\Omega = \int_0^L \left(k_G \nabla^2 w_0 - k_w w_0 - C_d \frac{\partial w_0}{\partial t} \right) \delta w_0 dx \tag{16}$$

where C_d, k_w and k_G are the external damper parameter, stiffness and shear modules in the visco-Pasternak foundation, respectively. By applying the variational method (i.e. $\delta V=0$), we obtain the following equation of motion

$$\begin{aligned} &-\frac{\partial^2 M_x}{\partial x^2} + B \frac{\partial^3 Q_x}{\partial x^3} + N_x \left(B^2 \frac{\partial^6 w_0}{\partial x^6} + \frac{\partial^2 w_0}{\partial x^2} + 2B \frac{\partial^4 w_0}{\partial x^4} \right) - I_m \left(\frac{\partial^4 w_0}{\partial x^2 \partial t^2} \right) \\ &-m_0 \left(\frac{\partial^2 w_0}{\partial t^2} + B^2 \frac{\partial^6 w_0}{\partial x^4 \partial t^2} + 2B \frac{\partial^4 w_0}{\partial x^2 \partial t^2} \right) + k_G \nabla^2 w_0 - k_w w_0 - C_d \frac{\partial w_0}{\partial t} = 0 \end{aligned} \tag{17}$$

where M_x, Q_x, and N_x are nonlocal stress resultants, respectively. The quantity N_x refers to the in-plane axial resultant, here neglected in what follows. The stress resultants in the local form can be expanded as follows:

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \int_A \begin{Bmatrix} \sigma_x z \\ \sigma_{xz} \end{Bmatrix} dA \tag{18}$$

By inserting Eq. (13) into Hookean stress-strain elasticity relation and substituting it into Eq. (18) the local stress resultants are presented as:

$$\begin{Bmatrix} M_x \\ Q_x \end{Bmatrix} = \begin{Bmatrix} -EI_c \dfrac{\partial^2 w_0}{\partial x^2} \\ AGB \dfrac{\partial^3 w_0}{\partial x^3} \end{Bmatrix} \tag{19}$$

As far as the nonlocal strain gradient model is concerned, the following relations are applied [65-66]

$$\left(1-\mu\nabla^2\right)\sigma_{ij} = C_{ijkl}\left(1-l^2\nabla^2\right)\varepsilon_{kl} \;\; ; \;\; \mu(nm^2) = \left(e_0 a\right)^2, \; \nabla^2 = \frac{\partial^2}{\partial x^2} \tag{20}$$

l and μ, being the length scale and the nonlocality parameters, respectively. Based on Eq. (20), the nonlocal stress resultants are governed by the following relations

$$\left(1-\mu\nabla^2\right)M_x=-\left(1-l^2\nabla^2\right)\left(EI_c\frac{\partial^2 w_0}{\partial x^2}\right) \tag{21}$$

$$\left(1-\mu\nabla^2\right)Q_x=\left(1-l^2\nabla^2\right)AG\left(B\frac{\partial^3 w_0}{\partial x^3}\right) \tag{22}$$

It is well known that the mechanical properties of viscoelastic models are usually defined by some linear relations, which means the formulated differential equations are completely time-dependent. In this paper, the linear Kelvin-Voigt model is considered within the equilibrium equation as detailed in the following procedure.

The stress-strain equation for the viscoelastic beam has been defined as

$$p^E\sigma=q^E\varepsilon \tag{23}$$

where p^E and q^E are the viscoelastic operators. The Kelvin-Voigt model incorporates the Newtonian damper and the Hookean elastic spring connected in a parallel shape [67]

$$p^E=1,\ q^E=E+g\frac{\partial}{\partial t}\Rightarrow\sigma(t)=E\left(1+g\frac{\partial}{\partial t}\right)\varepsilon(t) \tag{24}$$

In which g is an internal structural damping parameter.

By combining Eqs. (21), (22), (23), (24) with the Eq. (17), the free damped oscillation equation is obtained as

$$\begin{aligned}&\left(1-l^2\nabla^2\right)\left[\left(1+g\frac{\partial}{\partial t}\right)\left(EI_c\frac{\partial^4 w_0}{\partial x^4}+AGB^2\frac{\partial^6 w_0}{\partial x^6}\right)\right]\\&+\left(1-\mu\nabla^2\right)\left[-I_m\left(\frac{\partial^4 w_0}{\partial x^2\partial t^2}\right)-m_0\left(\frac{\partial^2 w_0}{\partial t^2}+B^2\frac{\partial^6 w_0}{\partial x^4\partial t^2}+2B\frac{\partial^4 w_0}{\partial x^2\partial t^2}\right)\right]\\&+\left(1-\mu\nabla^2\right)\left[k_G\nabla^2 w_0-k_w w_0-C_d\frac{\partial w_0}{\partial t}\right]=0\end{aligned} \tag{25}$$

3. Analytical solution

In this study, we assume a nanotube with simply-supported boundary conditions at both ends. Thus, the solution based on the Navier's approach is considered as follows [67]

$$w_0(x,t)=\sum_{m=1}^{\infty}W_m\exp(i\,\omega_n t)\sin\left(\frac{m\pi}{L}\right),\ i=\sqrt{-1} \tag{26}$$

where W_m and ω_n represent the displacement parameter and vibration frequency, and m is half-wave. By substitution of Eq. (26) into Eq. (25), after a mathematical manipulation, we get the following equation of motion

$$\left(-\omega^2\mathbf{M}+i\omega\mathbf{C}+\left(1+ig\omega\right)\mathbf{K}\right)\mathbf{W}=0 \tag{27}$$

where ***K***, ***C***, ***M*** refer to the stiffness, damping and the mass quantities, respectively.

4. Numerical examples

A preliminary check in terms of accuracy of the present formulation is done, by comparing its results with respect to other formulations available in literature. Some results are summarized in Tab. 1, while comparing our formulation with respect to the Euler-Bernoulli beam theory (EBT), the Timoshenko beam theory (TBT), and the Sinusoidal beam theory (SBT) [68-69]. According to this comparative evaluation, it is worth noticing the good agreement between our formulation with the SBT and TBT for which there was no use any shear correction factor. The current formulation makes the solution easier and simpler than both SBT and TBT because of the presence of only one variable in the displacement field. On the other hand, the EBT features some consistent differences with the other formulations. This is mainly related to the neglected transverse shear strains in EBT, which are instead important, especially for moderately thick and thick models. As also shown in Tab. 1, EBT-based results for L/h=40, are in good agreement with shear deformation models. This proved that whilst the model is a narrow beam, the transverse shear strains can be negligible.

As second investigation, we want to prove the capability of the present formulation to study transient vibration problems of carbon nanotubes. The material and mechanical properties of the selected single-walled carbon nanotube is reported in Tab. 2, whereas the main results are plotted in Fig. 2. More specifically, Fig. 2a indicates the effect of the external damping coefficient on the time-history. As clearly visible in Fig. 2, with the presence of the damping, the vibrational motion of the system is damped and the initial displacement amplitude is completely lost in a few nano seconds. In addition, the larger C_d parameter makes the vibrations damped without a delay. To study the effects of the elastic section of the visco-Pasternak base on the frequency response of the system, Fig. 2b is also plotted. This figure is presented with the same parameters mentioned before, with the only difference that the stiffening and shearing effects of the foundation have been halved. It is interesting to note that the number of periodic cycles of the system will be reduced to a complete damping, when the elastic effects of the foundation become weaker. This means that the stronger elastic base increases the natural frequency of the system, and consequently the number of cycles increases in the same time increment.

Figs. 3a and b show respectively the effects of the nonlocal parameter and the length scale on the frequency behavior of the nanotube system. As visible in Fig. 3, an increasing delay in periodic cycles occurs for lower values of the nonlocal parameter. As a result, smaller nonlocal coefficients result in longer vibration time of the system and the local system is damped later than the nonlocal system. From a physical point of view, nonlocality leads to a certain weakness of the stiffness of the nanotubes, thus leading to a decrease in the natural frequency and an overall increase in the rate of damping.

Figure 3b demonstrates quite reversible results, compared to the previous figure. It should be pointed out that the larger length scale parameter leads to an increase in damping time. Indeed,

an increasing the length scale increases the internal energy of the system, which leads, in turn, to a delay in the damping of the oscillations.

Figs. 4a and 4b describe the nanotubes with different thicknesses. It is worth mentioning that if the nanotube is narrower, the number of periodic cycles will go up. In fact, in thicker nanotubes, due to an higher elastic energy, we can notice a certain decrease in the number of amplitudes of motion, which is associated to a lower delay. This can be due to the ratio of mass values to the external damper.

The effect of internal damping is presented in Fig. 5 by examining several viscoelastic parameters. First, we compare the results of the internal viscosity with the external one. According to Fig. 5a, if the viscosity is removed, the system with the initial amplitude and without damping features an infinite cycle (without considering friction in the system). However, with the presence of both interior and exterior viscoelasticity, the system will be damped, with different rates of damping. To achieve an appropriate comparison, the condition based on a complete absence of the external damper is considered, while assuming an internal damping $g=5$ *Ns/m* (first) or an external damping $C_d=5$ *N.s/m* (second). The two cases are identical in values with a clear influence of the external viscosity compared to the internal one. As a matter of fact, for the present nanotube system, the effect of the external damping is greater than the one related to the internal damping. This result is related to the delay in damping for the case $C_d=0$, $g=5$ *N.s/m*, differently from the case $C_d=5$ *N.s/m*, $g=0$. This means that the external damping has a marked role in the vibrational systems, despite the great importance of the internal damping. Moreover, the amplitudes for case 1 is clearly lower than the ones for case 2, which enforces the relevance of the internal damping. On the other hand, Fig. 5b illustrates that the vibrations are damped in a shorter time for an increasing internal viscoelasticity parameter,

and the system will be damped faster. It is also worth noting that the amplitude of motion reduces for increasing values of *g*.

To investigate the influence of the initial displacement enforced to the system, Fig. 6 plots the time-history response for three different values of the initial amplitude. It can be seen that higher values of the initial displacement yield to increasing amplitudes, and more cycles are required to lose the initial energy, up to a complete standstill of the system. An interesting point is that an increasing value of the initial displacement results in a more pronounced reduction of the global domains, which means that the larger the initial displacement, the greater the effect of the external damper. To have a further validation between the internal and external damping Figs. 6b and 6c are also shown. These figures represent that the number of periodic cycles, for the same time-history, is about 4.5 cycles, when *Cd=0, g=5N.s/m*, and of about 5 cycles, when *Cd=5N.s/m, g=0*. These cases confirm that the internal damping makes the system reasonably slower, due to an increasing viscidity in the model's molecules.

5. Conclusions

This work focuses on the vibrational behavior of a carbon nanotube in the presence of internal and external viscoelasticity. Based on a modified Timoshenko's beam theory together with a nonlocal strain gradient model, we obtain the frequency equations in a nanoscale form. The Navier's solution procedure is here employed to solve analytically the partial differential equations of the problem, and to determine the time increment of the modeled system. The main conclusions can be summarized as follows:

- A weaker elastic foundation yields to a reduced number of periodic cycles.
- A decrease of the nonlocal parameter and an increase of the length scale parameter introduce a delay in the damping of the nanotube system.

- It is very important to use an internal damping in vibrating mechanical systems. It was found that the external damping increases the damping rate of the system, whereas the internal damping reduces significantly the amplitudes of the motion.

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Table 1. Comparison of nondimensional natural frequencies.

$\Omega_n = \omega_n L^2 \sqrt{\frac{\rho A}{EI}}$, *m= (0.9 to 1.1), E=1TPa, υ= 0.3, h=1nm,* k_s*=5/6 for TBT.*

L/h	$(e_0a)^2$	Present	Euler-Bernoulli theory (EBT) [69]	Timoshenko beam theory (TBT)		Sinusoidal beam theory (SBT)	
				[68]	[69]	[68]	[69]
5	0	9.2943	9.7112	9.2740	9.2740	9.2752	9.2752
	1	8.8587	9.2647	8.8477	8.8477	8.8488	8.8488
	2	8.4788	8.8747	8.4752	8.4752	8.4763	8.4763
	3	8.1495	8.5301	8.1461	8.1461	8.1472	8.1472
	4	7.8693	8.2228	7.8526	7.8526	7.8536	7.8536
10	0	9.7209	9.8293	9.7075	9.7075	9.7077	9.7077
	1	9.2666	9.3774	9.2612	9.2612	9.2614	9.2614
	2	8.8857	8.9826	8.8713	8.8713	8.8715	8.8715
	3	8.5483	8.6338	8.5269	8.5269	8.5271	8.5271
	4	8.2320	8.3228	8.2196	8.2196	8.2198	8.2198
20	0	9.8377	9.8595	9.8281	9.8281	9.8282	9.8282
	1	9.3840	9.4062	9.3763	9.3763	9.3764	9.3764
	2	8.9917	9.0102	8.9816	8.9816	8.9816	8.9816
	3	8.6456	8.6604	8.6328	8.6328	8.6329	8.6329
	4	8.3329	8.3483	8.3218	8.3218	8.3218	8.3218

Table 2. Mechanical characteristics of the nanotube [70-74]

$E=1$ TPa, $\nu=0.19$
$h=0.34$ nm, $d=0.7$ nm,
$\rho=2300$ kg/m^3

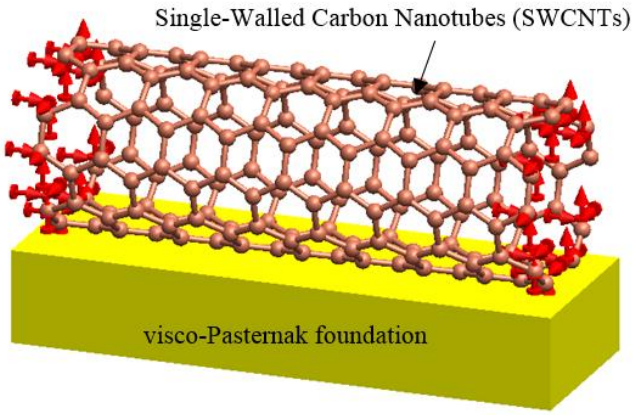
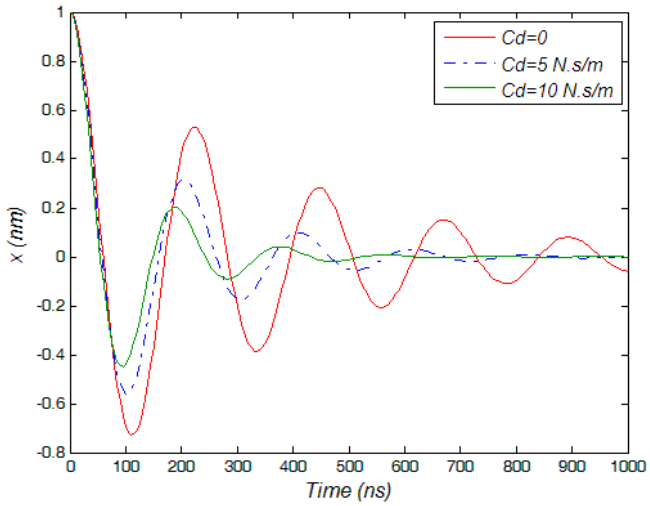
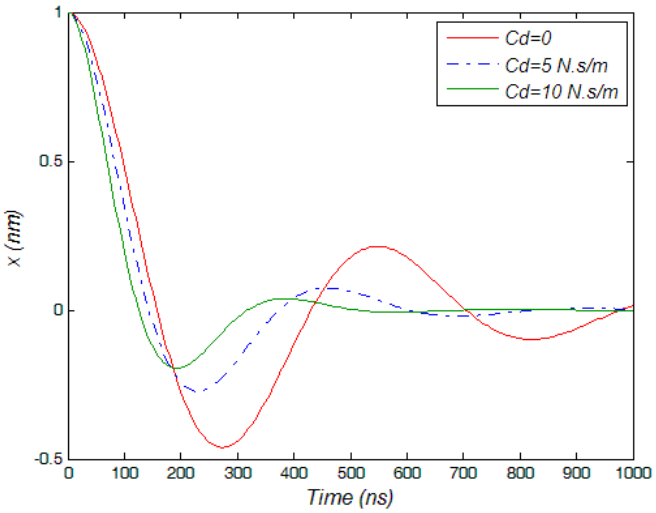


Fig. 1. Molecular scheme of a SWCNT resting on a visco-Pasternak matrix.

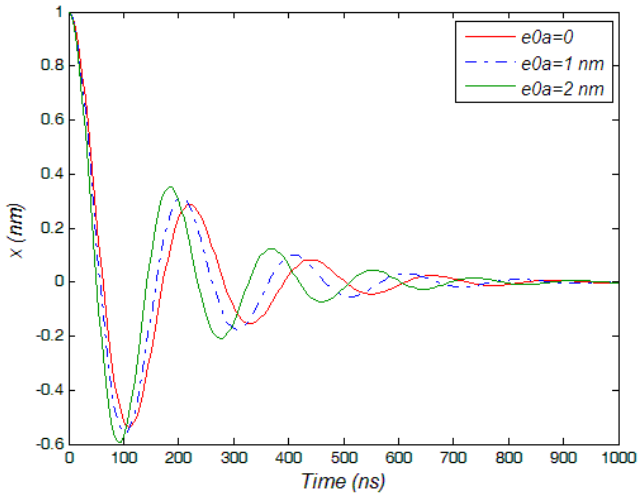


(a) k_G=1GN, k_w=1GPa

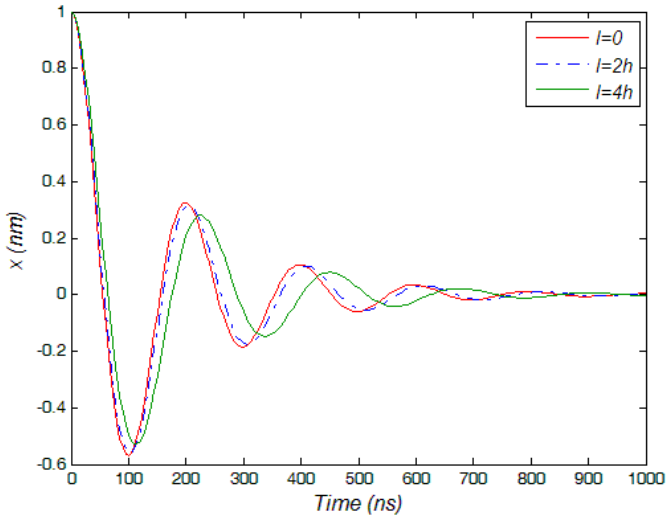


(b) k_G=0.5GN, k_w=0.5GPa

Fig. 2. Variation of the time-history with the external damping (e_0a=1nm, l=2h, m=1, g=5N.s/m, L=10d).

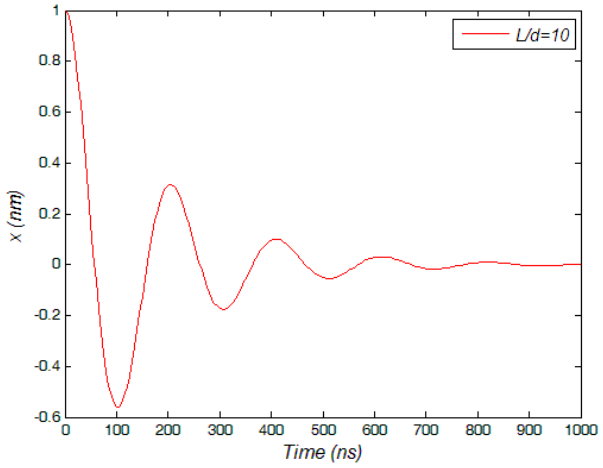


(a) $l=2h$

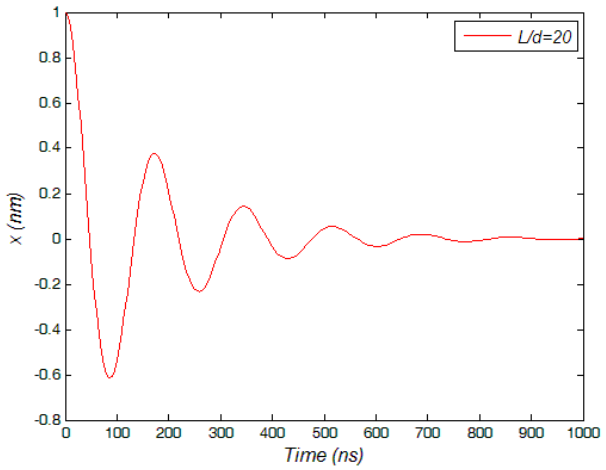


(b) $e_0a=1nm$

Fig. 3. Variation of the time-history with the length scale ($Cd=5N.s/m$, $m=1$, $k_G=1GN$, $k_w=1GPa$, $g=5N.s/m$, $L=10d$).

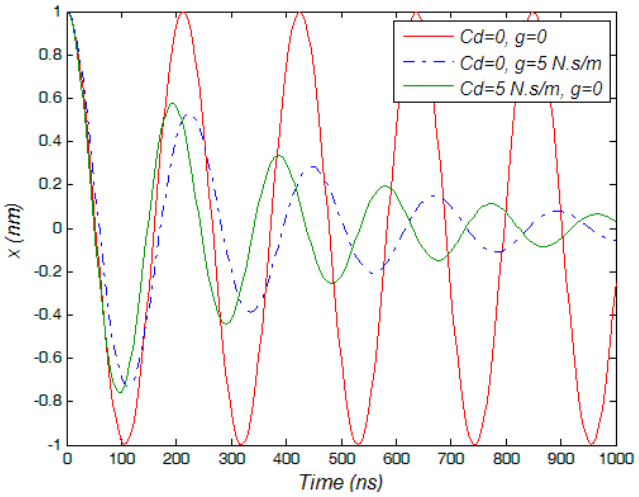


(a)

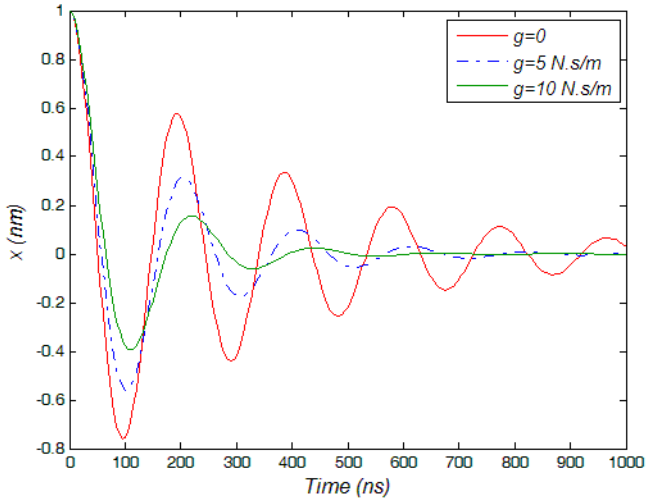


(b)

Fig. 4. Variation of the time-history with the length-to-thickness ratio (e_0a=1nm, l=2h, Cd=5N.s/m, m=1, k_G=1GN, k_w=1GPa, g=5N.s/m).

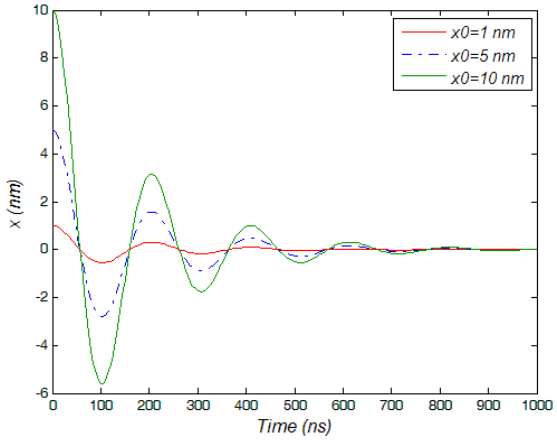


(a) *Cd=0; 5N.s/m*

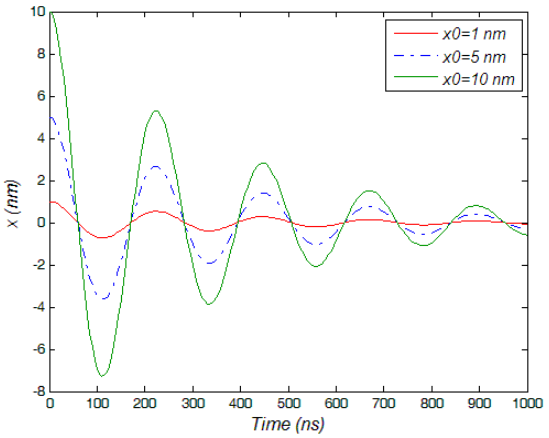


(b) *Cd=5N.s/m*

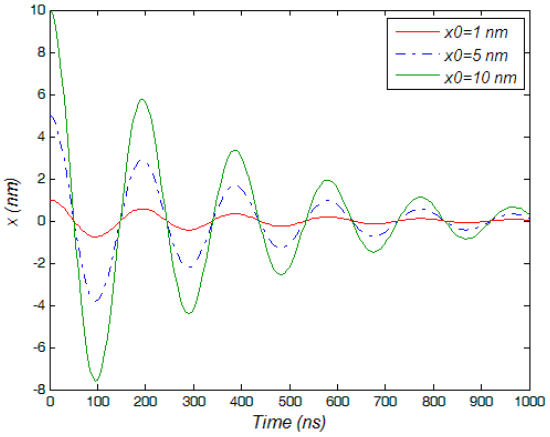
Fig. 5. Variation of the time-history with the external (a) and/or internal (b) damping coefficients (e_0a=1nm, l=2h, m=1, k_G=1GN, k_w=1GPa, L=10d, Cd=5N.s/m).



(a) *Cd=5N.s/m, g=5N.s/m*



(b) *Cd=0 N.s/m, g=5N.s/m*



(c) *Cd=5N.s/m, g=0N.s/m*

Fig. 6. Variation of the time-history with the initial amplitude (e_0a=1nm, l=2h, m=1, k_G=1GN, k_w=1GPa, L=10d).