UNUSUAL STREAMING IN CHEMICALLY REACTING GASES

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Nonlinear stimulation of the vorticity mode caused by losses in the momentum of sound in a chemically reacting gas, is considered. The instantaneous dynamic equation which describes the nonlinear generation of the vorticity mode, is derived. It includes a quadratic nonlinear acoustic source. Both periodic and aperiodic sound may be considered as the origin of the vorticity flow. In the non-equilibrium regime of a chemical reaction, sound and its nonlinear effects behave unusual. There may exist vortices whose direction of rotation is opposite to that of the vortices in the standard thermoviscous flows. This is illustrated by example relating to periodic sound. The theory and examples consider cases of both equilibrium and non-equilibrium regime of a chemical reaction.

INTRODUCTION BASIC EQUATIONS AND STARTING POINTS

Investigations of the acoustic stability of flow in the thermodynamically non-equilibrium gas began in connection with the advances in laser engineering and plasma aerodynamics. The hydrodynamics of the non-equilibrium fluids is one of new fields of modern hydrodynamics. The studies in this field started in sixties of the 20th century and now they are passing through the stage of the fundamental equations formulating and revealing new physical effects. Interest in non-equilibrium phenomena in the physics of gases was originated by observation of anomalous dispersion and absorption of ultrasonic waves in a gas having non-equilibrium internal degrees of freedom. A reason for these anomalies is the mechanism of retarded energy exchange between the internal and translational degrees of freedom of the molecules [1, 2].

This paper is devoted to vorticity flow in the field of sound. Its reason is nonlinear loss in the momentum of the sound wave. Aperiodic in time acoustic source can generate the mean motion, a phenomenon known as acoustic streaming [1, 2]. Although acoustic streaming was deeply studied both theoretically and experimentally for over a century, the spatial and temporal distribution of radiation force as function of diffraction, absorption and geometry of a flow, are still poor analyzed. The main difficulty is the nonlinear origin of the phenomenon. The second origin acoustic and non-acoustic modes interaction is absorption. It is evident that any relaxation process contributes to the vortices caused by sound and to the streaming, in particular. The relaxation processes of different types manifest themselves, among other attributes, through absorption.

We start from the linear determination of modes as specific types of gas motion in a gas where a chemical reaction takes place (Sec.1). The definition of any mode fixes the relations of the dynamic perturbations belonging to this mode. That is necessary for correct decomposition of equations governing sound and non-acoustic modes accounting for the interaction of all modes in a weakly nonlinear flow. That gives also possibility taking into account fluid compressibility – the traditional analysis is limited by incompressible fluid (see discussion on this topic in [3, 4]). Presented decomposition of different types of motion of the overall perturbation and every governing equations (witch are true also in a weakly nonlinear flow), is instantaneous and do not need time averaging over sound period, so it undoubtedly have an advantage over the traditional analysis, witch is valid for only periodic sound [2, 5]. Consequently, under some assumptions the equations can be simplified. For example, the diminutive parameter M (the Mach number) and a weak diffraction of a sound beam, will be employed, along with some other ones connected with chemical reaction.

1. DYNAMIC EQUATIONS. DISPERSION RELATIONS

The present study considers the simplest model of gas dynamics where a chemical reaction of the $A \to B$ type takes place. This model applies also for description of acoustical properties of reacting media with complex, branching reactions [6]. There are three equations, momentum, energy, and continuity ones:

$$\begin{gathered} \frac{d\rho}{dt} + \rho \nabla \mathbf{v} = 0 \\ \rho \frac{d\mathbf{v}}{dt} = -\nabla P \\ \frac{C_{V,\infty}}{R} \frac{dT}{dt} - \frac{T}{\rho} \frac{d\rho}{dt} = Q \end{gathered} \tag{1}$$

In the equations above, $\mathbf{v}$ denotes gas velocity, ρ, P denote density and pressure of a gas, T is temperature measured in Joules per molecule (actually the ordinary temperature multiplied by the Boltsmann constant k_B), $C_{V,\infty}, C_{P,\infty}$ are the "frozen" heat capacity at constant volume and constant pressure, respectively correspondent processes take place at infinitely high frequencies), $R = C_{P,\infty} - C_{V,\infty}$ is the universal gas constant, $Q = HmW / \rho$ designates the heat produced in a medium per one molecule due to a chemical reaction (W is the volume rate of formation of the reaction product B), H denotes the reaction enthalpy per unit mass of reagent A, and m denotes the averaged molecular mass. The relaxation equation of the mass fraction Y of reagent A, and the equation of state complement the system (1):

$$\frac{dY}{dt} = -\frac{W}{\rho}, \qquad P = \frac{\rho T}{m} \tag{2}$$

Eqs (1) do not account for standard attenuation due to shear viscosity and thermal conductivity.

1.1 EQUATIONS GOVERNING SOUND AND THE VORTICITY MODE IN A WEAKLY NONLINEAR FLOW

Let consider two-dimensional gas flow of infinitely small amplitude in the plane OXY. Every quantity ζ represents a sum of unperturbed value ζ_0 and its variation ζ', ($|\zeta'|<<|\zeta_0|$). Following Molevich [7, 8], we assume that the stationary quantities Y_0, T_0, P_0, ρ_0 are maintained by a transverse pumping, so that in the longitudinal direction pointed by plane OXY, the background is homogeneous. The governing system (1), with account for (2) within accuracy of quadratic nonlinear terms is rearranged in two dimensions into the following system:

$$
\begin{aligned}
&\frac{\partial v_x}{\partial t}+\frac{T_0}{m\rho_0}\frac{\partial \rho'}{\partial x}+\frac{1}{m}\frac{\partial T'}{\partial x}=-(\mathbf{v}\nabla)v_x+\frac{T_0\rho'}{m\rho_0}\frac{\partial\rho'}{\partial x}-\frac{T'}{m\rho_0}\frac{\partial\rho'}{\partial x},\\
&\frac{\partial v_y}{\partial t}+\frac{T_0}{m\rho_0}\frac{\partial \rho'}{\partial y}+\frac{1}{m}\frac{\partial T'}{\partial y}=-(\mathbf{v}\nabla)v_y+\frac{T_0\rho'}{m\rho_0}\frac{\partial\rho'}{\partial y}-\frac{T'}{m\rho_0}\frac{\partial\rho'}{\partial y},\\
&\frac{\partial T'}{\partial t}+(\gamma_\infty-1)\left(T_0\frac{\partial v_x}{\partial x}+T_0\frac{\partial v_y}{\partial y}-Q_T\frac{Q_0}{T_0}T'-Q_\rho\frac{Q_0}{\rho_0}\rho'-Q_Y\frac{Q_0}{Y_0}Y'\right)=\\
&=-(\mathbf{v}\nabla)T'-(\gamma_\infty-1)T'(\nabla\mathbf{v}),\\
&\frac{\partial Y'}{\partial t}+\frac{1}{Hm}\left(Q_T\frac{Q_0}{T_0}T'+Q_\rho\frac{Q_0}{\rho_0}\rho'+Q_Y\frac{Q_0}{Y_0}Y'\right)=-(\mathbf{v}\nabla)Y',\\
&\frac{\partial\rho'}{\partial t}+\rho_0\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)=-(\mathbf{v}\nabla)\rho'-\rho'(\nabla\mathbf{v})
\end{aligned}
\tag{3}
$$

where $\gamma_\infty=\dfrac{C_{P,\infty}}{C_{V,\infty}}$ denotes the frozen adiabatic exponent, and quantities Q_T, Q_ρ, Q_Y are evaluated in the equilibrium state:

$$
Q_T=\frac{T_0}{Q_0}\left(\frac{\partial Q}{\partial T}\right)_{T_0,\rho_0,Y_0},\quad Q_\rho=\frac{\rho_0}{Q_0}\left(\frac{\partial Q}{\partial \rho}\right)_{T_0,\rho_0,Y_0},\quad Q_Y=\frac{Y_0}{Q_0}\left(\frac{\partial Q}{\partial Y}\right)_{T_0,\rho_0,Y_0}
\tag{4}
$$

∇ in two dimensions denotes $\vec{i}\dfrac{\partial}{\partial x}+\vec{j}\dfrac{\partial}{\partial y}$ ($\vec{i}$ and $\vec{j}$ are correspondent basis vectors). Actually, the nonlinear right-hand parts of Eqs (3) include also terms involving the second-order derivatives of Q: $\dfrac{\partial^2 Q}{\partial T^2}$ and so on, which are not taken into account by the present study, confining the changes in $Q(\rho,T,Y)$ by differential and hence restricting the accuracy of conclusions.

1.2 DISPERSION RELATIONS

An excess pressure in the first two equations from (3) is expressed in terms of excess density and temperature in accordance with equation of state (the second one from (2)). Studies of motions of infinitely-small amplitudes begin usually with representing of all perturbations as planar waves:

$$
\varepsilon'(x,y,t)=\tilde{\varepsilon}(k_x,k_y,t)\exp i(\omega t-k_x x-k_y y)
\tag{5}
$$

Some intermediate steps are necessary to determine the heat capacities under constant pressure, C_P, or under constant volume, C_V. Both these quantities depend on frequency

$$C_P = C_{P,\infty} + mHR\left(\frac{\partial Y}{\partial T}\right)_P, \quad C_V = C_{V,\infty} + mHR\left(\frac{\partial Y}{\partial T}\right)_V \tag{6}$$

and participate in the dispersion equation, whose roots determine all possible types of motion in a reacting gas. From the fourth equation from the system (3) and the equation of state the equalities below follow:

$$\left(\frac{\partial Y}{\partial T}\right)_V = -\frac{Q_T}{Q_Y(1+i\omega\tau_c)}\frac{Y_0}{T_0}, \quad \left(\frac{\partial Y}{\partial T}\right)_P = \frac{(Q_\rho - Q_T)}{Q_Y(1+i\omega\tau_c)}\frac{Y_0}{T_0} \tag{7}$$

where $V = 1/\rho$ is the specific gas volume,

$$\tau_c = \frac{HmY_0}{Q_0 Q_Y} \tag{8}$$

is the characteristic duration of chemical reaction.

The dispersion equation determining two acoustic (wave) types of motion and three non-wave ones, takes finally the form:

$$\omega^2\left(\omega^3 - i\frac{C_{V,0}}{C_{V,\infty}\tau_c}\omega^2 - c_\infty^2\widetilde{\Delta}\omega + i\frac{C_{P,0}}{C_{P,\infty}\tau_c}c_\infty^2\widetilde{\Delta}\right) = 0 \tag{9}$$

where $\widetilde{\Delta} = k_x^2 + k_y^2$, $c_\infty = \sqrt{\gamma_\infty \frac{T_0}{m}}$ is the frozen linear sound velocity, and $C_{P,0}, C_{V,0}$ denote the low-frequency heat capacities

$$C_{P,0} = C_{V,\infty}\left(\gamma_\infty + \frac{(\gamma_\infty - 1)Q_0\tau_c(Q_\rho - Q_T)}{T_0}\right), \quad C_{V,0} = C_{V,\infty}\left(\gamma_\infty - \frac{(\gamma_\infty - 1)Q_0\tau_c Q_T}{T_0}\right) \tag{10}$$

The approximate roots of dispersion relations for acoustic branches in one dimension were firstly derived and adequately studied by Molevich [8]. There are five roots of dispersion relation in two-dimensional flow, two acoustic modes indexed by 1 and 2, and three non-wave ones. The third non-acoustic root refers to relaxation due to chemical reaction, its approximate value depends on a spatial scale of perturbation. The last two roots (the fourth denoting the thermal mode, and the fifth denoting the vorticity mode) equal zero

$$\omega_4 = 0, \quad \omega_5 = 0 \tag{11}$$

The vorticity mode appears as one of possible types of motions in flows exceeding one dimension. As for the both branches of sound, the dispersion relations depend on a ratio of sound period and the characteristic duration of chemical reaction τ_c.

1.3 DISPERSION RELATIONS FOR THE HIGH-FREQUENCY SOUND

In this section we consider only limiting case relates to the domain of acoustic frequency large compared to the reverse duration of chemical reaction:

$$\frac{1}{|\,\omega_{1,2}\tau_c\,|} \approx \frac{1}{|\,c_\infty\sqrt{\widetilde{\Delta}}\tau_c\,|} \equiv \delta_\infty << 1 \tag{12}$$

The next condition reminds that sound is a wave process, so that dispersion and attenuation during the sound period are small, where D denotes the dispersion:

$$|\,D\delta_\infty\,| << \left|\frac{C_{V,\infty}}{C_{V,0}}\right|, \quad D = \frac{c_\infty^2 - c_0^2}{c_\infty^2} \tag{13}$$

And $c_0 = \sqrt{\dfrac{C_{P,0}T_0}{C_{V,0}m}}$ denotes a linear sound velocity at very low frequencies. The inequality (13) is valid if

$$|\,Q_0(Q_\rho + (\gamma_\infty - 1)Q_T)\,| << \left|\frac{\gamma_\infty T_0}{\delta_\infty(\gamma_\infty - 1)\tau_c}\right| \tag{14}$$

In view of Eqs (12),(13), the high-frequency acoustic roots of the dispersion equation (9) in the leading order take the form

$$\omega_1 = c_\infty\sqrt{\widetilde{\Delta}} + i\frac{D}{2}\frac{C_{V,0}}{C_{V,\infty}\tau_c}, \quad \omega_2 = -c_\infty\sqrt{\widetilde{\Delta}} + i\frac{D}{2}\frac{C_{V,0}}{C_{V,\infty}\tau_c} \tag{15}$$

Amplitudes of excess acoustic quantities increase if

$$Q_0(Q_\rho + (\gamma_\infty - 1)Q_T) > 0 \tag{16}$$

and decrease otherwise. The inequality (16) determines the area of irreversibility of a chemical reaction, it establishes also the inequality as follows [8]:

$$c_\infty^2 - c_0^2 = \frac{T_0}{m}\left(\frac{C_{P,\infty}}{C_{V,\infty}} - \frac{C_{P,0}}{C_{V,0}}\right) = \frac{(\gamma_\infty - 1)Q_0(Q_\rho + (\gamma_\infty - 1)Q_T)T_0\tau_c}{m(Q_0Q_T\tau_c(\gamma_\infty - 1) - T_0)} < 0 \tag{17}$$

if $C_{V,0} > 0$. Therefore, the sign of D determines an equilibrium (positive D) or non-equilibrium, irreversible chemical reaction (negative D).

1.4 DEFINITION OF MODES

Substitution of approximate roots of the dispersion equation in linear part of Eqs (3), readily produces the relationships of the Fourier-transforms of perturbations specific for every mode. The relations for both acoustic branches determined by roots ω_1 and ω_2, are following:

$$\psi_i = \begin{pmatrix} \widetilde{v}_{x,i} \\ \widetilde{v}_{y,i} \\ \widetilde{T}_i' \\ \widetilde{Y}_i' \\ \widetilde{\rho}_i' \end{pmatrix} = \begin{pmatrix} \dfrac{\omega_i k_x}{\widetilde{\Delta}} \\ \dfrac{\omega_i k_x}{\widetilde{\Delta}} \\ T_0(\gamma_\infty - 1 - (\gamma_\infty - \gamma_0)\hat{R}) \\ \dfrac{c_\infty^2 - c_0^2}{H(\gamma_\infty - 1)}\hat{R} \\ \rho_0 \end{pmatrix} \frac{\widetilde{\rho}'_i}{\rho_0} \tag{18}$$

where i=1,2, and $\hat{R}$ denotes a operator applying at a scalar function $\phi(x,y,t)$:

$$\hat{R}\phi = \frac{C_{V,0}}{C_{V,\infty}\tau_c}\int_{-\infty}^{t}\phi e^{-(t-t')\frac{C_{V,0}}{C_{V,\infty}\tau_c}}dt' \tag{19}$$

The relations of perturbations in the vorticity mode are free of partial derivatives with respect to coordinates and therefore do not refer to the frequency. The latter is determined by relationships as follows:

$$\nabla \mathbf{v}_5 = 0, \quad \widetilde{T}_5' = 0, \quad \widetilde{Y}_5' = 0, \quad \widetilde{\rho}_5' = 0 \tag{20}$$

The velocity field of the both sound modes (also the third and fourth ones) are potential so $\nabla \times \mathbf{v_n} = \mathbf{0}$ n=1,..,4, and the last one is rotational in accordance to Eqs (20). The overall linear velocity is a sum of all specific parts:

$$\mathbf{v} = \sum_{n=1}^{5}\mathbf{v}_n \tag{21}$$

The linear flow may be uniquely decomposed into individual modes at any time. That may be proceeded by use of a set of matrix projectors. The matrix projectors were derived and exploited by one of the authors in some problems of nonlinear hydrodynamics in media with standard absorption [9, 10]. For example, in order to decompose the vorticity part from the overall vector of velocity, it is sufficient to apply the operator $\mathbf{P_v}$ on the vector of the Fourier-transforms of velocity components:

$$\widetilde{P}_v\begin{pmatrix}\widetilde{v}_x\\ \widetilde{v}_y\end{pmatrix} = \frac{1}{\widetilde{\Delta}}\begin{pmatrix}k_y^2 & -k_xk_y\\ -k_xk_y & k_y^2\end{pmatrix}\begin{pmatrix}\widetilde{v}_x\\ \widetilde{v}_y\end{pmatrix} \tag{22}$$

Operating in the (x,y) space $\mathbf{P_v}$ satisfies the equality

$$P_v\Delta = \begin{pmatrix}\dfrac{\partial^2}{\partial y^2} & -\dfrac{\partial^2}{\partial x\partial y}\\ -\dfrac{\partial^2}{\partial x\partial y} & \dfrac{\partial^2}{\partial x^2}\end{pmatrix} \tag{23}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, denotes the Laplacian operating in the (x,y) space.

Determination relation of other non-acoustic modes (relaxation and thermal modes) was derived by the authors in [11].

2. EQUATIONS GOVERNING SOUND AND THE VORTICITY MODE IN A WEAKLY NONLINEAR FLOW

2.1 DECOMPOSING OF EQUATIONS FOR INTENCE SOUND AND THE VORTICITY MODE. ACOUSTIC STREAMING

In studies of weakly nonlinear dynamics, we still fix linear relations of perturbations in accordance to Eqs (18, 20) and will consider every field perturbation as a sum of perturbations of different modes. The main idea is to decompose equations governing different modes by applying of correspondent projector at the system including weakly nonlinear terms like (3) [9, 10]. Every equation includes nonlinear terms of order not lower than M^2 relating to all modes and reflecting the nonlinear interactions of modes in a weakly nonlinear flow. The solution of the final dynamic equations depends on contribution of every mode in the overall field perturbation. Let the progressive in the positive direction sound be intense as compared to all other modes. That means that the characteristic amplitude of velocity associated with the first branch of sound in the considered domain is much greater than that of other modes:

$$\max|v_1| >> \max|v_n|, \quad n = 2,3,4,5, \tag{24}$$

We will keep only dominative terms correspondent to the rightwards progressive sound in the nonlinear terms in all formulas below. In view of relations specific for sound, the governing equations for an excess acoustic density ($\rho_a \equiv \rho_1$) in the leading order takes the forms:

$$\frac{\partial \rho_a}{\partial t} + c_\infty \sqrt{\Delta}\rho_a - c_\infty B\rho_a + \frac{1}{2}\left((\mathbf{v}_a \nabla)\rho_a + \gamma_\infty \rho_a (\nabla \mathbf{v})\right) = 0 \tag{25}$$

In order to simplify mathematical context and meaning the physically interesting case of quasi-planar sound propagating in the direction of axis OX, using assumption that all acoustic perturbations vary much faster in the direction of axis OX than along OY ($k_x >> k_y$) we can write equation (25) in following form

$$\frac{\partial \rho_a}{\partial t} + c_\infty \frac{\partial \rho_a}{\partial x} + \frac{c_\infty}{2}\frac{\partial^2}{\partial y^2}\int \rho_a dx - c_\infty B\rho_a + \frac{(\gamma_\infty + 1)c_\infty}{2\rho_0}\rho_a \frac{\partial \rho_a}{\partial x} = 0 \tag{26}$$

where

$$B = -\frac{D}{2c_\infty}\frac{C_{V,0}}{C_{V,\infty}\tau_c} \tag{27}$$

To decompose the dynamic equations for the velocity of the vorticity mode, it is sufficient to apply the matrix operator $\mathbf{P}_\mathbf{v}$ (Eq.(23)) at the momentum equation (two first equations from the system (3)).

As a result, all terms, correspondent to the potential velocity vector are reduced in the linear part. In the right-hand side we keep only acoustic terms. Applying of $\mathbf{P}_\mathbf{v}$ yield the dynamic equation for the vorticity mode in the field of intense sound, in two equivalent forms

$$\frac{\partial \mathbf{\Omega}}{\partial t} = \frac{1}{\rho_0}\nabla \times \left(\rho_a \frac{\partial \mathbf{v}_a}{\partial t}\right), \quad \frac{\partial \mathbf{v}_v}{\partial t} = \frac{1}{\rho_0}P_v\left(\rho_a \frac{\partial \mathbf{v}_a}{\partial t}\right) \tag{28}$$

where $\mathbf{\Omega}$ is the vorticity of a flow, $\mathbf{\Omega} = \nabla \times \mathbf{v}_v$ with $\mathbf{v}_v$ replacing $\mathbf{v}_5$. Accounting for Eqs (18), one gets finally the following equations in terms of $\mathbf{v}_v$ and $\mathbf{\Omega}$:

high-frequency

$$\frac{\partial \mathbf{v}_v}{\partial t} = -\frac{2Bc_\infty^3}{\rho_0^2}\mathbf{P}_v\left(\nabla\rho_a \int \rho_a dt\right), \quad \frac{\partial \mathbf{\Omega}}{\partial t} = \frac{2Bc_\infty^3}{\rho_0^2}\left(\nabla\rho_a \times \int \nabla\rho_a dt\right) \tag{29}$$

high-frequency quasi-planar

$$\frac{\partial \mathbf{v}_v}{\partial t} = \frac{2Bc_\infty^2}{\rho_0^2}\mathbf{P}_v\left(\nabla\rho_a \int \rho_a dx\right), \quad \frac{\partial \mathbf{\Omega}}{\partial t} = -\frac{2Bc_\infty^2}{\rho_0^2}\left(\nabla\rho_a \times \int \nabla\rho_a dx\right) \tag{30}$$

2.2 THE VORTICITY MODE GENERATED BY PERIODIC SOUND

The difficulty of describing of the vorticity mode cased by sound is obvious in view of nonlinearity, absorption and diffraction of both equations governing sound and the vorticity mode. The periodic on a transducer solution of the planar version of the second equation from Eqs (26)

$$\frac{\partial\rho_a}{\partial t}+c_\infty\frac{\partial\rho_a}{\partial x}-c_\infty B\rho_a+\frac{(\gamma_\infty+1)c_\infty}{2\rho_0}\rho_a\frac{\partial\rho_a}{\partial x}=0 \tag{31}$$

takes the form [2, 12]:

$$\rho_a(X,\tau)=\rho_A e^{B_{sh}X}\sum_{n=1}^{\infty}\frac{2J_n(nK(e^{B_{sh}X}-1))\sin(n\omega\tau)}{nK(e^{B_{sh}X}-1)} \tag{32}$$

where $\tau=t-x/c_\infty$ denotes the retarded time, ω is the frequency of the sound, $K=\frac{(\gamma+1)\omega\rho_A}{2\rho_0 c_\infty B}$, $x_{sh}=\frac{1}{B}\ln(1+1/K)$ is the distance of forming of the shock front of the sound, $X=x/x_{sh}$ is dimensionless coordinate, and $B_{sh}=Bx_{sh}$. J_n denotes the Bessel function of the order n. The solution (32) accounts for nonlinearity and absorption, it is valid for distances from a transducer before forming of the saw-like front, $0\le X<1$. It may be expanded in the case of weakly diffracted beam into the following solution:

$$\rho_a(X,Y,\tau)=\rho_A e^{B_{sh}X-Y^2}\sum_{n=1}^{\infty}\frac{2J_n(nK(e^{B_{sh}X}-1))\sin(n\omega\tau)}{nK(e^{B_{sh}X}-1)} \tag{33}$$

$Y=y/R$ denotes the dimensionless transversal coordinate, and R marks the characteristic transversal width of the sound beam. Substituting the solution (33) in the "high-frequency" equation for $\mathbf{\Omega}$ (Eq.(29)), and averaging the equation over the period of sound $2\pi/\omega$, one arrive at the solution for the $\langle\mathbf{\Omega}_t\rangle$, which equals the driving force of the vorticity mode, averaged over the sound period

$$\langle\mathbf{\Omega}_t\rangle=\frac{\omega}{2\pi}\int_t^{t+\frac{2\pi}{\omega}}\frac{\partial\mathbf{\Omega}}{\partial t}dt=\frac{2Bc_\infty^3}{\rho_0^2}\left\langle\nabla\rho_a\times\int\nabla\rho_a dt\right\rangle \tag{34}$$

Considering of the averaged quantities reduces the problem of generation of the vorticity mode to the problem of acoustic streaming in its classic meaning. The first two components $\langle\Omega_{x,t}\rangle$, $\langle\Omega_{y,t}\rangle$ are zero and the third one $\langle\Omega_{z,t}\rangle$ is nonzero as follows

$$E\langle\Omega_{z,t}\rangle=\frac{8Ye^{2(B_{sh}X-Y^2)}}{K^2(e^{B_{sh}X}-1)}\sum_{n=1}^{\infty}\frac{1}{n^2}J_n(nK(e^{B_{sh}X}-1))^2 \tag{35}$$

where

$$E=\frac{\rho_0^2 R}{\rho_A^2 Dc_\infty}\frac{C_{V,\infty}\tau_c}{C_{V,0}}=-\frac{R}{2M^2Bc_\infty^2} \tag{36}$$

The figure 1 shows distribution of the longitudinal force of acoustic streaming along axis OX for different quantities of B at different distances from the axis of the sound beam. In evaluations, only five first terms in the series (35) was taken into account.

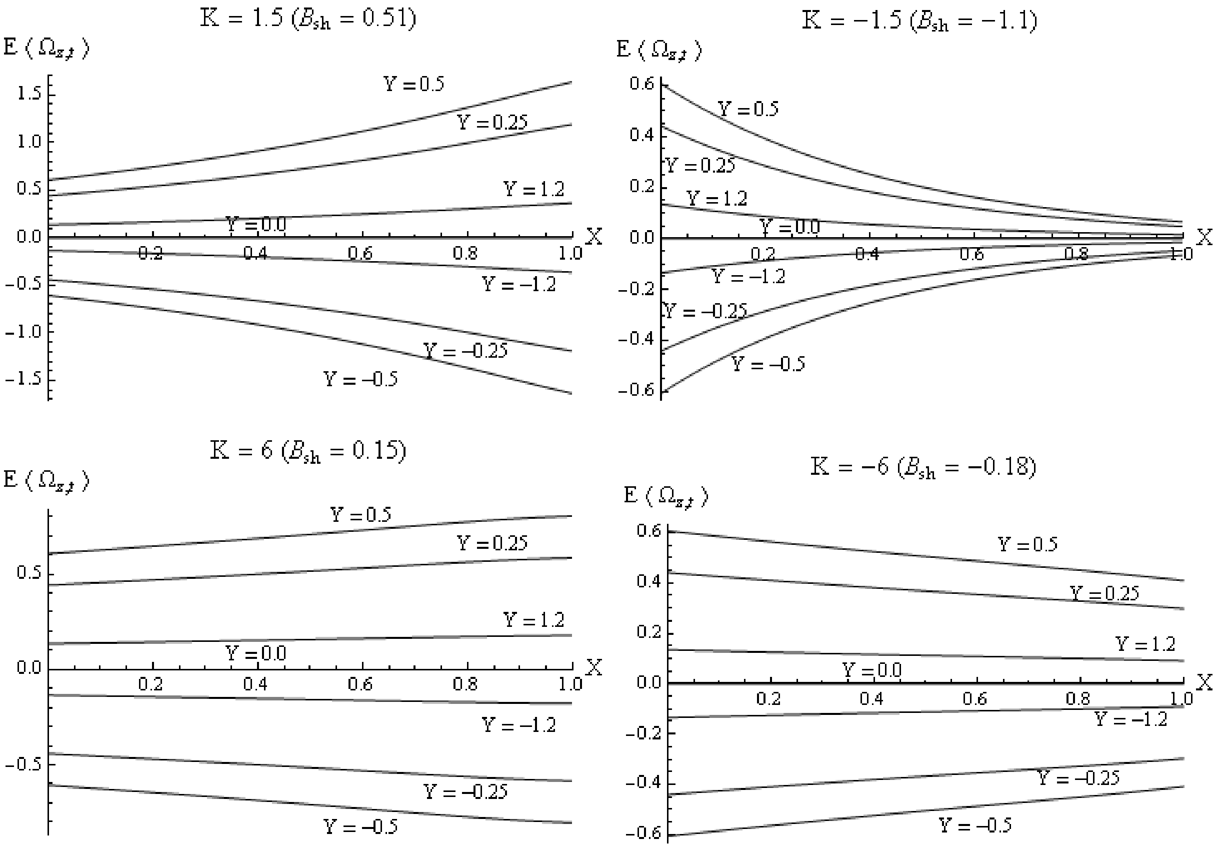

Fig. 1. Longitudinal force of acoustic streaming for different B_{sh} and dimensionless transversal distances from the sound beam, Y

The vorticity $\langle\Omega_z\rangle$ depends on time and may be calculated by use of $\langle\Omega_{z,t}\rangle$ by means of relation

$$\langle\Omega_z\rangle = \left(t + \frac{\pi}{\omega}\right)\langle\Omega_{z,t}\rangle \tag{37}$$

The curves of the Fig.1 prove that the sign of acoustic force of streaming depends on the sign of B by means of dependence of E on B. Therefore, the inversion of direction of streamlines occurs in the equilibrium regime of a chemical reaction (when $B < 0$) compared with non-equilibrium chemical reaction (when $B > 0$).

3. CONCLUDING REMARKS

The two-dimensional weakly nonlinear flow of chemically reacting gas is considered in this study. Equations describing dynamics of sound (26) and the vorticity mode (28), are derived within accuracy up to the terms of order M^2 inclusively. Accuracy of conclusions are restricted by considering of only first partial derivatives of heat release $Q(\rho, T, Y)$. Inclusion of higher order derivatives in Q would lead to correction in the dynamic equation for the dominative sound, but will not change the equation governing streaming. The conclusions are valid at confined temporal and spatial domains, where sound remains dominative with respect to other modes, vorticity and entropy.

This study does not take into account for the thermal and viscous (standard) attenuation of a reacting gas. The terms, reflecting these phenomena (originating from the stress tensor and energy flux associated with thermal conductivity), should complete the momentum and energy equations in the system (1).

Equation (26) is numerically not stable. In the nearest future, authors are going to give approximate illustrations of vorticity of a flow (30) using equation describing sound dynamics, accounting for the standard attenuation.

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