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# Uncertainty estimation of loop impedance measurement determined by the vector method

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**Abstract**

This paper presents a detailed analysis of the uncertainty estimation of loop impedance measurement determined by the vector method. The analysis includes the following estimates: resistance variance, voltage variance and time measurement variance. The paper presents a methodology for estimating the combined standard uncertainty of the loop impedance.

**Keywords**: loop impedance, vector method, measurement uncertainty, analysis, measurement, meter.

## Szacowanie niepewności pomiaru impedancji pętli zwarciowej wyznaczonej metodą wektorową

**Streszczenie**

Metoda wektorowa umożliwia wyznaczenie impedancji pętli zwarcia w oparciu o składowe ortogonalne napięcia. Autorka niniejszego referatu przeprowadziła analizę metrologiczną tej metody stosując teorię błędów [1, 2, 3, 4, 5]. Aktualnie przeprowadzane są szczegółowe analizy szacowania niepewności pomiaru impedancji pętli zwarciowej metodą wektorową. W niniejszym artykule zaprezentowano metodę wektorową, zgodnie z którą impedancja pętli zwarciowej wyznaczana jest na podstawie pomiaru fazorów dwóch napięć [2, 3, 4, 5]: napięcia $\underline{E}$ panującego w miejscu badanym przy odłączonym obciążeniu oraz napięcia $\underline{U}$ – jakie występuje przy włączonym obciążeniu pomiarowym o znanej wartości rezystancji $R_0$. Referat zawiera wyniki szczegółowego szacowania niepewności pomiaru impedancji pętli zwarcia uwzględniając wariancję rezystora obciążenia pomiarowego $u^2(R_0)$, wariancję pomiaru napięć $u^2(E)$ i $u^2(U)$ oraz wariancję wyznaczenia chwil czasowych $u^2(t_1)$, $u^2(t_2)$, $u^2(t_3)$, $u^2(t_4)$. Wyznaczona wartość niepewności standardowej $u_c$, potwierdza, iż metoda wektorowa jest znacznie bardziej dokładna od metod klasycznych wyznaczania impedancji pętli zwarcia. Wyniki oszacowanej niepewności pomiaru są zbieżne z wynikami uzyskanymi przez autorkę referatu z eksperymentu, w którym wykorzystano prototyp miernika impedancji pętli realizującego metodę wektorową [4].

**Słowa kluczowe**: impedancja pętli, metoda wektorowa, niepewność pomiaru.

## 1. Introduction

The vector method is a method for determining loop impedance based on the orthogonal voltage components. The advantages of this method are as follows: much more accurate measurement of loop impedance in comparison with classical methods as well as pure resistant measurement load. Thanks to that, a meter used in this method is characterized by a smaller size and lower weight than the one used in classical methods. The paper presents the results of total uncertainty estimation of loop impedance measurement determined by the vector method.

## 2. The vector method

A measured value of loop impedance is expressed with the formula: $\underline{Z} = R + j\,X$, where $R$ stands for resistance and $X$ stands for inductive reactance. Fig. 1 shows the equivalent tested circuit.
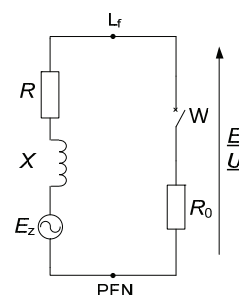


Fig. 1.    The principle of loop impedance measurement [4]
Rys. 1.    Zasada pomiaru impedancji pętli zwarciowej [4]

This circuit consists of a series connection of an ideal sinusoidal supplying voltage source $E_z$ (with constant amplitude and frequency $f$), the loop impedance $Z$ and resistance load $R_0$ [2, 3, 4, 5].

A measured value of loop impedance by the vector method is calculated on the basis of: the orthogonal voltage components [2, 3, 4, 5] the voltage $\underline{E}$ measured with the disconnected load $R_0$, and the voltage $\underline{U}$ - measured when the load $R_0$ is switched on, and equals one of the known resistance values $R_0$.

Voltage vector measurement: the voltage $\underline{E}$ before, and the voltage $\underline{U}$ after switching on the load resistance $R_0$ - consists in determining the orthogonal components of the two voltages: $E_b$, $E_c$, $U_b$, $U_c$ (Fig. 2).
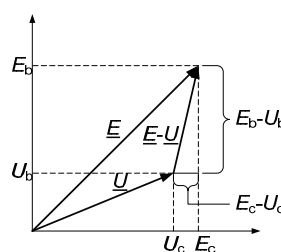


Fig. 2.    The orthogonal voltage diagram – the vector method [5]
Rys. 2.    Fazorowy wykres napięć - metoda wektorowa [5]

On the basis of the four momentary voltage values, one can calculate the values of the voltage vectors $\underline{E}$ and $\underline{U}$, using the following equations:

$$E_c + j\,E_b = E\cos\omega t + j\,E\sin\omega t = \underline{E}\,, \qquad (1)$$

$$U_c + j\,U_b = U\cos\omega t + j\,U\sin\omega t = \underline{U}\,. \qquad (2)$$

The voltage vector $\underline{E}$ is determined on the basis of two momentary values of no load voltage source: $E_b$ i $E_c$. The first value of $E_b$ obtained at time $t_1$ is determined by the following formula:

$$E_b = e(t_1) = \sqrt{2}\,E\sin\omega t_1\,. \qquad (3)$$

The second sample voltage $E_c$ obtained at time $t_2$ shifted by $0.25T$ ($T$- the signal period) over the first $t_1$, that is $t_2=t_1+0.25T$:

$$E_c = e(t_2) = \sqrt{2}\, E\, cos\omega t_1 . \qquad (4)$$

The same procedure is used when determining the phasor $\underline{U}$, which is calculated on the basis of two momentary voltage values: $U_b$ and $U_c$ – with the resistance $R_0$ (Fig. 1), whereas these values are shifted relative to $E_b$ and $E_c$ for $mT$ ($m = 1,2,3…$), that is $t_3=t_1+mT$. This delay has been introduced in order to eliminate the transient states caused by the switching measurement load $R_0$. The voltage $U_b$ taken at time $t_3$ is defined by the following dependence:

$$U_b = u(t_3) = \sqrt{2}\, \frac{ER_0}{\sqrt{(R+R_o)^2 + X^2}}\, sin\omega(t_1+\varphi), \qquad (5)$$

where $\varphi$ is a phase angle for the tested circuit of Fig. 1, defined as:

$$tg\varphi = \frac{X}{R + R_0} . \qquad (6)$$

While the voltage $U_c$, determined at time $t_4=t_1+0.25T+mT$ (analogous to the voltage value $E_c$), is described by the formula:

$$U_c = u(t_4) = \sqrt{2}\, \frac{ER_0}{\sqrt{(R+R_o)^2 + X^2}}\, cos\omega(t_1+\varphi). \qquad (7)$$

On the basis of the equations describing the voltage state of the tested circuit (Fig. 1), and knowing the dependencies determining the instantaneous values of voltage $E_b$, $E_c$, $U_b$ and $U_c$, a loop impedance module can be determined [2, 3, 4] as follows:

$$Z_w = R_0 \sqrt{\frac{(E_c-U_c)^2 + (E_b-U_b)^2}{U_c^2 + U_b^2}} \qquad (8)$$

## 3. Uncertainty estimation of loop impedance measurement

The combined standard uncertainty of impedance $Z_w$ was determined on the basis of a measurement function, which depends on the following parameters: $Z_w = f(R_0, E, U, t_1, t_2, t_3, t_4)$.

According to the principle of uncertainty propagation [6, 9], the combined standard uncertainty $u_c$ of determining the loop impedance is defined as follows (assuming no correlation between the uncertainties of the measured values):

$$u_c = \sqrt{\begin{array}{l} \left|\frac{\partial Z_w}{\partial R_0}\right|^2 u^2(R_0) + \left|\frac{\partial Z_w}{\partial E}\right|^2 u^2(E) + \\[2mm] + \left|\frac{\partial Z_w}{\partial U}\right|^2 u^2(U) + \left|\frac{\partial Z_w}{\partial t_1}\right|^2 u^2(t_1) + \\[2mm] + \left|\frac{\partial Z_w}{\partial t_2}\right|^2 u^2(t_2) + \left|\frac{\partial Z_w}{\partial t_3}\right|^2 u^2(t_3) + \\[2mm] + \left|\frac{\partial Z_w}{\partial t_4}\right|^2 u^2(t_4) \end{array}} \qquad (9)$$

The uncertainty analysis was conducted under the following assumptions:

- the measured loop impedance Z consists of resistance and inductive reactance. In practice, the values of the loop impedance are usually smaller than 1 Ω, hence it was assumed that Z is equal to 1Ω.
- the supplying voltage source $E_z$ is an ideal sine with constant amplitude and frequency during the measurement. Typically, in a low voltage network, the supply voltage is equal to 230V of frequency 50 Hz.
- the tested circuit is linear when the load $R_0$ is disconnected,
- the voltage $\underline{U}$ is measured in steady state; m=6.
- the measurement current should be of the largest possible value, therefore load $R_0$ equal to 10 Ω was used for computing in the meter prototype.
- the limiting error of the resistor $R_0$ includes the temperature drift.

The first step of the uncertainty analysis was to estimate the sensitivity of the vector method to main error sources: the load resistor $R_0$, voltages measurement $\underline{E}$ and $\underline{U}$, determination of the instantaneous moments of $t_1$, $t_2$, $t_3$, $t_4$. The weight coefficients in equation (9) (partial derivatives) were determined and are given in Table 1.

Tab. 1. Values of partial derivatives counted for the following data: $E_z$=230 V, $Z$=1 Ω, $R_0$=10 Ω, $t_1$=10 µs
Tab. 1. Wartości pochodnych cząstkowych wyznaczone dla danych: $E_z$=230 V, $Z$=1 Ω, $R_0$=10 Ω, $t_1$=10 µs

| | Partial derivatives | $\phi$ [°] | | |
|---|---|---|---|---|
| | | 0 | 45 | 75 |
| 1 | $\left\|\frac{\partial Z_w}{\partial R_0}\right\|^2 [-]$ | 1.00E-02 | | |
| 2 | $\left\|\frac{\partial Z_w}{\partial E}\right\|^2 \left[\frac{\Omega^2}{V^2}\right]$ | 2.23E-03 | 1.56E-03 | 4.83E-04 |
| 3 | $\left\|\frac{\partial Z_w}{\partial U}\right\|^2 \left[\frac{\Omega^2}{V^2}\right]$ | 1.81E-04 | 1.22E-03 | 3.11E-04 |
| 4 | $\left\|\frac{\partial Z_w}{\partial t_1}\right\|^2 \left[\frac{\Omega^2}{s^2}\right]$ | 2.70E-02 | 7.10E-02 | 6.40E-02 |
| 5 | $\left\|\frac{\partial Z_w}{\partial t_2}\right\|^2 \left[\frac{\Omega^2}{s^2}\right]$ | 2.70E-02 | 1.92E-03 | 2.63E-03 |
| 6 | $\left\|\frac{\partial Z_w}{\partial t_3}\right\|^2 \left[\frac{\Omega^2}{s^2}\right]$ | 2.70E-02 | 7.50E-02 | 7.50E-02 |
| 7 | $\left\|\frac{\partial Z_w}{\partial t_4}\right\|^2 \left[\frac{\Omega^2}{s^2}\right]$ | 2.70E-02 | 2.62E-03 | 8.60E-04 |

The angle $\phi$, stands for the argument of the loop impedance $Z$, and is defined by the following formula:

$$\phi = arctg\frac{X}{R} . \qquad (10)$$

The next step of the uncertainty analysis was to estimate the following variances:
a) the load measurement resistor $u^2(R_0)$,
b) voltage measurement $u^2(E)$ and $u^2(U)$,
c) determination of the instantaneous moments of $u^2(t_1)$, $u^2(t_2)$, $u^2(t_3)$, $u^2(t_4)$.

The resistance variance $u^2(R_0)$ was estimated under the assumption of the uniform probability distribution [7] of the fabrication tolerance of the load measurement resistance $R_0$ as:

$$u^2(R_0) = \left(\frac{\Delta R_0}{\sqrt{3}}\right)^2, \qquad (11)$$

where $\Delta R_0$ is the limiting error of the resistor $R_0$, resulting from the fabrication tolerance of the load measurement resistor $R_0$. It is associated with the accuracy class specified by a resistor producer.

When $R_0$ is equal to 10 $\Omega$, and it is made with the fabrication tolerance of 0.1%, the variance $u^2(R_0)$ is equal to 0,03 m$\Omega^2$.

The voltage variance $u^2(E)$ defined as a sum of the voltage variance $u^2(\Delta E_{przetw})$ resulting from the processing error of the measurement transducer and the voltage variance $u^2(\Delta E_{DAQ})$ determined by the Data Aquision Card, can be given by:

$$u^2(E) = u^2(\Delta E_{przetw}) + u^2(\Delta E_{DAQ}). \qquad (12)$$

The voltage variance $u^2(\Delta E_{przetw})$ resulting from the processing error of the voltage transducer was determined under the assumption of the uniform probability distribution of the sensor processing error $\Delta E_{przetw}$

$$u^2(\Delta E_{przetw}) = \left( \frac{\sqrt{(E_{in})^2 + (\delta k \cdot E_{in})^2} - E_{in}}{\sqrt{3}} \right)^2. \qquad (13)$$

where $\delta k$ is a determined percentage of the rms value of the input voltage $E_{in}$ (this value is specified by a transducer producer).

The voltage variance $u^2(\Delta E_{DAQ})$ was determined under the assumption of the uniform probability distribution of the a/c conversion error $\Delta E_{DAQ}$, as:

$$u^2(\Delta E_{DAQ}) = \left( \frac{\Delta E_{DAQ}}{\sqrt{3}} \right)^2, \qquad (14)$$

where the limiting error $\Delta E_{DAQ}$ is defined by the following formula [4]:

$$\Delta E_{DAQ} = \pm \begin{bmatrix} (E_{inDAQ} \cdot \delta E_{\%R}) + \\ + (\Delta E_{offset} + \Delta E_{noise+quant} + \Delta E_{temp}) \end{bmatrix}. \qquad (15)$$

It is assumed that the a/c conversion error $\Delta E_{DAQ}$ depends on: the input signal level $E_{inDAQ}$, the relative error $\delta E_{\%R}$ (depending on the actual measured value), the additive error of zero offset $\delta E_{offset}$, quantization and noise error $\delta E_{noisequant}$ and the temperature drift $\delta E_{temp}$.

When considering the following hardware configuration of the measurement system: a meter for measuring the loop impedance as a voltage transducer - the transducer LEM CV-500, a data acquisition card - the National Instruments PCI – 6024E, the voltage variance $u^2(\Delta E_{przetw})$ equals 0.018 $\mu V^2$ and variance $u^2(E_{DAQ})$ for the input voltage $E_{inDAQ}$=10 V is 0.125 mV$^2$.

The total uncertainty voltage $u(E)$ calculated by means of formula (12), for the previously assumed conditions of the measurement, is $\pm$ 11.2 mV, while the measurement voltage is equal to 230 V. Due to the fact that the voltage $U$ - with the load resistor $R_0$ connected - is measured and processed by the same circuits as the measured voltage $E$, it was assumed that the variance $u^2(U) = u^2(E)$.

In order to estimate the total measurement uncertainty of the loop impedance by the vector method, it was also necessary to determine the time measurement variance $u^2(t_1)$, $u^2(t_2)$, $u^2(t_3)$, $u^2(t_4)$, in which the voltage samples were taken. The moments of the time variance, in the general form $u^2(t_l)$, were estimated by means of the uniform probability distribution of the time measurement error, as:

$$u^2(t_l) = \left( \frac{\Delta t_l}{\sqrt{3}} \right)^2, \qquad (16)$$

where $l$ =1, 2, 3, 4 appropriate for the sample voltages $E_b$, $E_c$, $U_b$, $U_c$.

The accuracy of determining the time instants in which the momentary voltage values are measured, depends on the period measurement error $\Delta T$, the sampling time $T_p$ and location of the samples [4]:

$$\Delta t_l = \frac{t_l}{N} \Delta T + T_p. \qquad (17)$$

The equation (16) shows that the limiting error grows linearly with increase in the distance from the first sample. Assuming that $T_p$=0.5$\mu$s, $N$ = 20000 (the period contains 200 samples, which is sufficient for an ideal sine signal), $\Delta T$=0.3$\mu$s and concluding that $t_1$=20$T_p$, $t_2$=70$T_p$, $t_3$=1220$T_p$, $t_4$=1270$T_p$, gives $\Delta t_1$=$\Delta t_2$=0.5$\mu$s, $\Delta t_3$= $\Delta t_4$=0.5002$\mu$s. The variance of the measurement time $u^2(t_1)$ is equal to 80 fs$^2$.

The combined standard uncertainty of the loop impedance measurement $u_c$ was determined using equation (9) by means of computer simulations in the MathCad, for the following instrument configuration: the load resistor $R_0$ with a resistance of 10 $\Omega$ and maximum relative error 0.1%, LEM CV-500 transducer, National Instruments PCI – 6024E Data Acquision Card.

Fig. 3 illustrates the dependence of the measurement uncertainty $u_c$ on the argument of the loop impedance $\phi$.

The value of measurement uncertainty $u_c$ decreases with increase in the loop impedance argument $\phi$. The smallest value of the measurement uncertainty $u_c$ $\pm$0.658 m$\Omega$, for argument $\phi$ equal to 75$^o$, when the measurement is taken near the transformer (inductive nature of the loop impedance $Z$).

The maximum value of the impedance measurement uncertainty $u_c$ $\pm$0.916 m$\Omega$, occurs in the case of a fault loop resistance, that is when argument $\phi$ equals 0$^o$.



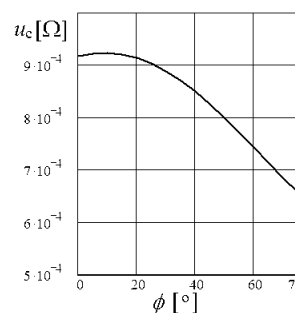Fig. 3. Graph $u_c$=f($\phi$), for data $E_z$=230 V, $Z$=1 $\Omega$, $R_0$=10 $\Omega$
Rys. 3. Przebieg $u_c$=f($\phi$), dla danych $E_z$=230 V, $Z$=1 $\Omega$, $R_0$=10 $\Omega$

Fig. 4. shows the elements of the impedance measurement uncertainty $u_c$, according to formula (9), connected with: the load measurement resistor $R_0$, voltage measurement $E$ and $U$ and determination of the instantaneous moments of $t_1$, $t_2$, $t_3$, $t_4$.
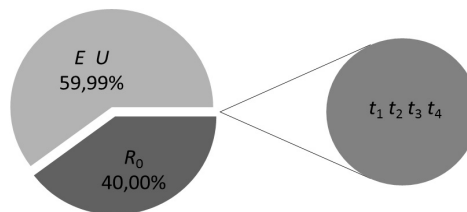


Fig. 4. Share of particular components of impedance measurement uncertainty for data $E_z$=230V, $Z$=1 $\Omega$, $R_0$=10 $\Omega$, $\phi$=0°
Rys. 4. Udział poszczególnych składowych niepewności pomiaru $u_c$ impedancji pętli zwarcia, dla danych $E_z$=230 V, $Z$=1 $\Omega$, $R_0$=10 $\Omega$, $\phi$=0°

The most essential uncertainty factor is the uncertainty of voltage $\underline{E}$ and $\underline{U}$ measurements, which proves that the voltage measurement system of the meter prototype should be characterized by high precision.

The load resistor should also be chosen as a high precision one.

## 4. Conclusions

This paper presents a methodology for estimating the combined standard uncertainty of loop impedance by the vector method. The vector method allows determining the loop impedance based on the orthogonal voltage components, which makes a loop impedance measurement much more accurate than in the case of conventional methods. The estimated value of the combined standard uncertainty $u_c$, confirms that the vector method is more accurate than the classical ones [2, 3].

The results of the uncertainty estimated from MathCad simulations were compared with the experimental results obtained by the author of this paper. A loop impedance meter prototype implemented with the vector method was used in the experiment. The meter prototype consisted of: the LEM CV-500 transducer, the PCI 6024E data acquisition card and the load resistor 10 Ω of accuracy class 0.1. The simulation and experimental results (when $E_z$=230 V, $Z$=1 Ω) are convergent. For these measurement conditions, the loop impedance was calculated as (1000.00±1.83) mΩ for the coverage factor $k = 2$ at the confidence level of 95%.

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