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W-dominance: Tradeoff-inspired dominance relation for preference-based evolutionary multi-objective optimization

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ABSTRACT

The paper presents a method of incorporating decision maker preferences into multi-objective meta-heuristics. It is based on tradeoff coefficients and extends their applicability from bi-objective to multi-objective. The method assumes that a decision maker specifies a priori each objective's importance as a weight interval. Based on this, w-dominance relation is introduced, which extends Pareto dominance. By replacing reference points with weight intervals the method eliminates the need for any knowledge concerning expected solutions. Instead, decision maker reflects his context-independent policy regarding objectives. The proposed w-dominance was incorporated into selected multi-objective metaheuristics. Following this, three new metrics were designed. The metrics include prescreening true Pareto Front and final population according to w-dominance relation. Based on preliminary tests, Vector Angle Evolutionary Algorithm (VaEA) was selected as the best match for w-dominance. W-dominance-extended VAEA (wVAEA) was compared in a series of simulations with four state-of-the-art reference point-based multi-objective algorithms. The results show that wVaEA outperforms the four representative algorithms for selected benchmark problems.

1. Introduction

Multi-Objective Evolutionary Algorithms (MOEA) and other Multi-Objective Meta Heuristics (MOMH) become more popular because of two facts. First, the majority of real-world optimization problems are de facto multi-objective ones. Second, reducing a multi-objective problem to a single objective by means of aggregated functions (or turning some objectives into constraints) results in losing solutions which could be preferred by a decision maker (DM). On the other hand, sticking to regular Pareto dominance in EMO has the disadvantage of processing and returning some solutions undesired from DM's point of view. Thus in turn may severely increase processing time [1], in some situations making the optimization process unacceptably long-lasting. Therefore, it is common in EMO to take into account DM preferences. Owing to this, the EMO algorithm may focus on that part of the objective space which is essential to DM and speed up the convergence towards the true Pareto Front (PF).

The approaches to incorporating DM preferences can be classified in a number of ways, depending which criterion is considered. The three commonly used criteria are:

(1) The time of eliciting DM preferences: the elicitation can be done a priori (before the optimization process), interactively (during the optimization process) or a posteriori (after the main optimization process is finished).
(2) The way of expressing DM preferences: among others, DM preferences can be specified by means of reference points or reference vectors, preference relations, comparisons of solutions, outranking, knee points and tradeoffs.
(3) Incorporating DM preferences into algorithms: the preferences can be handled by means of dominance relations, decomposition algorithms and indicator-based methods.

As for preference elicitation, each of the three approaches has its advantages, which make it a good fit for some application cases, as well as some limitations. An a priori approach is useful if the DM knows the preferences in advance. It may also be a necessity, if interaction with DM is impossible due to the optimization's working environment, e.g. in case of real time systems, where solutions have to be generated and applied automatically and within strict time limits. However, if DM can be engaged, interactive approach [2] may be recommended, as it enables DM to adapt their preferences based on newly obtained solutions and their coordinates in the space of objectives. This approach can result in solutions, which meet DM's needs best, though this gain may come at the cost of a longer overall optimization time due to interaction. Finally, DM can relate to an already generated solution a posteriori – in a Multi-Criteria Decision Making phase, which is done after the main op-

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timization process. However, the drawback of the last approach is that the optimization phase cannot benefit from DM preferences by focusing on DM's needs and reducing computational time.

In terms of the form that DM preferences can take, reference points (RP) are particularly popular [1]. RPs are points in the objective space which represent desired and probably feasible solutions [3–5]. Possible applications of RPs to MOEA and other MOMH include among others dominance relation [6,7], non-dominated solution sorting [8,9] and crowding distances [10,11]. Apart from specifying a single RP, it is also possible for DM to give a reference vector [10]. Both a single RP and a reference vector may reflect DMs requirements as well as aspiration levels [12] or expectations. Another RP-related approach is specifying directly a Region of Interest (ROI) or target region [13]. DM may express their satisfaction with solutions within ROI by means of density function [14], desirability function [15,16] or a combinations of the above two [17]. Furthermore, in case of decomposition-oriented EMO algorithms, a DM-supplied aspiration-level vector may be used to map uniformly distributed reference points to their new positions, thus focusing the optimization process on ROI [18]. Other, not RP-related approaches to handling DM's preferences include comparison of solutions [19], preference relations, outranking, knee points [1,20] and finally – tradeoffs [9, 21–23]. Of these, both preference relations and outranking [24,25] utilize the same concept of comparing the objectives [26], however the transitive relation is assumed in the former [27], while non-transitivity is allowed in the latter [25,28]. As for comparison of solutions, pair-wise comparisons may be used to construct a DM preference cone [2], which may be updated interactively to fit DM's needs and limit the search space of the optimization process.

When it comes to incorporating DM preferences into MOMH also a few approaches can be applied. Of them, the most popular are dominance-based, decomposition-based and indicator-based ones. In the first case, preference-oriented relations extend the range of dominance beyond regular Pareto dominance and thus enable the algorithm to compare some of Pareto non-dominated solutions. Dominance relations may be based on RPs and various approaches are possible here. Among others, an RP-based dominance relation may take into account Euclidean distance and angle information between the solutions and RPs to evaluate the solutions in terms of their convergence and diversity, respectively [29]. As for decomposition-based algorithms, they usually rely on weight vectors or related concepts. Here DM preferences are reflected by the distribution of weight vectors, which are more densely placed in the DM's ROI [30,31] and this mechanism may be additionally tuned to avoid premature convergence [32]. Indicator-based methods apply preference information by means of quality indicators. Depending on how well various solutions match DM preferences, they have different impact on the indicator value, which results in directing the optimization process. Finally, it is worth noting, that combinations of two or more approaches to incorporating DM preferences are also possible.

According to the three division criteria given above, the method proposed here can be classified as a priori, tradeoff-inspired and dominance-based. It extends the concept of [21], but unlike in [21], here tradeoff coefficients are not used directly, but are replaced with weight intervals. Those weight intervals turn out to be easier to handle than coefficients and allow for a generalization from bi-objective to multi-objective as is later explained in Section 2.3. The method introduces w-dominance relation, which takes into account DM-specified weight intervals and can be checked in linear time with regard to the number of objectives. As opposed to RP-based approaches, it does not require any knowledge concerning expected solutions. W-dominance relation was added to over 40 MOMHs, whose performance was then tested leading to the choice of Vector Angle Evolutionary Algorithm (VaEA) [33] as most promising. The w-dominance-equipped VaEA (wVaEA) was then compared to four state-of-the art RP-based EMO algorithms. The results shown that wVaEA outperforms the other algorithms for the majority of selected problems, especially in terms of convergence. Given the algorithm's performance and the fact that DM does not need to have any knowledge on the feasible objective values, w-dominance can be considered an alternative to other preference-based approaches, especially for problems where it is impossible to estimate the true Pareto Front (PF) a priori with reasonable accuracy.

The rest of this paper is organized as follows. Next section shows motivation for developing the proposed method as well as some literature backgrounds. After this, w-dominance is described in detail, including three newly proposed metrics, which make it possible to compare w-dominance-equipped algorithms with other preference-based MOMHs. Experimental studies are then presented, followed by a discussion of their results. Finally, conclusions are provided in the last section. Appendix 1 includes comparison results of w-dominance-equipped VaEA (wVaEA) with the original VaEA algorithm.

2. Backgrounds

2.1. Dominance-based approaches to incorporating DM preferences into MOMH

Dominance-based algorithms use relations, which extend the range of dominance beyond regular Pareto approach and thus are able to compare some of Pareto non-dominated solutions. Dominance relations do not have to be based on DM preferences – e.g. the popular epsilon-dominance [34] is not. As for preference-oriented dominance relations they include, among others g-dominance [35], r-dominance [7] and p-dominance [36], all of which utilize reference points. The RP-based dominance relations may compare solutions based on division of objectives' space (g-dominance [35]) or their Euclidean distance to an RP. The Euclidean distance may take into account weights assigned to objectives (r-dominance [7]) and may be enhanced by indicators like preference radius (p-dominance [36]) or preference angle [29]. Those additional indicators aim at combining algorithms' good convergence (resulting from using Euclidean distances) with diversity. Some other variants of dominance relations use local search strategies [37] and user preference indicators [8] to improve performance of NSGA-II and R-NSGAII respectively.

2.2. Route optimization – limitations of RP-based approaches and motivation for applying tradeoff-oriented methods

Arguably, the most popular recent thread in specifying DM preferences is by means of RPs. They are easy to use provided that DM is either able to specify their values in the objective space in advance or that it is possible to engage DM in the interactive preference elicitation during the optimization process. For example, when optimizing design of some devices DM usually can point desired and feasible combinations of objective values. This is especially true in case of market-available products, whose performance parameters are widely known. Also, in such case, DM is able to relate to the newly obtained solutions, which makes it possible to handle DM preferences interactively.

Unfortunately there is also a class of real-world multi-objective optimization problems where neither of the above is possible, that is RPs cannot be specified in advance because of insufficient knowledge and the interactive process is impossible due to working conditions of the optimization process. Such problems include real time route optimization in land and marine transport, where possible objective values are not known a priori and interactive process is impossible because of computational time constraints. Typical objectives in transport are minimization of travel time, fuel consumption and risk. In route planning of road vehicles all three objective values depend on the start and destination points, vehicle parameters, time of the year, loading conditions and current congestion levels including temporary traffic jams. Therefore, even if historic data are available, they may not be relevant for the current case. Also, a driver transiting through a country or a continent may not wish to be troubled with entering new RPs for each leg of a route. The same is true for routing of transoceanic vessels [38], which is heavily affected by seasonal environmental changes (including tropical cyclones), ships' speed characteristics for various loading conditions and up-to-date weather forecasts. Whenever a new forecast is made available, the weather routing system should re-run the optimization with the ship's current position as a starting point. Thus, using RP-based approach would lead to involving a navigator more often than it is actually possible. Similarly, in case of managing a fleet of vessels, a ship-owner would have to specify RPs for each combination of a vessel, route endpoints, time of the year etc., which, again, would be extremely tiresome. Finally, in ship collision avoidance [39], particular encounter conditions are usually unique and navigator has a limited time for decision making, which means that entering RPs is not feasible.

Summing up, RPs are of limited use in route optimization and related problems, which makes impossible or impractical to apply RP-based dominance relations mentioned in the previous subsection. Instead, DM will be interested in specifying his general, context-independent policy towards objectives. This need is addressed by tradeoff approaches, though, unfortunately tradeoffs also have their limitations as is described in the following subsection.

2.3. Tradeoff-oriented MOMH

Within tradeoff based approaches to incorporate DM preferences it is possible to distinguish objective (problem structure-based) tradeoffs and subjective (DM's-preference based) ones [1]. In terms of input data the latter may either take numerical or linguistic values, which can then be transformed into objective weightings, weight intervals or coefficients by means of fuzzy preference relations, as has been shown in [22,40]. A guided MOEA utilizing the subjective tradeoff approach with DM-specified coefficients has been presented in [21]. The authors define there the tradeoffs as values reflecting how much the DM is ready to sacrifice some objectives for improving the other. Such information can be specified in units, e.g. DM may decide that improvement by a single unit in one objective is worth at most degradation by n units in another objective. This coefficients-based approach is well-suited for bi-objective optimization but, as stated in [1], the approach presented in [21] cannot be directly applied to more than two objectives. For a larger number of objectives this tradeoff approach requires a matrix of tradeoff coefficients given for each combination of two objectives. Each coefficient's value $c_{i,j}$ reflects the fact that according to the DM a single unit improvement in $f_i(x)$ is worth at most $\frac{w_i^{min}}{w_j^{max}}$ units of degradation in $f_j(x)$. This has two disadvantages. First, providing such a matrix would be tiresome for DM and would require consistency check. The coefficients $c_{i,j}$ have to be consistent, that is, the following condition has to be satisfied for all $\frac{m(m-1)(m-2)}{6}$ combinations of i, j and k: $c_{ij}c_{jk}c_{ki=}c_{ik}c_{ji}c_{kj}$, where m is the number of objectives. If any of those conditions was not met, we would get a contradiction: a tradeoff not allowed by a single coefficient could be obtained by a sequence of two tradeoffs. Furthermore, even if we are given a matrix, which satisfies the consistency condition, it is still impossible to apply it directly in the algorithm. In practice, when comparing two solutions, we encounter different objective values for all or nearly all objectives, not just the two addressed by a given coefficient. The above issues contribute to the fact that the tradeoff from Branke et al. [21] cannot be generalized to more than two objectives.

In general, it can be stated that tradeoffs' main drawback is the difficulty to apply tradeoff preferences during optimization process. As for tradeoffs' main advantage, it is the fact that they are independent on the particular occurrence of an optimization problem and thus can be provided a priori. Considering the above, we can state that it is desirable to propose a tradeoff-based method that would offer DM the possibility to specify their context-independent preferences a priori in such a way that they could be easily incorporated into the algorithm and applied throughout the optimization process. This can be done by extending the method from Branke et al. [21] to more than two objectives and replacing troublesome matrix of coefficients with some more DM-friendly solution. The current paper addresses both of those issues by combining tradeoffs with a flexible weight intervals-based dominance relation. The proposed approach is a compromise between popular weighted average (which is both too knowledge-demanding and too limiting for DM) and unfocused Pareto-optimization approach (which does utilize DM preferences). Instead of requesting precise weight values assigned to objectives, the proposed method can accept very vague DM-given information and still benefit from them. As it turns out, even quite wide weight intervals can significantly limit the objectives' space and make the optimization process more focused.

3. Proposed weight interval-based tradeoff in MOMH (w-dominance)

In this section we describe in detail our approach to quantification of DM's preferences in MOMH. It extends any Pareto-dominance-based MOMH to preference-based one, by applying weight intervals and a new dominance relation – w-dominance. We have developed an early version of this method [38] specifically for weather routing of sea-going vessels, using SPEA2 [41] algorithm. Here we present w-domination in its mature stage, generalized for any kind of application and any Pareto-preference-based MOMH. We introduce three new metrics to compare performance of a w-dominance equipped MOMH with any other preference-based MOMH. The following subsections elaborate on that.

3.1. A generalized weighted average objective function

Using weighted averages of objective functions instead of multi-objective optimization has two disadvantages. One of them is that DM is usually not able to specify the weights precisely. The other, that a lot of potentially interesting solutions cannot be found that way (e.g. non-convexities of the true Pareto Front). The approach we present here is based on the observation that DM may specify weights as intervals instead of single values. The mean value of each interval reflects DM's assessment of the objective's importance and the interval's width – DM's uncertainty concerning this assessment. Furthermore, the wider the intervals, the deeper the optimization algorithm will search into non-convex parts of the true Pareto Front (PF).

Now let us consider the i-th objective and denote weight interval w_i assigned by DM to it:

$$w_i \in \langle w_i^{min}, w_i^{max} \rangle, \tag{1}$$

where $w_i^{min}, \in \langle 0,1 \rangle$, $w_i^{max} \in \langle 0,1 \rangle$ and $w_i^{min} \leq \ w_i^{max}$. This results in: $\sum_{i=1}^{m} w_i \in \langle 0, m \rangle$, where m is the number of objectives. If needed, $\sum_{i=1}^{m} w_i$ can be scaled down to $\langle 0,1 \rangle$ range: it is enough to divide each of the DM-specified w_i^{min}, w_i^{max} by $\sum_{i=1}^{m} w_i^{max}$.

Further on, a generalized weighted average objective function will be used, where each w_i given by (1) will remain an unspecified value from the $\langle w_i^{min}, w_i^{max} \rangle$ range:

$$f(x) = \sum_{i=1}^{m} w_i f_i(x), \tag{2}$$

where m is the number of objectives and $f_i(x)$ is a normalized i-th objective function.

Despite the use of weights the above generalized weighted average objective function (2) does not mean a loss of generality, because it is an equivalent of regular Pareto optimization if: $w_1^{min}, w_2^{min}, \ \ldots, \ w_n^{min} = 0$. This results from the fact that according to DM a single unit improvement in $f_i(x)$ is worth at most $\frac{w_i^{min}}{w_j^{max}}$ units of degradation in $f_j(x)$. If all w_{i}^{min} are equal to 0 than the quotient $\frac{w_i^{min}}{w_j^{max}}$ is also 0 for all i, which means that DM does not accept any degradation in any objective and thus a tradeoff is not possible.

On the other hand, if $w_i^{min} = w_i^{max}$ for all i then the weight intervals are replaced by precise weight values and thus the proposed aggregated objective function (2) would become a typical weighted average.

The above two cases ($\forall_i w_i^{min} = 0$ or $\forall_i w_i^{min} = w_i^{max}$) are the extreme variants, which are covered by the proposed method, but do not bring any practical benefits. Therefore it is expected of DM to specify: $0 < w_i^{min} < w_i^{max}$. Once w_i^{min} and w_i^{max} are known, the dominance rules can be proposed, which extend the range of classical Pareto dominance.

3.2. An example of preferences elicitation in the proposed approach

Let us now look at the example of preferences elicitation in case of autonomous ship. International Maritime Organization (IMO) specifies four degrees of autonomy for Maritime Autonomous Surface Ship (MASS). The last of them is: "Fully autonomous ship: The operating system of the ship is able to make decisions and determine actions by itself ". Let us focus on this fully automated MASS. One of frequent operations that it will have to perform is avoiding collisions with other ships. In general, vessels' behavior in encounter situations is governed by International Regulations for Preventing Collisions at Sea (COLREGs) [42]. Among others, COLREGs point out which vessel is obliged to give way and what types of maneuvers are preferred. The detailed choice and execution of a sequence of maneuvers is however up to the navigator, which in case of a MASS means – a navigation system installed onboard and acting as navigator. It is a multiobjective optimization problem, where the objectives may be as follows:

- minimization of collision risk index (covering collisions with ships and static obstacles as well as groundings),
- minimization of cargo damage risk due to large and instant course alterations,
- minimization of extra fuel consumption due to evasive maneuvers and getting back on course.

Let us denote those three objectives by: f_1, f_2 and f_3 respectively. Obviously, some tradeoff between them is needed, but in case of a fully automated MASS it is not possible to consult an external DM every time another vessel is encountered. RPs cannot be used because the objective values will depend on a particular encounter (motion parameters of encountered vessels, surrounding traffic, limitations of a waterway etc.). Instead, a certain policy of tradeoffs between all objectives has to be set a priori for the system to use it during the exploitation. The objectives have been sorted in the order of their importance: avoiding collisions is critical because collisions may involve casualties in the other vessel's crew, environmental damages and losing a vessel. Minimization of extra fuel consumption is least important here, though it is still desired as long as may be done without significant degradation in the first two objectives. Thus, an example of DM's policy expressed in weight intervals assigned to each objective may be as follows: $w_1 \in \langle 0.9,\ 1\rangle,\ \ w_2 \in \langle 0.4,\ 0.7\rangle,\ \ w_3 \in \langle 0.1,\ 0.3\rangle$.

The weight intervals above are averaged values obtained by the authors from navigators of conventional manned vessels. As we can see, navigators are sure of the critical importance of the first objective (collision risk index). As for the second objective (cargo damage risk), the numerical values can be interpreted as moderate to significant importance and the third (extra fuel consumption) – as little to moderate importance. During the optimization process, the objectives will be normalized in each generation (based on the objective values obtained throughout the population) and following this, the w-dominance can be checked as described in the following section 3.3. Owing to w-dominance a significant reduction in objective's space is possible, when compared to Pareto dominance. This reduction of objective's space depends strictly on the widths of DM-specified intervals and it is possible to estimate it in advance, based on those widths. For weight intervals given in the above example, the percentages of solutions Pareto-dominated and w-dominated by a given solution Y are provided in Table 1.

As can be seen, even though navigators have a vague idea of relative weights of the second and third objective (intervals: $\langle 0.4,\ 0.7\rangle$ and $\langle 0.1,\ 0.3\rangle$), w-dominance can still bring a massive reduction of objective's space, when compared to Pareto-dominance. For given weight intervals even an average solution $(0.5,\ 0.5, 0.5)$ w-dominates nearly 43% of all objectives' space, while only 12.5% are Pareto-dominated by it. Furthermore, a solution approaching utopia $(0.1, 0.1, 0.1)$ w-dominates nearly all of objectives' space (98.4%), despite the fact that only 72.9% of them are Pareto-dominated. The latter means that in case of w-dominance only 1.6% of objectives' space would be taken into account after obtaining solution $(0.1, 0.1, 0.1)$, while Pareto dominance would still leave us with 27.3% of objective space to handle.

3.3. Extending the range of dominance by application of weight intervals – a conditions for w-dominance check

First, we may observe that the following always holds when taking into account (2):

$$\sum_{i=1}^{m} w_i^{min} f_i(x) \le f(x) \le \sum_{i=1}^{m} w_i^{max} f_i(x). \tag{3}$$

In minimization Pareto MOP x dominates y if and only if:

$$\exists_i \left(f_i(x) < f_i(y)\right) \text{ and } \forall_i \left(f_i(x) \le f_i(y)\right). \tag{4}$$

For the generalized weighted average objective function as given in (2), the rule (4) still implies dominance. However, in cases when (4) is not satisfied, thus x does not dominate y, another relation extending Pareto-dominance can be introduced, namely **w-dominance.** It is defined that a solution x **w-dominates** solution y if:

$f(x) < f(y)$, where $f(x)$ is the generalized weighted average function given by (2).

Because each w_i is an unspecified value from $\langle w_i^{min}, w_i^{max}\rangle$ range it is impossible to check condition $f(x) < f(y)$ in a direct way. However, there is also another condition, which is sufficient for w-dominance ($f(x) < f(y)$) and even easier to be fulfilled. To formulate it we need first to present the condition of $f(x) < f(y)$ as:

$$\sum_{i=1}^{m} w_i f_i(y) - \sum_{i=1}^{m} w_i f_i(x) > 0, \tag{5}$$

which can also be presented as:

$$\sum_{i=1}^{m} w_i \left(f_i(y) - f_i(x)\right) > 0 \tag{6}$$

To simplify further formulas let us denote:

$$d_i(x, y) = f_i(y) - f_i(x). \tag{7}$$

Since $w_i \in \langle w_i^{min}, w_i^{max}\rangle$, we may observe that:

$$\left(d_i(x, y) \ge 0\right) \Rightarrow \left(w_i d_i(x, y) \ge w_i^{min} d_i(x, y)\right) \tag{8}$$

and similarly

$$\left(d_i(x, y) \le 0\right) \Rightarrow \left(w_i d_i(x, y) \ge w_i^{max} d_i(x, y)\right). \tag{9}$$

Let us now introduce a function: $g_i(x, y)$:

$$g_i(x, y) = \begin{cases} w_i^{min} d_i(x, y), & \text{for } d_i(x, y) \ge 0 \\ w_i^{max} d_i(x, y), & \text{for } d_i(x, y) < 0 \end{cases} \tag{10}$$

Taking into account (8) and (9) we can see that for this function the following holds:

$$w_i d_i(x, y) \ge g_i(x, y). \tag{11}$$

Consequently, the following also holds:

$$\left(\sum_{i=1}^{m} g_i(x, y) > 0\right) \Rightarrow \left(\sum_{i=1}^{m} w_i d_i(x, y) > 0\right). \tag{12}$$

Table 1
The percentage of solutions, which are Pareto-dominated and w-dominated by example solutions in the <0;1>-normalized objective space if weight intervals <0.9, 1>, <0.4, 0.7>, <0.1, 0.3> are assigned to the objectives for w-dominance.

Solution Y	(0.5, 0.5, 0.5)	(0.4, 0.4, 0.4)	(0.3, 0.3, 0.3)	(0.2, 0.2, 0.2)	(0.1, 0.1, 0.1)
Percentage of solutions dominated by solution Y					
Pareto dominated	12.5%	21.6%	34.3%	51.2%	72.9%
w-dominated (including Pareto dominated)	42.6%	60.3%	77.2%	90.9%	98.4%
w-dominated but not Pareto dominated (our gain in objective space reduction)	30.1%	38.7%	42.9%	39.3%	25.5%

Therefore, the following condition is sufficient for the proposed w-dominance rules:

$$\sum_{i=1}^{m} g_i(x, y) > 0. \tag{13}$$

Checking the above condition (13) has a linear computational complexity regarding the number of objectives, just as regular Pareto dominance has. Therefore, it can be incorporated into most MOMHs without affecting their computational complexity.

As mentioned in section 3.1, if $w_1^{min}, w_2^{min}, \ \ldots, \ w_n^{min} = 0.$, then (13) cannot be fulfilled because, based on (12), its left side will not include any positive elements. Therefore, in such theoretical case the range of Pareto-dominance will not be extended by w-dominance and the algorithm will not make any use of the weight intervals.

3.4. Adding w-dominance to a MOMH

The proposed tradeoff relation can be incorporated to practically any MOMH by means of replacing regular Pareto-dominance with a w-dominance condition (13). In case of MOMHs using non-dominated sorting algorithm (e.g. NSGA-inspired algorithms) the non-w-dominated sorting needs to be introduced, e.g. by modifying the Efficient Non-dominated Sort with Sequential Search (ENS-SS) [43].

However, applying the dominance rule of (13) with $g_i(x, y)$ defined as in (10) may result in too fast convergence and a loss of diversity within population. Therefore, we use a modified, generalized version of $g_i(x, y)$ throughout the evolutionary process. It is defined as:

$$g_i(x, y) = \begin{cases} w_i^{min} d_i(x, y)\left(1 - e\frac{k}{n}\right) \text{ for } d_i(x, y) \geq 0 \\ w_i^{max} d_i(x, y)\left(1 + e\frac{k}{n}\right) \text{ for } d_i(x, y) < 0 \end{cases}, \tag{14}$$

where:

k – current generation number,
n – number of all generations,
e – diversity encouragement factor from (0,1) range.

Owing to this modification, the dominance acts as if $\langle w_i^{min}, w_i^{max} \rangle$ interval was much wider for initial generations and then linearly narrowed down to the interval specified by DM. If necessary, k and n values can be replaced with: a number of fitness evaluations already performed and a total number of allowed fitness evaluations, respectively. As for diversity encouragement factor e, it has been found in the course of preliminary simulations that it is best to set it to around 0.5. Values smaller than 0.3 may result in the abovementioned too fast convergence and a loss of diversity within population. On the other hand, values larger than 0.7 decrease the impact of DM preferences and make the optimization process unfocused. Therefore, values between 0.4 and 0.6 are recommended for use in practice, with the lower bound accentuating DM preferences and the upper bound – diversity. The value of $e = 0.5$ has been used throughout the experimental studies presented in section 4.

3.5. Proposed w-dominance metrics

As recognized in [44], classic metrics for evaluation performance of MOMHs fail to address properly the problem of considering DMs preferences. Therefore in [44] new metrics are proposed, whose main concept is prescreening of the original solution set and trimming the Pareto Front (PF), so that only solutions within Region of Interest (ROI) are taken into account for performance assessment. Since our preference-based method does not use RPs, R-Metrics from [44] (e.g. R-IGD) cannot be directly applied here. However, the main idea of R-Metrics – prescreening solution set and available representation of PF – can still be used. Thus, we propose wIGD, wGD and wHV metrics. They are based on commonly used metrics of IGD, GD and HV respectively, but (like R-Metrics) they prescreen the original solution set and PF: only non w-dominated solutions are taken into account. The metrics aim to offer the possibility of a fair comparison between w-dominance and RP-based methods. In order to fulfill this, we have to handle the below described problems.

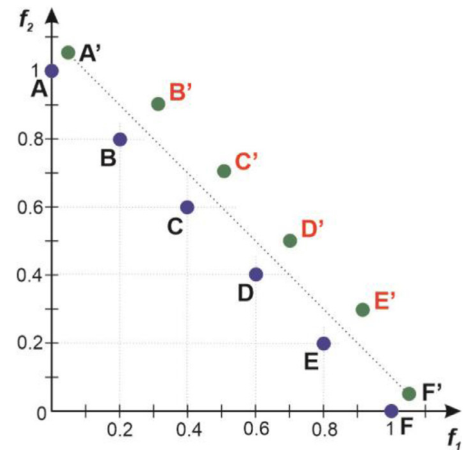

Fig. 1. wIGD: problem of prescreening solution set and true PF. True PF points marked in blue, obtained solutions marked in green. W-dominated solutions additionally marked with red letters.

As opposed to RP-based methods, in case of w-dominance, based on DM-specified weight intervals we are always able to decide unambiguously if a certain solution is w-dominated by another or not. Considering the motivation behind DM-specified weight intervals it seems natural to only take into account non-w-dominated solutions and non-w-dominated points of the true PF. However, when applying this policy, the problems of excluding certain points affect the final metrics values much more than in case of RP-based approach. This issue is illustrated below.

Let us assume the bi-objective case of minimizing functions f_1 and f_2 and the weight intervals of <0.5, 1> for f_1 and <0.25, 0.5> for f_2. The following phenomenon can then be observed for wIGD and wGD metrics.

3.5.1. wIGD

In Fig. 1 we show the available representation of true PF as blue dots (A to F). Those true PF points are approximated by solutions A' to F' (green dots) of the obtained solution set. Neither of true PF points is w-dominated by any other, hence they can all be taken into account. However, as for solution set, points A' and F' w-dominate the other four (C' to E') so neither of those other four points would be taken into account. As a result, points B to E of the true PF will not have their equivalents in the prescreened solution set, leading to poor value of the wIGD metric, de-

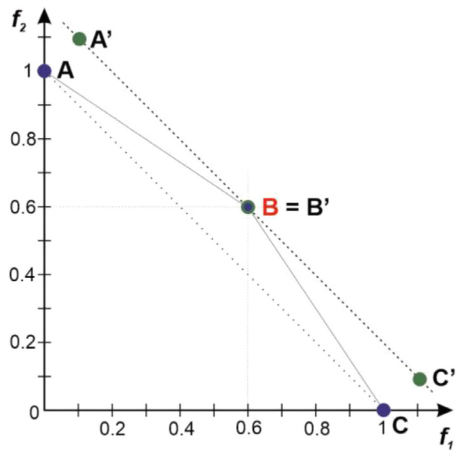

Fig. 2. wGD: problem of prescreening solution set and true PF. True PF points marked in blue, obtained solutions marked in green. W-dominated true PF point B additionally marked with red letter.

spite the fact, that points B' to E' are actually reasonably close to points B to E. The question arises here: should points B' to E' be considered after all? Unfortunately, taking them into account would contradict DM's preferences because those solutions are w-dominated and thus should not be among recommendations for DM. Therefore, despite the above mentioned problem, we prescreen both the solution set and true PF for wIGD assessment throughout further simulation experiments.

3.5.2. wGD

In Fig. 2 we again show the true PF points as blue dots (A to C). Those true PF points are approximated by solutions A' to C' of the solution set. Neither of the solutions is w-dominated by any other, hence they can all be taken into account. However, as for true PF, point B is w-dominated by point A. Unfortunately, if we exclude it, the point B' will be assigned a large distance despite the fact that it actually lies exactly on the true PF and is itself not w-dominated. Ironically, if the EMO algorithm did not find the non-w-dominated solution B', the metric value would be better. Therefore to avoid such paradoxes, we have decided that only non-w-dominated solutions will be valid, but we will consider their distances to the closest point in true PF regardless if this true PF point is w-dominated or not. Concluding: we only prescreen solution set and we do not prescreen true PF for wGD assessment.

3.5.3. wHV

The wHV metric is based on HV, however, similarly to previous two metrics, only non-w-dominated solutions are considered for evaluation and regular Pareto dominance is replaced with w-dominance. The metric is not directly affected by the problem described in section 3.5.1 (Fig. 1), because distances between solution set and true PF are not used there. However, the metric values may still suffer if there are very few non-w-dominated solutions in the final set.

4. Experimental studies

This section presents results of w-dominance application experiments and comparison of w-dominance-enhanced EMO with the other state-of-the-art RP-based EMOs, as well as with original VaEA algorithm.

4.1. Selected algorithms

In general, w-dominance can be combined with nearly every MOMH, though it is designed to enhance those of the, which are not originally preference-based. In the course of our research we first added w-dominance relation to over 40 EMO and other MOMH algorithms implemented in PLATEMO platform [45]. We have focused on the algorithms which were not initially preference-based so as not to mix w-dominance with other preference-based approaches within one algorithm. Tested algorithms included: Decomposition-Based Multiobjective Evolutionary Algorithm With the ε-Constraint Framework (DMOEA-eC) [46], Generic Front Modeling-based Multiobjective Evolutionary Algorithm (GFM-MOEA) [47], Large Scale Multiobjective Optimization Framework (LSMOF) [48], Multiobjective Evolutionary Algorithm Based on Decomposition (MOEA/D) [49], Nondominated Sorting and Local Search (NSLS) [50], Sparse multi-objective Evolutionary Algorithm (SparseEA) [51], Two-Archive Algorithm for Many-Objective Optimization (Two_Arch2) [52], Vector Angle Evolutionary Algorithm (VaEA) [33] and Weighted Optimization Framework-enhanced SMPSO (WOF-SMPSO) [53]. During preliminary tests it has been found that w-dominance worked particularly well with decomposition-oriented algorithms including MOEA/D. This can be attributed to the fact that w-dominance makes it possible to limit the range of weight vectors thus focusing on those parts of the true Pareto front, which satisfy DM preferences. Of all decomposition-based and decomposition-related algorithms enhanced by w-dominance we registered the best results for VaEA. VaEA is an algorithm, which uses the maximum-vector-angle-first rule for environmental selection to obtain good coverage of the true PF by the solution set. The above rule is supplemented by removing worst-convergence solutions, which results in a stronger selection pressure toward the true PF. As shown in [33], the combination of the above mechanisms leads to very good convergence without sacrificing diversity.

W-dominance-extended VaEA (wVaEA) outperformed the other w-dominance-enhanced algorithms for the majority of tried benchmark problems from DTLZ and WFG suites. Therefore, we selected wVaEA as a w-dominance-based EMO algorithm for further comparisons: this time with some state-of-the-art preference-based EMO algorithms. When selecting the reference algorithms we took into account both preference-based and those, which are known for overall good performance and can be turned into preference-based by appropriate choice of RPs (like NSGAIII [3], A-NSGAIII [54] and ar-MOEA [29]). Eventually, for comparison with wVAEA we selected four successful and recent preference-based and RP-based algorithms, which showed very good performance in the initial simulations: hpaEA [55], RPD-NSGAII [56], RVEA [30] and PICEA-g [57].

4.2. Experimental settings

Experiments described in the following sections were aimed at:

- validating the w-dominance enhanced VaEA (wVaEA),
- comparison of wVaEA's results with the four selected reference-point-based EMOs (hpaEA, RPD-NSGAII, RVEA and PICEA-g).

The algorithms were tested on two strongly different cases of DM preferences expressed as weight interval settings.

- Case 1 assumed comparable importance of all objectives, with $\langle w_i^{min}, w_i^{max}\rangle$ equal for all objectives, but dependent on the number of objectives m:

$$w_i^{min} = 0.5 - \frac{1}{m+1}, \tag{15}$$

$$w_i^{max} = 0.5 + \frac{1}{m+1}. \tag{16}$$

As can be seen, the weight interval's width decreases with the total number of objectives m and will vary: from $w_i \in \langle 0.2.5,\ 0.75\rangle$ for 3 objectives, to $w_i \in \langle 0.4,\ 0.6\rangle$ for 9 objectives.

Owing to this, there will still be a considerable percentage of w-dominated solutions even for a large number of objectives m.

- Case 2 assumed cascaded diminishing of objective importance depending on the objective number i and given by (17) and (18).

$$w_i^{min} = \left(\sqrt{2}\right)^{-i-2}, \tag{17}$$

$$w_i^{max} = \left(\sqrt{2}\right)^{1-i}. \tag{18}$$

As can be seen, for case 2: $\frac{w_i^{min}}{w_{i+1}^{max}} = \frac{1}{2}$, $\frac{w_i^{max}}{w_{i+1}^{min}} = 4$, which means that DM would be ready to "buy":

- 1 unit of improvement in i-th objective for $\frac{1}{2}$ unit of degradation in (i+1)th objective,
- 1 unit of improvement in (i+1)-th objective for $\frac{1}{4}$ unit of degradation in ith objective.

A fair quantitative comparison of wVAEA with RP-based algorithms is hard to make because DM preferences are not only handled differently, but also are specified in different formats. Thus, a crucial aspect of setting up the experiments for the four RP-based algorithms is the initial assignment of RPs. Here, all RPs were generated by PLATEMO, which ensures a uniform distribution of RPs over a true PF for each benchmark problem. However, we have applied additional filtering so as to include only the non-w-dominated RPs. Owing to this, all RP-based methods were aiming for the same tradeoffs as the tested wVAEA algorithm. All the experiments were conducted in MatLab 2018b with PLATEMO [45] platform set up for five abovementioned algorithms (wVaEA against hpaEA, RPD-NSGAII, RVEA and PICEA-g as reference EMOs).

As we mentioned in section 2.2, the main motivation behind introducing w-dominance is the inability of DM to provide RPs for some optimization problems – this includes real-time multiobjective optimization in transport. Also, due to the nature of those problems, they require complex modelling, data acquisition and data processing. Therefore it is not possible to provide a concise and unambiguous experimental comparison based on those practical applications. Hence we have decided to compare w-dominance with other preference-based methods using commonly recognized benchmarks problems: DTLZ1 – 7 and WFG1 – 9. They are configured with 3, 5, 7 and 9 objectives ($m = 3, 5, 7, 9$) to show that w-dominance is resistant to the curse-of-dimensionality. W-dominance metrics, namely wIGD, wGD and wHV, introduced previously in section 3.5, were utilized to quantitatively compare wVaEA results with the other four EMOs. The default PLATEMO's sum test (Wilcoxon's rank sum test) was used to make the comparisons at a significance level of 0.05 over 20 independent runs. The optimization software during the experiments was ran at a PC machine equipped with i7–6700 CPU@2.6 GHz, 16 GB RAM.

4.3. Parameter settings

For all the tested benchmark problems reaching the maximum number of FEs was utilized as the condition of optimization termination. The FEs number required to gain acceptable level of results might vary depending on the problem and number of objectives (m). Table 2 presents the maximum FEs assumed in each test case as a result of previous trial-and-error tests and recommendations given in [58].

Population size in all experiments was between 100 and 200, depending on the number of objectives, as shown in Table 3.

For offspring generation simulated binary crossover and polynomial mutation were used. Default PLATEMO settings for offspring generation parameters, recommended also by other researches, e.g. [59,60], were applied, namely:

- distribution index of crossover: 20,
- distribution index of mutation: 20,
- crossover probability: 1.0,
- mutation probability: 1/d, where d is the number of decision variables.

The w-dominance parameter e, depicting diversity encouragement factor, was set to 0.5, as described earlier in section 3.4.

Table 2
Maximum number of FEs in each test case in the experiments.

Problem	m	max FEs
DTLZ1	3; 5	40 000
	7; 9	50 000
DTZL2	3	10 000
	5; 7; 9	40 000
DTZL3	3; 5	100 000
	7; 9	120 000
DTLZ4	3; 5; 7; 9	50 000
DTLZ5	3	10 000
	5; 7; 9	40 000
DTLZ6	3	50 000
	5; 7; 9	60 000
DTLZ7	3	20 000
	5	50 000
	7; 9	120 000
WFG1	3; 5	100 000
	7; 9	200 000
WFG2	3	30 000
	5	50 000
	7; 9	150 000
WFG3	3	30 000
	5	80 000
	7; 9	200 000
WFG4	3	30 000
	5; 7; 9	100 000
WFG5	3; 5	100 000
	7; 9	150 000
WFG6	3	100 000
	5	120 000
	7; 9	200 000
WFG7	3	100 000
	5; 7; 9	120 000
WFG8	3	100 000
	5	150 000
	7; 9	300 000
WFG9	3; 5	100 000
	7; 9	200 000

Table 3
Population size (number of individuals) depending on the number of objectives.

Number of objectives	3	5	7	9
Population size	100	120	150	200

4.4. General results on DTLZ and WFG problems

During the experiments the w-dominance enhanced VaEA (wVaEA) algorithm has been validated and compared with four reference RP-based EMO algorithms, namely: hpaEA, RPD-NSGAII, RVEA and PICEA-g. All the algorithms were compared by their performance for DTLZ1–7 and WFG1–9 benchmark problems. Two cases of w-dominance weight settings (section 4.2) have been applied.

Tables 4–9 present results obtained for:

- wGD (Tables 4 and 5 for Case 1 & 2, respectively),
- wIGD (Tables 6 and 7 for Case 1 & 2, respectively),
- wHV (Tables 8 and 9 for Case 1 & 2, respectively).

In each table the best result for a particular benchmark problem has been shaded.

For all considered metrics it is visible (Tables 4–9) that wVaEA outperforms the four reference EMOs for the majority of test cases. The superiority of wVaEA is especially evident for wGD and wIGD. wVaEA wins 106 out of the 128 comparisons (a total of Case 1 and Case 2) for wGD and 113 out of the 128 comparisons for wIGD. In case of wHV wVaEA wins only 95 out of the 128 comparisons, however, in the majority of 33 lost comparisons wVaEA gets results close to those that the winners obtained (marked with "="). VaEA's original property of good

Table 4
Case 1: mean and standard deviation of wGD values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	m	wVaEA	hpaEA	RPD NSGAII	RVEA	PICEAg
DTLZ1	3	1,8914E-04 (1,30e-5)	2,1040E-04 (6,88e-5) =	2,5322E-04 (6,01e-5) -	2,0532E-04 (3,15e-5) -	4,8798E-04 (3,36e-4) -
	5	1,5582E-03 (3,95e-5)	3,2230E-03 (6,68e-3) =	1,7656E-03 (1,14e-4) -	1,6458E-03 (6,44e-5) -	1,7511E-03 (5,35e-4) =
	7	2,8241E-03 (5,54e-5)	7,9740E-02 (1,47e-1) -	3,2855E-03 (1,20e-4) -	3,2282E-03 (1,05e-4) -	2,7885E-03 (3,06e-4) =
	9	3,3394E-03 (7,70e-5)	2,2054E-01 (1,41e-1) -	3,7887E-03 (1,45e-4) -	3,3239E-03 (1,85e-4) =	3,4856E-03 (3,27e-4) =
DTZL2	3	6,5493E-04 (2,82e-5)	2,1038E-03 (3,39e-4) -	1,9290E-03 (7,89e-4) -	2,5732E-03 (9,58e-4) -	2,4072E-03 (1,02e-3) -
	5	1,3832E-06 (5,84e-7)	3,6200E-04 (1,69e-4) -	8,4681E-03 (5,83e-3) -	7,0576E-06 (4,07e-6) -	1,0445E-02 (3,28e-3) -
	7	1,0373E-06 (1,70e-7)	7,5618E-03 (5,33e-3) -	1,9966E-02 (4,89e-3) -	4,8097E-05 (1,29e-5) -	1,8976E-02 (6,07e-3) -
	9	1,8152E-06 (9,29e-7)	2,2406E-02 (3,95e-3) -	1,7574E-02 (9,31e-3) -	4,3643E-04 (1,56e-4) -	2,2339E-02 (4,58e-3) -
DTZL3	3	6,3568E-04 (1,17e-4)	2,3462E-03 (4,22e-4) -	1,4048E-03 (3,31e-4) -	2,3810E-03 (2,34e-4) -	2,3058E-03 (1,82e-3) -
	5	7,6718E-04 (6,33e-4)	5,1135E-03 (2,54e-3) -	6,7450E-04 (4,49e-4) =	1,6418E-03 (2,57e-3) =	1,1375E-02 (5,99e-3) -
	7	2,8626E-04 (1,86e-4)	6,6022E-01 (4,35e-1) -	1,2161E-03 (1,87e-3) -	1,4915E-03 (1,56e-3) -	2,5338E-02 (1,39e-2) -
	9	2,2126E-04 (1,25e-4)	8,9901E+00 (3,96e+0) -	2,5297E-03 (2,89e-3) -	1,7270E-03 (1,55e-3) -	3,5841E-02 (2,35e-2) -
DTLZ4	3	5,8961E-04 (2,36e-5)	1,5347E-03 (6,60e-4) -	2,3136E-03 (1,93e-3) -	2,2760E-03 (1,31e-5) -	2,2303E-03 (1,61e-3) -
	5	9,1037E-07 (3,97e-8)	1,6909E-04 (2,31e-4) -	1,7966E-02 (9,73e-3) -	3,8193E-06 (9,62e-6) -	1,7102E-02 (8,68e-3) -
	7	9,0771E-07 (3,48e-8)	2,6656E-03 (1,14e-3) -	1,5977E-02 (1,72e-2) -	8,2155E-06 (1,99e-5) -	1,5624E-02 (4,76e-3) -
	9	9,4422E-07 (1,24e-8)	1,8873E-02 (5,12e-3) -	1,1231E-02 (7,14e-3) -	1,4002E-05 (6,89e-6) -	1,7995E-02 (4,42e-3) -
DTLZ5	3	4,3152E-05 (9,08e-6)	5,0372E-04 (1,93e-4) -	7,1036E-04 (1,01e-3) -	1,6799E-02 (1,22e-2) -	5,9370E-04 (2,35e-4) -
	5	6,5514E-08 (5,26e-8)	7,7668E-02 (4,17e-2) -	3,0911E-03 (5,11e-3) -	2,5389E-06 (3,92e-6) -	2,3274E-02 (1,87e-2) -
	7	3,3707E-06 (1,76e-6)	2,1933E-01 (1,11e-1) -	4,8732E-04 (2,63e-4) -	5,6447E-05 (7,43e-5) -	7,0808E-02 (4,44e-2) -
	9	2,2158E-06 (1,56e-6)	3,6301E-01 (8,89e-2) -	7,6870E-03 (4,22e-3) -	1,4540E-03 (1,53e-3) -	6,4160E-02 (6,88e-2) -
DTLZ6	3	5,4017E-06 (2,45e-7)	1,3908E-05 (1,98e-6) -	3,3253E-07 (8,80e-7) +	2,0952E-02 (1,93e-2) -	1,2961E-05 (1,95e-6) -
	5	1,0480E-17 (2,35e-18)	2,6222E-01 (2,92e-1) -	6,2870E-08 (1,95e-7) =	6,1232E-17 (1,26e-32) -	5,1299E-02 (7,43e-2) -
	7	5,5897E-18 (0,00e+0)	1,1495E+00 (5,82e-1) -	1,9108E-08 (4,46e-8) -	6,1232E-17 (1,26e-32) -	1,7740E-01 (1,21e-1) -
	9	4,3298E-18 (0,00e+0)	2,4571E+00 (8,45e-1) -	5,7432E-06 (1,40e-5) -	1,7020E-02 (5,78e-2) -	1,7394E-01 (1,43e-1) -
DTLZ7	3	4,0296E-04 (3,10e-5)	1,9160E-03 (5,05e-4) -	1,8800E-03 (5,92e-4) -	9,1268E-03 (3,24e-3) -	1,4043E-03 (7,78e-4) -
	5	5,8625E-03 (9,31e-4)	1,1786E-02 (3,17e-3) -	1,9005E-02 (5,05e-3) -	1,7593E-02 (3,68e-3) -	8,2192E-03 (7,50e-4) -
	7	8,5450E-03 (2,18e-4)	1,0745E-01 (6,61e-2) -	1,9076E-02 (1,04e-3) -	4,5490E-02 (2,80e-2) -	1,6547E-02 (1,20e-3) -
	9	7,5460E-03 (1,44e-4)	1,7233E-01 (9,35e-2) -	2,2720E-02 (3,17e-3) -	7,9189E-02 (5,62e-2) -	3,0907E-02 (5,86e-3) -
WFG1	3	1,4666E-03 (8,00e-5)	2,4316E-03 (7,74e-4) -	3,4557E-03 (2,53e-3) -	5,2055E-03 (1,11e-3) -	5,4369E-03 (1,59e-3) -
	5	1,0736E-02 (2,57e-4)	3,4022E-02 (1,09e-2) -	5,1049E-02 (2,43e-2) -	3,2371E-02 (6,24e-3) -	4,0432E-02 (1,20e-2) -
	7	2,4461E-02 (3,06e-4)	8,1278E-02 (3,38e-2) -	1,2221E-01 (1,03e-1) -	4,3986E-02 (5,18e-3) -	1,1287E-01 (3,65e-2) -
	9	2,6935E-02 (1,94e-4)	1,7988E-01 (4,86e-2) -	8,2172E-02 (1,20e-2) -	7,1047E-02 (1,63e-2) -	2,2466E-01 (7,27e-2) -
WFG2	3	4,0147E-03 (6,40e-4)	1,4675E-02 (9,67e-3) -	4,4859E-02 (4,74e-3) -	5,2043E-02 (5,18e-3) -	3,4501E-02 (1,66e-2) -
	5	9,5584E-03 (3,49e-4)	3,4895E-02 (5,15e-3) -	3,6954E-02 (4,77e-3) -	2,9796E-02 (2,15e-3) -	4,5552E-02 (9,79e-3) -
	7	2,3608E-02 (4,24e-4)	1,5738E-01 (7,38e-2) -	1,4456E-01 (5,32e-2) -	7,9550E-02 (1,41e-2) -	1,1912E-01 (4,78e-2) -
	9	2,7084E-02 (1,98e-4)	1,5652E-01 (2,97e-2) -	1,7674E-01 (4,34e-2) -	1,3742E-01 (3,82e-2) -	2,4760E-01 (1,05e-1) -
WFG3	3	1,7675E-03 (3,07e-4)	1,9053E-02 (7,05e-3) -	2,7756E-02 (8,98e-3) -	5,8785E-02 (2,53e-2) -	8,4471E-03 (3,31e-3) -
	5	2,8576E-01 (4,21e-3)	1,6632E-01 (3,34e-2) +	6,4237E-01 (1,39e-1) -	7,3808E-01 (1,04e-1) -	1,1566E-01 (4,31e-2) +
	7	1,0339E-01 (6,44e-3)	2,7865E-01 (5,45e-2) -	9,7401E-01 (1,62e-1) -	1,4827E+00 (9,28e-1) -	1,4555E-01 (6,36e-2) -
	9	4,1175E-02 (1,08e-2)	3,7079E-01 (1,14e-1) -	6,9496E+00 (7,97e-1) -	6,8096E+00 (1,02e+0) -	1,3779E-01 (5,87e-2) -
WFG4	3	1,1616E-03 (1,85e-4)	9,1123E-03 (1,80e-3) -	3,3415E-03 (1,21e-3) -	8,2476E-03 (1,58e-3) -	6,4329E-03 (3,81e-3) -
	5	1,9478E-05 (4,01e-5)	5,6359E-02 (3,35e-2) -	5,2238E-04 (2,52e-4) -	2,5821E-03 (8,81e-4) -	5,4041E-02 (3,81e-2) -
	7	1,9509E-05 (2,73e-6)	1,7013E-01 (9,93e-2) -	1,6872E-03 (7,57e-4) -	9,4182E-03 (2,04e-3) -	2,0640E-01 (7,36e-2) -
	9	2,6398E-05 (5,84e-6)	1,9198E-01 (3,02e-2) -	9,3451E-03 (7,08e-3) -	1,9647E-02 (5,64e-3) -	3,3081E-01 (1,28e-1) -
WFG5	3	7,1652E-03 (6,58e-5)	3,0195E-02 (3,18e-3) -	3,3454E-02 (5,46e-3) -	3,5508E-02 (2,33e-3) -	3,4116E-02 (4,67e-3) -
	5	6,6997E-02 (4,44e-2)	1,2222E-01 (3,11e-2) -	9,3656E-02 (1,77e-2) -	1,0862E-01 (1,01e-2) -	1,3877E-01 (3,78e-2) -
	7	1,2075E-02 (5,31e-7)	1,9680E-01 (7,34e-2) -	1,6153E-01 (5,87e-2) -	1,3274E-01 (3,26e-4) -	3,2196E-01 (9,91e-2) -
	9	1,0714E-01 (4,06e-2)	3,9277E-01 (1,13e-1) -	3,1029E-01 (1,40e-1) -	1,5491E-01 (1,41e-3) -	5,3910E-01 (1,70e-1) -
WFG6	3	9,8107E-03 (2,35e-3)	4,7502E-02 (8,90e-3) -	4,0411E-02 (1,05e-2) -	4,2414E-02 (1,59e-2) -	4,4224E-02 (1,28e-2) -
	5	9,4398E-02 (3,96e-2)	1,4147E-01 (3,79e-2) -	1,2435E-01 (4,68e-2) =	1,2157E-01 (3,34e-2) =	1,4627E-01 (4,71e-2) -
	7	7,0671E-02 (8,10e-2)	2,1376E-01 (6,76e-2) -	1,8173E-01 (4,54e-2) -	1,8298E-01 (4,64e-2) -	2,9054E-01 (1,16e-1) -
	9	8,2604E-02 (9,00e-2)	3,6705E-01 (1,22e-1) -	2,0685E-01 (3,85e-2) -	2,1032E-01 (5,71e-2) -	5,4586E-01 (2,94e-1) -
WFG7	3	8,2179E-04 (2,02e-5)	9,3488E-04 (1,78e-3) -	5,7891E-06 (3,87e-6) +	1,7603E-04 (9,11e-5) +	3,2765E-03 (3,04e-3) -
	5	5,1343E-06 (4,39e-6)	3,6804E-02 (3,85e-2) -	8,3847E-06 (1,83e-6) -	3,6195E-04 (1,66e-4) -	7,6147E-02 (5,30e-2) -
	7	6,3807E-06 (6,60e-6)	1,7841E-01 (5,54e-2) -	1,3142E-04 (1,04e-4) -	3,0139E-03 (9,73e-4) -	2,9699E-01 (1,37e-1) -
	9	2,3014E-05 (1,41e-5)	2,7590E-01 (1,24e-1) -	1,3264E-03 (5,20e-4) -	7,5037E-03 (1,75e-3) -	5,4842E-01 (2,88e-1) -
WFG8	3	1,4990E-02 (6,61e-4)	6,6135E-02 (4,55e-3) -	6,1809E-02 (1,30e-2) -	6,7857E-02 (8,10e-4) -	6,8871E-02 (1,28e-3) -
	5	2,0975E-01 (1,41e-2)	2,0454E-01 (1,23e-1) +	2,1159E-01 (2,32e-2) -	2,2062E-01 (5,41e-4) -	2,1733E-01 (2,85e-2) =
	7	3,8863E-01 (1,78e-1)	1,8155E-01 (2,21e-2) +	3,8595E-01 (3,99e-2) =	4,9679E-01 (2,36e-1) =	3,3389E-01 (1,17e-1) =
	9	4,6632E-01 (8,44e-2)	3,8171E-01 (1,99e-1) =	9,0615E-01 (3,95e-1) -	2,9152E-01 (2,15e-1) +	4,0787E-01 (1,13e-1) =
WFG9	3	2,0409E-03 (4,23e-4)	1,8355E-02 (2,28e-3) -	1,8596E-02 (3,94e-3) -	1,7041E-02 (2,10e-3) -	1,3180E-02 (3,20e-3) -
	5	3,1162E-02 (4,48e-2)	1,7223E-01 (5,83e-2) -	1,7985E-01 (4,98e-2) -	1,0093E-01 (6,51e-2) -	1,2333E-01 (4,21e-2) -
	7	3,1738E-02 (1,91e-2)	3,3151E-01 (1,19e-1) -	5,3228E-01 (1,55e-1) -	1,2278E-01 (9,75e-2) -	2,7828E-01 (1,15e-1) -
	9	3,9716E-02 (2,48e-2)	6,7777E-01 (2,85e-1) -	7,2691E-01 (2,71e-1) -	1,7171E-01 (4,28e-2) -	6,1718E-01 (1,86e-1) -

'+', '-' and '=' denote that the result is statistically significantly better, worse and comparable to that obtained by wVaEA, respectively.

Table 5
Case 2: mean and standard deviation of wGD values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	m	wVaEA	hpaEA	RPD NSGAII	RVEA	PICEAg
DTLZ1	3	1,9153E-04 (1,39e-5)	2,1063E-04 (6,87e-5) =	2,4043E-04 (4,78e-5) -	2,0532E-04 (3,15e-5) -	4,3044E-04 (2,55e-4) -
	5	1,5958E-03 (5,38e-5)	2,0326E-03 (1,41e-4) -	1,9070E-03 (1,43e-4) -	1,7269E-03 (3,94e-5) -	7,6311E-03 (9,93e-3) -
	7	2,8112E-03 (6,65e-5)	3,0261E-03 (1,81e-4) -	2,6783E-03 (1,64e-4) +	2,7184E-03 (3,40e-4) =	1,1743E-02 (1,32e-2) -
	9	3,0698E-03 (8,78e-5)	5,0934E-01 (3,78e-1) -	3,6446E-03 (3,45e-4) -	1,0173E-02 (2,08e-2) -	5,4342E-03 (2,42e-3) -
DTZL2	3	4,3658E-04 (2,43e-5)	1,4621E-03 (4,97e-4) -	2,1474E-03 (1,48e-3) -	1,5343E-03 (6,73e-4) -	3,7981E-03 (2,81e-3) -
	5	2,6293E-03 (1,17e-4)	1,5335E-03 (1,03e-3) +	7,5299E-06 (4,36e-6) +	3,0410E-03 (2,05e-4) -	2,4437E-02 (1,81e-2) -
	7	4,4064E-03 (2,31e-4)	4,9801E-03 (3,95e-3) -	3,0901E-03 (4,80e-3) +	6,4573E-05 (3,28e-5) +	5,2425E-02 (3,31e-2) -
	9	5,9868E-03 (2,18e-4)	3,3260E-02 (1,00e-2) -	8,2467E-03 (1,36e-2) -	2,5582E-03 (5,93e-3) +	6,1259E-02 (3,37e-2) -
DTZL3	3	4,5879E-04 (7,14e-5)	1,3301E-03 (7,83e-4) -	1,4264E-03 (6,14e-4) -	8,1109E-04 (5,98e-4) =	2,0796E-03 (7,98e-4) -
	5	2,6136E-03 (1,26e-4)	9,2911E-03 (5,35e-3) -	2,4116E-03 (3,26e-3) +	3,5570E-03 (6,90e-4) -	8,2041E-02 (1,29e-1) -
	7	4,7907E-03 (3,83e-4)	3,6631E-01 (4,03e-1) -	5,5193E-03 (5,78e-3) =	6,8748E-03 (6,96e-3) =	1,2301E-01 (1,28e-1) -
	9	5,6991E-03 (1,33e-3)	4,0455E+00 (1,37e+0) -	1,8453E-02 (1,30e-2) -	7,7806E-02 (1,41e-1) =	3,4094E-01 (4,22e-1) -
DTLZ4	3	3,9754E-04 (6,20e-6)	5,2126E-04 (4,45e-4) =	3,0840E-03 (4,24e-3) -	3,2369E-04 (1,09e-5) +	4,1453E-03 (3,73e-3) -
	5	2,5835E-03 (1,30e-4)	2,3902E-03 (4,10e-3) +	5,5418E-06 (2,52e-6) +	3,2281E-03 (8,06e-4) -	2,9392E-02 (2,40e-2) -
	7	4,6561E-03 (5,31e-4)	1,1071E-02 (7,52e-3) =	6,0394E-03 (7,79e-3) =	5,3221E-06 (2,79e-6) +	5,0671E-02 (4,66e-2) -
	9	5,9210E-03 (3,50e-4)	2,8589E-02 (1,17e-2) -	1,2251E-02 (1,44e-2) =	6,3979E-05 (4,45e-5) +	9,9529E-02 (4,43e-2) -
DTLZ5	3	5,6703E-05 (2,04e-5)	3,0278E-04 (2,06e-4) -	6,9616E-05 (3,88e-5) =	1,8450E-02 (1,26e-2) -	8,3426E-04 (9,15e-4) -
	5	1,7731E-07 (1,55e-7)	1,0001E-01 (7,98e-2) -	7,3547E-06 (4,39e-6) -	4,5700E-06 (3,74e-6) -	4,9941E-02 (2,57e-2) -
	7	1,8010E-07 (1,55e-7)	3,7215E-01 (2,35e-1) -	4,4568E-02 (2,24e-2) -	6,9098E-03 (9,08e-3) -	1,0540E-01 (6,04e-2) -
	9	1,0713E-06 (1,51e-6)	5,2544E-01 (2,03e-1) -	1,4851E-03 (8,12e-4) -	3,4758E-02 (2,02e-2) -	1,2144E-01 (7,31e-2) -
DTLZ6	3	1,0986E-17 (3,16e-33)	2,1046E-07 (5,70e-7) -	2,0588E-17 (2,06e-17) =	1,9292E-02 (1,75e-2) -	6,3394E-17 (1,26e-32) -
	5	6,1232E-18 (0,00e+0)	4,3543E-01 (2,24e-1) -	7,2763E-09 (1,40e-8) -	6,1232E-17 (1,26e-32) -	1,9668E-01 (1,10e-1) -
	7	5,5897E-18 (0,00e+0)	3,3298E+00 (5,84e-1) -	1,6386E-03 (7,33e-3) -	6,1232E-17 (1,26e-32) -	2,5963E-01 (2,34e-1) -
	9	4,3298E-18 (0,00e+0)	3,4245E+00 (7,82e-1) -	1,8780E-08 (4,23e-8) -	5,8160E-02 (1,83e-1) -	3,3612E-01 (1,74e-1) -
DTLZ7	3	4,6233E-04 (3,98e-5)	1,7334E-03 (4,27e-4) -	1,7618E-03 (4,94e-4) -	8,1925E-03 (3,06e-3) -	1,3092E-03 (6,57e-4) -
	5	4,9133E-03 (3,11e-4)	1,0408E-02 (2,74e-3) -	1,4169E-02 (1,03e-3) -	1,7581E-02 (2,48e-3) -	6,1436E-03 (9,90e-4) -
	7	8,8584E-03 (3,85e-4)	2,0949E-02 (2,64e-3) -	2,1187E-02 (2,94e-3) -	4,5985E-02 (1,26e-2) -	1,3894E-02 (8,26e-4) -
	9	1,0304E-02 (3,20e-4)	1,1807E-01 (3,72e-2) -	3,8289E-02 (1,23e-2) -	5,7532E-02 (8,50e-3) -	2,8813E-02 (3,53e-3) -
WFG1	3	1,1650E-03 (6,65e-5)	1,8768E-03 (3,30e-4) -	2,8988E-03 (2,74e-3) -	6,2491E-03 (1,34e-3) -	2,9755E-03 (8,01e-4) -
	5	8,1369E-02 (3,47e-3)	1,1010E-01 (3,15e-2) -	1,1163E-01 (7,41e-2) =	5,2950E-02 (5,23e-3) +	1,2436E-01 (1,80e-2) -
	7	2,1210E-02 (6,80e-4)	6,4307E-02 (2,09e-2) -	4,5389E-02 (2,03e-2) -	4,5422E-02 (1,06e-2) -	3,8568E-02 (3,36e-3) -
	9	1,7888E-01 (5,97e-3)	3,0266E-01 (1,03e-1) -	1,6568E-01 (3,40e-3) +	1,8856E-01 (2,24e-2) =	3,1471E-01 (2,68e-2) -
WFG2	3	8,8368E-03 (8,56e-4)	2,0460E-02 (5,28e-3) -	2,3844E-02 (3,25e-3) -	3,2454E-02 (5,58e-3) -	3,7814E-02 (5,21e-3) -
	5	1,2452E-02 (5,25e-4)	2,5634E-02 (3,33e-3) -	2,3075E-02 (1,59e-3) -	2,0810E-02 (1,54e-3) -	2,9708E-02 (4,18e-3) -
	7	2,2737E-02 (1,14e-3)	7,7701E-02 (1,27e-2) -	4,0805E-02 (1,92e-3) -	5,5870E-02 (1,14e-2) -	6,0227E-02 (6,94e-3) -
	9	3,0756E-02 (3,34e-3)	6,6164E-02 (4,82e-3) -	4,2024E-02 (4,96e-3) -	8,6019E-02 (9,72e-3) -	1,0031E-01 (1,18e-2) -
WFG3	3	1,3069E-03 (3,43e-4)	1,3950E-02 (1,99e-3) -	2,1154E-02 (3,91e-3) -	4,4312E-02 (8,50e-3) -	5,1662E-03 (1,64e-3) -
	5	2,5906E-03 (1,01e-3)	6,0025E-02 (8,86e-3) -	1,1476E-01 (1,32e-1) -	8,0657E-02 (2,92e-2) -	5,7493E-02 (1,23e-2) -
	7	5,2865E-04 (1,97e-4)	2,1422E-02 (1,11e-2) -	6,7815E-02 (1,12e-2) -	2,6334E-01 (2,76e-2) -	1,1318E-01 (3,44e-2) -
	9	2,1646E-04 (6,02e-5)	5,8324E-02 (1,79e-2) -	1,8622E-01 (4,12e-2) -	3,9757E-01 (3,60e-2) -	9,3292E-02 (3,75e-2) -
WFG4	3	1,7587E-03 (2,58e-4)	7,3539E-03 (1,21e-3) -	5,0379E-03 (9,52e-4) -	7,3319E-03 (1,30e-3) -	8,2443E-03 (1,88e-3) -
	5	1,7303E-02 (7,96e-4)	5,3436E-02 (1,28e-2) -	3,0398E-02 (3,30e-3) -	3,8204E-02 (3,02e-4) -	4,2411E-02 (6,14e-3) -
	7	3,3823E-02 (1,73e-3)	1,2417E-01 (2,98e-2) -	4,0444E-02 (2,12e-2) =	5,2567E-02 (7,32e-3) -	1,0833E-01 (1,66e-2) -
	9	5,6821E-02 (3,21e-3)	2,7142E-01 (4,95e-2) -	5,6632E-02 (1,15e-2) =	1,1350E-01 (3,67e-2) -	1,8485E-01 (1,81e-2) -
WFG5	3	7,1674E-03 (3,54e-5)	1,9531E-02 (1,10e-3) -	2,5120E-02 (3,77e-3) -	2,6004E-02 (5,84e-4) -	2,5040E-02 (2,60e-3) -
	5	2,0825E-02 (7,23e-4)	5,1174E-02 (6,08e-3) -	3,9240E-02 (3,81e-3) -	4,6431E-02 (8,97e-4) -	5,2502E-02 (8,09e-3) -
	7	3,7281E-02 (1,49e-3)	1,1228E-01 (1,41e-2) -	6,2834E-02 (1,99e-2) -	7,4363E-02 (1,27e-2) -	9,9792E-02 (7,69e-3) -
	9	5,6816E-02 (2,39e-3)	3,1443E-01 (7,03e-2) -	6,0162E-02 (1,62e-2) =	1,4402E-01 (7,78e-3) -	1,9287E-01 (3,29e-2) -
WFG6	3	8,7527E-03 (2,09e-3)	3,4391E-02 (6,65e-3) -	3,3041E-02 (8,38e-3) -	3,1696E-02 (1,08e-2) -	3,4490E-02 (8,91e-3) -
	5	2,2951E-02 (2,54e-3)	6,7692E-02 (1,17e-2) -	5,5579E-02 (6,21e-3) -	5,6675E-02 (1,10e-2) -	5,3284E-02 (6,22e-3) -
	7	3,9159E-02 (1,54e-3)	1,0391E-01 (1,23e-2) -	6,5881E-02 (1,51e-2) -	1,0173E-01 (3,20e-2) -	9,7576E-02 (9,23e-3) -
	9	6,0041E-02 (1,78e-3)	2,7883E-01 (6,67e-2) -	9,3446E-02 (1,40e-2) -	1,6737E-01 (3,33e-2) -	1,6858E-01 (2,47e-2) -
WFG7	3	1,4185E-03 (2,18e-5)	4,3301E-03 (7,72e-4) -	4,8752E-03 (1,28e-3) -	3,9846E-03 (1,40e-4) -	7,0835E-03 (1,60e-3) -
	5	1,7892E-02 (8,22e-4)	5,0839E-02 (1,07e-2) -	3,2975E-02 (3,07e-3) -	3,8402E-02 (4,00e-4) -	4,7242E-02 (5,50e-3) -
	7	3,5413E-02 (1,70e-3)	1,0659E-01 (2,14e-2) -	3,5100E-02 (1,00e-2) +	3,8830E-02 (2,22e-2) -	9,7985E-02 (8,51e-3) -
	9	5,5134E-02 (1,42e-3)	2,4984E-01 (5,71e-2) -	7,8274E-02 (1,75e-2) -	7,6099E-02 (5,19e-2) =	1,7198E-01 (2,55e-2) -
WFG8	3	8,1950E-03 (4,38e-4)	4,6731E-02 (2,37e-3) -	5,2941E-02 (1,10e-2) -	4,9259E-02 (2,68e-3) -	5,7392E-02 (7,15e-3) -
	5	3,6386E-02 (8,95e-4)	6,7162E-02 (1,61e-2) -	8,1243E-02 (2,90e-2) -	7,4019E-02 (1,40e-3) -	1,2024E-01 (1,91e-2) -
	7	8,0426E-02 (2,33e-3)	1,0841E-01 (1,31e-2) -	1,8598E-01 (3,73e-2) -	1,4155E-01 (1,13e-2) -	1,6839E-01 (2,90e-2) -
	9	5,5313E-02 (1,96e-3)	2,6908E-01 (4,93e-2) -	1,0090E-01 (1,50e-2) -	1,2864E-01 (7,54e-3) -	2,3150E-01 (4,15e-2) -
WFG9	3	2,7763E-03 (1,66e-4)	1,1199E-02 (1,06e-3) -	1,4813E-02 (3,45e-3) -	1,2099E-02 (9,54e-4) -	9,7815E-03 (2,50e-3) -
	5	2,2124E-02 (1,54e-3)	5,8437E-02 (7,34e-3) -	7,7212E-02 (3,45e-2) -	4,8352E-02 (9,28e-3) -	4,6996E-02 (5,22e-3) -
	7	4,5241E-02 (6,13e-3)	1,2511E-01 (2,23e-2) -	2,0485E-01 (8,09e-2) -	1,0298E-01 (4,42e-2) -	1,0555E-01 (2,02e-2) -
	9	6,4171E-02 (5,42e-3)	3,4613E-01 (4,46e-2) -	1,2552E-01 (7,65e-2) -	1,3501E-01 (2,51e-2) -	2,1493E-01 (3,00e-2) -

'+', '-' and '=' denote that the result is statistically significantly better, worse and comparable to that obtained by wVaEA, respectively.

Table 6
Case 1: mean and standard deviation of wIGD values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	m	wVaEA	hpaEA	RPD NSGAII	RVEA	PICEAg
DTLZ1	3	2,1291E-02 (3,77e-4)	1,9636E-02 (2,36e-4) +	2,9062E-02 (1,53e-3) -	2,0617E-02 (1,01e-4) +	8,0923E-02 (1,91e-2) -
	5	6,7254E-02 (7,52e-4)	7,9261E-02 (4,63e-2) =	8,5766E-02 (4,70e-3) -	6,8533E-02 (1,05e-3) -	1,5928E-01 (2,68e-2) -
	7	9,6980E-02 (8,24e-4)	3,0512E-01 (2,46e-1) -	1,2288E-01 (9,25e-3) -	9,4616E-02 (1,40e-3) +	2,4547E-01 (3,02e-2) -
	9	1,0435E-01 (6,07e-4)	6,7214E-01 (3,03e-1) -	1,7699E-01 (2,30e-2) -	1,3325E-01 (1,46e-2) -	2,3245E-01 (2,69e-2) -
DTZL2	3	1,0779E-02 (6,02e-4)	6,6555E-02 (2,36e-3) -	6,8562E-02 (1,56e-3) -	6,9248E-02 (5,90e-4) -	6,1259E-02 (1,11e-2) -
	5	3,0953E-06 (1,29e-6)	6,6226E-04 (2,61e-4) -	1,2768E-02 (9,69e-3) -	1,4936E-05 (8,72e-6) -	1,2938E-01 (3,37e-2) -
	7	2,8093E-06 (3,91e-7)	1,6308E-01 (1,28e-1) -	8,0678E-02 (4,12e-2) -	1,1579E-04 (2,91e-5) -	2,4447E-01 (7,69e-2) -
	9	5,6241E-06 (2,71e-6)	3,4394E-01 (8,34e-2) -	1,4151E-01 (6,93e-2) -	1,1011E-03 (3,09e-4) -	3,5036E-01 (1,61e-1) -
DTZL3	3	9,6345E-03 (9,71e-4)	6,7291E-02 (2,20e-3) -	2,3538E-01 (1,42e-1) -	6,8748E-02 (1,73e-4) -	5,4203E-01 (1,13e-2) -
	5	1,7264E-03 (1,40e-3)	1,1139E-02 (5,68e-3) -	1,5108E-03 (1,04e-3) =	3,6582E-03 (5,76e-3) =	1,9056E-01 (1,33e-1) -
	7	7,6628E-04 (4,98e-4)	2,0221E+00 (1,26e+0) -	3,1077E-03 (4,39e-3) -	3,8246E-03 (4,03e-3) -	6,7667E-01 (1,82e-1) -
	9	8,5285E-03 (3,51e-2)	1,8653E+01 (6,12e+0) -	1,1582E-01 (1,28e-1) -	5,1082E-03 (4,63e-3) +	8,8620E-01 (1,08e-1) -
DTLZ4	3	9,1095E-03 (2,67e-4)	3,3634E-01 (2,88e-1) -	6,8873E-02 (1,72e-3) -	6,8604E-02 (1,65e-6) -	3,1128E-01 (2,90e-1) -
	5	2,0534E-06 (8,87e-8)	4,4282E-01 (1,91e-1) -	9,2143E-02 (7,88e-2) -	2,8290E-02 (8,71e-2) -	2,8036E-01 (1,90e-1) -
	7	2,4556E-06 (8,92e-9)	3,1771E-01 (2,39e-1) -	9,8486E-02 (1,19e-1) -	1,0118E-02 (4,52e-2) -	3,3502E-01 (1,18e-1) -
	9	2,8399E-06 (1,53e-8)	2,0884E-01 (1,27e-1) -	1,5164E-01 (1,00e-1) -	4,0519E-05 (2,03e-5) -	3,5312E-01 (6,65e-2) -
DTLZ5	3	8,7260E-04 (4,86e-5)	6,1897E-03 (9,92e-4) -	5,0667E-02 (5,98e-3) -	1,4292E-01 (1,74e-1) -	5,3567E-03 (5,44e-4) -
	5	2,4938E-07 (1,76e-7)	3,3466E-01 (2,65e-1) -	2,5106E-01 (3,43e-1) -	7,0711E-01 (3,36e-6) -	6,6721E-02 (1,52e-1) -
	7	3,3707E-06 (1,76e-6)	2,1853E-01 (1,11e-1) -	5,0892E-04 (2,75e-4) -	5,6447E-05 (7,43e-5) -	7,0847E-02 (4,71e-2) -
	9	2,2281E-06 (1,55e-6)	3,3666E-01 (1,08e-1) -	7,7700E-03 (4,16e-3) -	1,4540E-03 (1,53e-3) -	7,0939E-02 (6,15e-2) -
DTLZ6	3	6,6884E-04 (1,19e-5)	6,0720E-03 (4,78e-4) -	4,6786E-01 (2,00e-1) -	2,5737E-01 (2,30e-1) -	8,3846E-03 (1,35e-2) -
	5	1,4506E-16 (5,06e-32)	5,6335E-01 (3,55e-1) -	6,0172E-01 (2,57e-1) -	7,0711E-01 (2,28e-16) -	2,9823E-01 (3,17e-1) -
	7	6,1232E-17 (1,26e-32)	1,8706E+00 (8,64e-1) -	1,6782E-07 (3,91e-7) -	6,1232E-17 (1,26e-32) =	2,0321E-01 (1,14e-1) -
	9	6,1232E-17 (1,26e-32)	4,3530E+00 (1,04e+0) -	2,3178E-02 (5,91e-2) -	1,7020E-02 (5,78e-2) =	1,9038E-01 (1,35e-1) -
DTLZ7	3	2,3660E-02 (4,83e-4)	7,9505E-02 (3,28e-2) -	6,3540E-02 (2,61e-2) -	8,6302E-02 (1,34e-2) -	1,7959E-01 (1,24e-1) -
	5	3,2291E-01 (1,03e-1)	7,5743E-01 (3,40e-1) -	1,1378E+00 (2,17e-1) -	1,5475E+00 (2,52e-1) -	1,3218E+00 (3,07e-1) -
	7	2,3354E-02 (4,00e-3)	9,7480E-01 (7,42e-1) -	4,3195E+00 (6,02e-1) -	7,2452E-01 (5,80e-1) -	9,1411E+00 (9,63e-2) -
	9	3,7408E-02 (5,14e-3)	9,1153E-01 (4,83e-1) -	7,3603E+00 (7,62e-1) -	1,4647E+00 (1,13e+0) -	1,2258E+01 (1,35e-1) -
WFG1	3	7,2131E-02 (8,40e-3)	6,3397E-01 (1,66e-1) -	3,5703E-01 (4,40e-1) -	1,9378E-01 (7,69e-2) -	1,3665E-01 (1,55e-2) -
	5	9,9621E-02 (2,62e-3)	2,9428E-01 (7,47e-2) -	6,7227E-01 (4,68e-1) -	4,5294E-01 (1,97e-2) -	3,1544E-01 (8,31e-2) -
	7	2,6568E-01 (6,02e-3)	3,7464E-01 (6,43e-2) -	7,9262E-01 (5,16e-1) -	4,6913E-01 (1,24e-2) -	5,2045E-01 (6,19e-2) -
	9	3,6421E-01 (6,62e-3)	5,1177E-01 (8,04e-2) -	4,7641E-01 (1,41e-2) -	3,2869E-01 (2,36e-2) +	7,3968E-01 (1,18e-1) -
WFG2	3	1,5363E-02 (1,13e-3)	8,1817E-02 (2,65e-2) -	7,8745E-02 (5,11e-3) -	2,0512E-01 (1,13e-1) -	1,3648E-01 (3,51e-2) -
	5	8,7693E-02 (4,02e-3)	3,2928E-01 (4,20e-2) -	3,3885E-01 (1,48e-2) -	3,3281E-01 (3,97e-2) -	5,2801E-01 (8,71e-2) -
	7	2,5403E-01 (9,54e-3)	4,4222E-01 (3,18e-2) -	4,8609E-01 (2,06e-1) -	4,3352E-01 (4,54e-2) -	8,2338E-01 (1,58e-1) -
	9	3,6282E-01 (5,39e-3)	5,2944E-01 (3,84e-2) -	5,0512E-01 (8,76e-3) -	4,7429E-01 (5,91e-2) -	1,0440E+00 (2,45e-1) -
WFG3	3	2,7623E-02 (3,30e-3)	1,2521E-01 (2,22e-2) -	1,6944E-01 (1,40e-2) -	2,6310E-01 (2,98e-2) -	5,5959E-02 (5,97e-3) -
	5	1,8658E-01 (1,52e-2)	5,3798E-01 (8,71e-2) -	8,8609E-01 (1,41e-1) -	1,0730E+00 (3,33e-1) -	2,4959E-01 (2,98e-2) -
	7	1,7428E-01 (2,51e-2)	8,3446E-01 (1,72e-1) -	1,7490E+00 (6,18e-2) -	4,0160E+00 (1,85e+0) -	4,8347E-01 (6,29e-2) -
	9	8,6356E+00 (3,27e-2)	1,3245E+00 (2,92e-1) +	1,0353E+01 (3,71e-1) -	1,0342E+01 (3,19e-1) -	5,5775E-01 (1,13e-1) +
WFG4	3	3,7640E-03 (1,03e-3)	4,2904E-01 (1,73e-1) -	5,3541E-01 (5,12e-4) -	5,3422E-01 (1,44e-3) -	4,7035E-01 (1,47e-1) -
	5	2,0208E-05 (4,00e-5)	3,4512E+00 (1,17e+0) -	5,7048E-04 (2,78e-4) -	2,5821E-03 (8,81e-4) -	7,7133E-01 (6,67e-1) -
	7	2,0069E-05 (1,84e-6)	7,6693E+00 (1,66e+0) -	1,6872E-03 (7,57e-4) -	9,4182E-03 (2,04e-3) -	4,9929E+00 (2,50e+0) -
	9	2,9303E-05 (3,21e-6)	1,1009E+01 (1,04e+0) -	9,3451E-03 (7,08e-3) -	1,9647E-02 (5,64e-3) -	8,7761E+00 (2,22e+0) -
WFG5	3	8,7135E-02 (4,16e-3)	1,3381E-01 (8,51e-3) -	5,2030E-01 (9,86e-2) -	5,4215E-01 (2,07e-4) -	3,8556E-01 (1,90e-1) -
	5	1,1180E-01 (8,71e-6)	7,1438E-01 (6,76e-1) -	1,1843E-01 (2,32e-2) -	1,1190E-01 (5,17e-5) -	9,8462E-01 (4,50e-1) -
	7	1,3227E-01 (5,82e-6)	3,4384E+00 (1,42e+0) -	1,9825E-01 (1,50e-1) -	1,3274E-01 (3,26e-4) -	2,3288E+00 (7,04e-1) -
	9	1,5004E-01 (1,14e-4)	4,3550E+00 (1,16e+0) -	3,9307E-01 (3,02e-1) -	1,5491E-01 (1,41e-3) -	3,5508E+00 (1,81e+0) -
WFG6	3	1,1770E-01 (2,83e-2)	5,0914E-01 (1,15e-1) -	4,6085E-01 (1,73e-1) -	5,4510E-01 (5,03e-3) -	3,5288E-01 (1,89e-1) -
	5	1,4167E-01 (3,82e-2)	6,7446E-01 (8,08e-1) -	1,4890E-01 (4,40e-2) =	1,4392E-01 (3,78e-2) =	8,7139E-01 (5,54e-1) -
	7	1,6587E-01 (4,68e-2)	2,9824E+00 (1,11e+0) -	1,8173E-01 (4,54e-2) =	1,8298E-01 (4,64e-2) =	1,0803E+00 (8,45e-1) -
	9	1,9527E-01 (3,16e-2)	4,7096E+00 (1,15e+0) -	2,0892E-01 (3,32e-2) =	2,1032E-01 (5,71e-2) =	3,3782E+00 (2,25e+0) -
WFG7	3	3,3691E-04 (9,38e-5)	4,6309E-01 (1,78e-1) -	5,3564E-01 (7,56e-7) -	5,3566E-01 (9,41e-6) -	4,3924E-01 (1,57e-1) -
	5	1,0475E-05 (2,98e-6)	1,5411E+00 (1,35e+0) -	9,7604E-06 (2,45e-6) =	4,1702E-04 (2,04e-4) -	5,6636E-01 (5,29e-1) -
	7	1,4079E-05 (2,74e-6)	4,2796E+00 (1,20e+0) -	1,3343E-04 (1,04e-4) -	3,0139E-03 (9,73e-4) -	1,9825E+00 (8,95e-1) -
	9	2,5172E-05 (1,29e-5)	7,5250E+00 (1,61e+0) -	1,3264E-03 (5,20e-4) -	7,5037E-03 (1,75e-3) -	2,7013E+00 (1,14e+0) -
WFG8	3	3,3447E-01 (6,63e-3)	5,9083E-01 (1,42e-2) -	5,1607E-01 (1,00e-1) -	5,7846E-01 (4,38e-3) -	5,8383E-01 (2,28e-2) -
	5	8,9598E-01 (1,17e-3)	2,1969E+00 (9,53e-1) -	9,7088E-01 (8,13e-2) -	9,0387E-01 (4,23e-3) -	1,1371E+00 (3,17e-1) -
	7	1,6550E+00 (2,26e-1)	5,3219E+00 (4,95e-1) -	1,7482E+00 (1,98e-1) -	1,1157E+00 (7,14e-1) +	6,5100E+00 (9,05e-1) -
	9	2,2351E+00 (2,97e-1)	5,3026E+00 (2,66e+0) -	1,3577E+00 (5,31e-1) +	4,2826E-01 (3,04e-1) +	9,3043E+00 (6,89e-1) -
WFG9	3	2,5941E-02 (7,31e-3)	1,2268E-01 (7,28e-2) -	3,6227E-01 (2,15e-1) -	1,0469E-01 (1,00e-1) -	3,5394E-01 (2,08e-1) -
	5	5,6879E-02 (8,41e-2)	2,9819E-01 (2,98e-1) -	3,4495E-01 (2,60e-1) -	1,0334E-01 (7,54e-2) -	1,0737E+00 (5,05e-1) -
	7	4,9752E-02 (1,50e-2)	2,7960E+00 (1,51e+0) -	8,0031E-01 (2,14e-1) -	1,2278E-01 (9,75e-2) -	2,2089E+00 (6,14e-1) -
	9	6,3518E-02 (2,18e-2)	3,8923E+00 (1,10e+0) -	1,2932E+00 (8,95e-1) -	1,7171E-01 (4,28e-2) -	6,0860E+00 (2,44e+0) -

'+', '-' and '=' denote that the result is statistically significantly better, worse and comparable to that obtained by wVaEA, respectively.

Table 7
Case 2: mean and standard deviation of wIGD values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	m	wVaEA	hpaEA	RPD NSGAII	RVEA	PICEAg
DTLZ1	3	2,1326E-02 (3,49e-4)	1,9636E-02 (2,36e-4) +	2,9116E-02 (1,52e-3) -	2,0617E-02 (1,01e-4) +	8,0710E-02 (1,68e-2) -
	5	5,0994E-02 (1,18e-3)	6,3332E-02 (4,57e-3) -	7,9770E-02 (8,99e-3) -	5,2569E-02 (1,36e-3) -	1,6687E-01 (4,17e-2) -
	7	5,3659E-02 (1,79e-3)	5,3109E-02 (2,45e-3) =	6,1543E-02 (2,64e-3) -	1,0854E-01 (3,17e-2) -	1,6959E-01 (6,19e-2) -
	9	4,7238E-02 (2,02e-3)	8,8560E-01 (6,01e-1) -	8,0848E-02 (4,90e-3) -	1,0870E-01 (5,56e-2) -	1,1864E-01 (2,25e-2) -
DTZL2	3	3,5197E-03 (6,26e-4)	4,4326E-02 (5,93e-3) -	5,5436E-02 (4,03e-2) -	7,9484E-02 (1,09e-1) -	8,5361E-02 (1,16e-1) -
	5	1,3516E-01 (6,45e-2)	4,4910E-01 (6,13e-2) -	4,4177E-01 (6,45e-2) -	3,4195E-01 (4,84e-2) -	2,0820E-01 (6,50e-2) -
	7	2,0818E-01 (8,97e-2)	2,1762E-01 (9,14e-2) -	3,5992E-01 (1,14e-1) -	4,2896E-01 (3,75e-5) -	3,4780E-01 (1,33e-1) -
	9	3,1830E-01 (1,09e-1)	3,9866E-01 (1,74e-2) -	3,5356E-01 (4,77e-6) -	2,8371E-01 (1,45e-1) +	3,5163E-01 (1,55e-1) =
DTZL3	3	3,6835E-03 (1,02e-3)	4,5545E-02 (3,29e-3) -	1,9081E-01 (1,51e-1) -	4,6496E-02 (3,89e-4) -	2,0243E-01 (1,28e-1) -
	5	1,4473E-01 (5,74e-2)	4,7389E-01 (3,88e-2) -	3,8415E-01 (1,03e-1) -	3,2036E-01 (6,96e-2) -	3,4102E-01 (1,65e-1) -
	7	2,4638E-01 (6,93e-2)	1,0348E+00 (1,11e+0) =	2,9777E-01 (1,05e-1) -	3,0130E-01 (1,30e-1) -	5,0233E-01 (2,20e-1) -
	9	2,8404E-01 (1,45e-1)	8,0793E+00 (2,65e+0) -	3,5380E-01 (3,76e-4) -	3,7511E-01 (2,85e-1) -	6,4487E-01 (3,05e-1) -
DTLZ4	3	2,3816E-03 (1,16e-4)	2,7062E-01 (2,99e-1) -	8,8525E-02 (8,77e-2) -	4,6164E-02 (2,58e-5) -	2,4858E-01 (3,31e-1) -
	5	1,1282E-01 (7,72e-2)	7,2445E-01 (2,21e-1) -	4,7319E-01 (2,57e-6) -	3,4981E-01 (3,51e-2) -	3,8247E-01 (1,92e-1) -
	7	1,8349E-01 (1,10e-1)	4,7869E-01 (2,93e-1) -	3,3324E-01 (1,19e-1) -	3,5819E-01 (1,11e-1) -	4,8573E-01 (1,89e-1) -
	9	3,1824E-01 (1,09e-1)	4,5088E-01 (1,69e-1) -	3,5356E-01 (6,09e-7) -	2,6527E-01 (1,57e-1) =	4,8821E-01 (1,91e-1) -
DTLZ5	3	9,4396E-05 (3,92e-5)	5,0754E-04 (5,01e-4) -	1,0147E-04 (5,35e-5) =	2,0444E-02 (1,36e-2) -	1,0655E-03 (1,30e-3) -
	5	1,8278E-07 (1,54e-7)	1,0008E-01 (7,98e-2) -	8,0867E-06 (5,66e-6) -	4,5700E-06 (3,74e-6) -	4,2006E-02 (2,65e-2) -
	7	1,8342E-07 (1,52e-7)	3,7215E-01 (2,35e-1) -	4,5484E-02 (2,27e-2) -	6,9098E-03 (9,08e-3) -	9,3546E-02 (6,04e-2) -
	9	1,1065E-06 (1,51e-6)	5,2544E-01 (2,03e-1) -	1,5075E-03 (7,92e-4) -	3,4758E-02 (2,02e-2) -	9,6796E-02 (7,84e-2) -
DTLZ6	3	8,6127E-17 (2,53e-32)	2,1046E-07 (5,70e-7) -	8,6127E-17 (2,53e-32) =	2,0681E-02 (1,67e-2) -	8,6127E-17 (2,53e-32) =
	5	6,1232E-17 (1,26e-32)	6,0634E-01 (3,61e-1) -	4,2412E-08 (7,56e-8) -	6,1232E-17 (1,26e-32) =	1,4390E-01 (1,17e-1) -
	7	6,1232E-17 (1,26e-32)	6,9679E+00 (1,20e+0) -	1,6386E-03 (7,33e-3) =	6,1232E-17 (1,26e-32) =	2,3452E-01 (2,34e-1) -
	9	6,1232E-17 (1,26e-32)	4,1232E+00 (6,15e-1) -	2,3357E-08 (4,51e-8) -	5,8541E-02 (1,83e-1) -	2,5929E-01 (2,06e-1) -
DTLZ7	3	2,7155E-02 (5,46e-4)	8,7788E-02 (4,67e-2) -	7,6180E-02 (4,94e-2) -	9,0263E-02 (1,01e-2) -	2,3908E-01 (1,86e-1) -
	5	2,4140E-01 (9,15e-2)	7,7541E-01 (4,04e-1) -	6,0571E-01 (7,14e-2) -	5,4359E-01 (2,95e-2) -	1,4019E+00 (5,04e-1) -
	7	2,5451E-01 (9,97e-2)	5,4406E-01 (2,72e-2) -	5,3112E-01 (3,76e-2) -	6,5782E-01 (5,95e-2) -	1,7385E+00 (6,96e-2) -
	9	1,9284E-01 (4,95e-2)	9,0377E-01 (8,20e-2) -	8,8340E-01 (6,50e-2) -	8,6234E-01 (7,10e-2) -	1,8972E+00 (6,27e-2) -
WFG1	3	9,5434E-02 (3,47e-3)	8,4938E-01 (2,48e-1) -	3,8006E-01 (4,75e-1) -	1,5093E-01 (1,03e-2) -	1,4824E-01 (9,89e-3) -
	5	8,9072E-01 (1,03e-1)	1,4542E+00 (2,66e-1) -	6,5352E-01 (1,63e-1) +	5,4684E-01 (4,85e-2) +	1,0015E+00 (3,01e-1) =
	7	4,4932E-01 (1,80e-2)	1,3662E+00 (6,22e-1) -	1,2717E+00 (5,87e-1) -	7,1321E-01 (7,79e-2) -	7,3405E-01 (4,57e-2) -
	9	3,4863E+00 (3,17e-1)	4,4166E+00 (9,01e-1) -	4,0648E+00 (8,63e-2) -	3,1947E+00 (6,18e-1) +	4,4962E+00 (2,77e-1) -
WFG2	3	2,8296E-02 (9,14e-4)	8,1992E-02 (1,10e-2) -	7,3199E-02 (3,07e-3) -	1,1402E-01 (1,23e-2) -	1,2374E-01 (1,15e-2) -
	5	2,2324E-01 (6,46e-3)	5,3010E-01 (8,54e-2) -	5,7605E-01 (6,40e-3) -	4,9725E-01 (2,52e-2) -	5,1707E-01 (5,06e-2) -
	7	3,8203E-01 (1,19e-2)	1,4719E+00 (1,15e-1) -	9,3200E-01 (6,22e-2) -	8,0568E-01 (1,05e-1) -	9,0600E-01 (1,04e-1) -
	9	4,8341E-01 (3,46e-2)	1,2570E+00 (1,80e-1) -	6,2419E-01 (1,02e-1) -	1,0957E+00 (2,53e-1) -	1,1451E+00 (1,54e-1) -
WFG3	3	2,4662E-02 (2,33e-3)	1,1989E-01 (2,08e-2) -	1,5802E-01 (9,37e-3) -	2,3563E-01 (2,01e-2) -	5,1461E-02 (4,01e-3) -
	5	4,8722E-02 (6,14e-3)	3,2247E-01 (3,43e-2) -	7,1911E-01 (6,39e-2) -	5,0641E-01 (6,45e-2) -	2,2114E-01 (1,89e-2) -
	7	5,5105E-02 (3,44e-3)	6,7890E-01 (3,31e-1) -	6,4240E-01 (2,78e-2) -	9,1298E-01 (6,32e-2) -	4,4288E-01 (5,03e-2) -
	9	5,1758E-02 (3,59e-3)	8,3860E-01 (3,00e-1) -	1,7254E+00 (3,85e-2) -	1,4140E+00 (7,31e-2) -	5,8855E-01 (8,58e-2) -
WFG4	3	1,5754E-02 (2,27e-3)	2,4333E-01 (9,87e-2) -	2,5869E-01 (9,32e-2) -	2,8583E-01 (1,02e-1) -	3,2239E-01 (1,46e-1) -
	5	1,4064E-01 (1,94e-2)	1,9642E+00 (4,79e-1) -	2,0634E+00 (7,68e-2) -	1,4127E+00 (1,31e-1) -	1,3578E+00 (3,47e-1) -
	7	4,1877E-01 (1,67e-2)	5,3746E+00 (5,13e-1) -	3,5980E+00 (1,17e-1) -	5,2075E+00 (1,43e-1) -	2,9051E+00 (7,92e-1) -
	9	3,7059E-01 (2,56e-1)	9,0107E+00 (5,05e-1) -	3,1964E+00 (2,49e-1) -	7,8432E+00 (2,60e-1) -	4,4835E+00 (6,15e-1) -
WFG5	3	8,4002E-02 (1,29e-4)	2,0845E-01 (1,59e-2) -	6,9713E-01 (7,28e-1) -	2,8044E-01 (7,46e-2) -	2,9365E-01 (1,23e-1) -
	5	2,0377E-01 (3,44e-2)	1,4877E+00 (1,34e-1) -	2,1604E+00 (1,57e-1) -	1,5003E+00 (8,31e-2) -	1,1698E+00 (2,91e-1) -
	7	4,8237E-01 (1,65e-2)	4,6506E+00 (3,68e-1) -	3,5751E+00 (2,13e-1) -	5,3182E+00 (1,32e-1) -	2,0602E+00 (1,74e-1) -
	9	4,2864E-01 (2,40e-1)	8,8932E+00 (6,90e-1) -	3,7047E+00 (3,96e-1) -	7,4581E+00 (8,93e-1) -	3,5969E+00 (7,16e-1) -
WFG6	3	1,0265E-01 (2,58e-2)	3,2168E-01 (1,05e-1) -	3,6647E-01 (1,33e-1) -	3,1055E-01 (9,37e-2) -	3,3264E-01 (1,28e-1) -
	5	2,2909E-01 (5,13e-2)	1,7063E+00 (2,19e-1) -	2,1080E+00 (6,83e-3) -	1,4870E+00 (1,52e-2) -	1,2155E+00 (3,37e-1) -
	7	5,1983E-01 (5,77e-2)	4,7629E+00 (3,60e-1) -	3,6066E+00 (1,40e-1) -	5,2508E+00 (1,16e-1) -	2,0976E+00 (1,65e-1) -
	9	3,3515E-01 (2,31e-1)	8,4911E+00 (7,33e-1) -	3,1627E+00 (3,46e-2) -	7,7360E+00 (7,56e-1) -	3,0760E+00 (4,40e-1) -
WFG7	3	9,1386E-03 (4,59e-4)	2,7697E-01 (1,04e-1) -	3,9662E-01 (1,55e-1) -	3,1841E-01 (1,08e-1) -	3,6155E-01 (1,72e-1) -
	5	1,2813E-01 (6,03e-3)	1,8090E+00 (3,79e-1) -	2,0421E+00 (1,17e-1) -	1,4728E+00 (7,89e-2) -	1,3465E+00 (4,27e-1) -
	7	4,4895E-01 (8,84e-2)	4,7978E+00 (4,60e-1) -	3,6090E+00 (1,00e-1) -	5,2674E+00 (7,49e-2) -	2,0322E+00 (1,98e-1) -
	9	3,2834E-01 (3,68e-1)	9,1522E+00 (7,22e-1) -	3,2424E+00 (5,16e-1) -	7,9205E+00 (1,06e-3) -	3,2547E+00 (8,26e-1) -
WFG8	3	3,3063E-01 (1,02e-1)	6,7556E-01 (8,68e-2) -	1,1109E+00 (3,89e-1) -	1,1199E+00 (5,09e-1) -	1,3385E+00 (5,22e-1) -
	5	7,8590E-01 (1,45e-1)	2,3788E+00 (3,29e-1) -	2,2378E+00 (2,77e-1) -	1,7383E+00 (4,45e-2) -	2,4216E+00 (4,72e-1) -
	7	2,1201E+00 (2,25e-1)	4,9438E+00 (2,34e-1) -	3,7813E+00 (1,91e-1) -	5,0816E+00 (2,07e-1) -	4,2209E+00 (6,85e-1) -
	9	3,7310E+00 (1,45e-1)	8,6430E+00 (7,36e-1) -	6,3013E+00 (1,04e-2) -	7,8798E+00 (1,74e-1) -	5,5405E+00 (4,84e-1) -
WFG9	3	6,0658E-02 (1,06e-2)	2,0040E-01 (1,54e-2) -	6,9090E-01 (6,42e-1) -	2,4816E-01 (6,66e-2) -	2,8749E-01 (1,61e-1) -
	5	2,9699E-01 (4,83e-2)	1,6761E+00 (2,24e-1) -	2,3628E+00 (5,48e-1) -	1,6006E+00 (3,17e-1) -	1,2107E+00 (2,55e-1) -
	7	8,1181E-01 (2,02e-1)	5,0119E+00 (2,54e-1) -	3,7391E+00 (7,16e-1) -	5,0151E+00 (3,21e-1) -	2,2753E+00 (3,34e-1) -
	9	1,6068E+00 (4,89e-1)	9,2447E+00 (4,72e-1) -	8,4303E+00 (1,23e+0) -	6,7162E+00 (4,75e-1) -	4,2867E+00 (7,35e-1) -

'+', '-' and '=' denote that the result is statistically significantly better, worse and comparable to that obtained by wVaEA, respectively.

Table 8
Case 1: mean and standard deviation of wHV values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	m	wVaEA	hpaEA	RPD NSGAII	RVEA	PICEAg
DTLZ1	3	9,4496E-01 (2,46e-4)	9,4508E-01 (5,96e-4) =	9,4262E-01 (5,41e-4) -	9,4502E-01 (4,28e-4) =	9,2603E-01 (5,05e-3) -
	5	9,9830E-01 (4,58e-5)	9,9549E-01 (1,17e-2) -	9,9810E-01 (6,49e-5) -	9,9830E-01 (6,32e-5) =	9,9679E-01 (5,68e-4) -
	7	9,9998E-01 (4,46e-6)	9,6532E-01 (8,20e-2) -	9,9997E-01 (4,14e-6) -	9,9998E-01 (4,60e-6) =	9,9993E-01 (1,38e-5) -
	9	1,0000E+00 (3,66e-7)	9,6903E-01 (6,43e-2) -	1,0000E+00 (5,23e-7) =	1,0000E+00 (3,08e-7) =	1,0000E+00 (4,10e-7) =
DTZL2	3	8,6758E-01 (3,42e-4)	8,6657E-01 (4,79e-4) -	8,6662E-01 (8,06e-4) -	8,6680E-01 (3,32e-4) -	8,6395E-01 (1,46e-3) -
	5	9,9413E-01 (7,47e-5)	9,9415E-01 (6,89e-5) =	9,9379E-01 (2,41e-4) -	9,9413E-01 (7,11e-5) =	9,9162E-01 (7,12e-4) -
	7	9,9991E-01 (1,15e-5)	9,9988E-01 (1,85e-5) -	9,9987E-01 (2,03e-5) -	9,9990E-01 (1,17e-5) =	9,9975E-01 (6,55e-5) -
	9	1,0000E+00 (1,24e-6)	1,0000E+00 (1,36e-6) =	1,0000E+00 (1,03e-6) =	1,0000E+00 (8,87e-7) =	1,0000E+00 (1,38e-6) -
DTZL3	3	8,6699E-01 (8,20e-4)	8,6648E-01 (1,02e-3) =	8,6670E-01 (7,01e-4) =	8,6681E-01 (8,82e-4) =	8,6480E-01 (1,01e-3) -
	5	9,9411E-01 (9,44e-5)	9,9383E-01 (1,83e-4) -	9,9410E-01 (9,13e-5) =	9,9404E-01 (1,86e-4) =	9,9216E-01 (8,07e-4) -
	7	9,9990E-01 (1,25e-5)	6,4633E-01 (4,02e-1) -	9,9990E-01 (1,09e-5) =	9,9990E-01 (1,04e-5) =	9,9978E-01 (4,49e-5) -
	9	1,0000E+00 (8,01e-7)	0,0000E+00 (0,00e+0) -	1,0000E+00 (9,68e-7) =	1,0000E+00 (9,33e-7) =	1,0000E+00 (2,73e-6) -
DTLZ4	3	8,6805E-01 (3,70e-4)	8,2974E-01 (4,93e-2) -	8,6676E-01 (9,97e-4) -	8,6764E-01 (3,14e-4) -	8,2962E-01 (4,71e-2) -
	5	9,9414E-01 (6,14e-5)	9,9180E-01 (1,44e-3) -	9,9261E-01 (1,17e-3) -	9,9407E-01 (3,53e-4) =	9,9105E-01 (1,15e-3) -
	7	9,9991E-01 (9,77e-6)	9,9989E-01 (2,10e-5) -	9,9987E-01 (5,37e-5) -	9,9991E-01 (8,43e-6) =	9,9981E-01 (4,14e-5) -
	9	1,0000E+00 (8,34e-7)	1,0000E+00 (6,71e-7) -	1,0000E+00 (1,54e-6) =	1,0000E+00 (5,87e-7) =	1,0000E+00 (1,84e-6) -
DTLZ5	3	6,3166E-01 (4,06e-4)	6,3168E-01 (4,60e-4) =	6,3180E-01 (4,08e-4) =	6,2299E-01 (4,19e-3) -	6,3136E-01 (7,16e-4) =
	5	8,0593E-01 (4,49e-4)	7,2828E-01 (5,34e-2) -	8,0571E-01 (4,53e-4) =	8,0572E-01 (3,78e-4) =	7,8292E-01 (1,63e-2) -
	7	3,3359E-01 (5,60e-4)	1,6371E-01 (9,81e-2) -	3,3307E-01 (6,16e-4) -	3,3322E-01 (3,32e-4) -	2,5773E-01 (6,43e-2) -
	9	3,3329E-01 (3,74e-4)	5,2534E-02 (5,96e-2) -	3,2814E-01 (2,80e-3) -	3,3243E-01 (1,08e-3) -	2,6321E-01 (5,27e-2) -
DTLZ6	3	6,3203E-01 (5,02e-4)	6,3210E-01 (3,25e-4) =	6,3169E-01 (5,47e-4) =	6,2455E-01 (5,68e-3) -	6,3201E-01 (4,54e-4) =
	5	8,0593E-01 (3,81e-4)	4,7230E-01 (2,87e-1) -	7,8291E-01 (1,02e-1) =	8,0577E-01 (5,46e-4) =	7,5862E-01 (5,42e-2) -
	7	3,3324E-01 (4,46e-4)	3,8683E-03 (1,73e-2) -	3,3338E-01 (4,53e-4) =	3,3336E-01 (3,58e-4) =	1,1119E-01 (9,52e-2) -
	9	3,3339E-01 (5,02e-4)	0,0000E+00 (0,00e+0) -	2,9047E-01 (1,04e-1) =	3,2200E-01 (3,86e-2) =	1,6207E-01 (1,11e-1) -
DTLZ7	3	5,7237E-01 (6,55e-4)	5,5308E-01 (1,63e-2) -	5,6658E-01 (1,20e-2) -	5,6171E-01 (2,64e-3) -	5,1895E-01 (4,72e-2) -
	5	6,4954E-01 (1,52e-2)	5,8831E-01 (4,24e-2) -	6,4945E-01 (4,82e-3) -	6,5164E-01 (2,01e-3) +	4,9246E-01 (2,89e-2) -
	7	6,0115E-02 (2,36e-4)	2,9530E-02 (1,72e-2) -	3,1750E-05 (1,09e-4) -	3,9675E-02 (1,20e-2) -	0,0000E+00 (0,00e+0) -
	9	5,0640E-02 (2,29e-4)	1,9944E-02 (1,06e-2) -	0,0000E+00 (0,00e+0) -	2,4393E-02 (1,46e-2) -	0,0000E+00 (0,00e+0) -
WFG1	3	9,2563E-01 (2,92e-4)	8,9489E-01 (1,14e-2) -	8,8557E-01 (8,74e-2) -	9,1890E-01 (1,05e-3) -	9,2092E-01 (1,01e-3) -
	5	8,1020E-01 (6,88e-4)	7,0415E-01 (6,69e-2) -	5,0487E-01 (2,33e-1) -	6,3944E-01 (4,03e-2) -	7,1708E-01 (5,59e-2) -
	7	9,8506E-01 (2,25e-4)	9,7059E-01 (6,89e-3) -	8,3879E-01 (1,71e-1) -	9,3638E-01 (6,92e-3) -	9,3301E-01 (2,86e-2) -
	9	9,9909E-01 (3,42e-5)	9,7468E-01 (7,11e-2) -	9,9567E-01 (6,12e-4) -	9,9376E-01 (7,46e-3) -	9,7290E-01 (2,28e-2) -
WFG2	3	8,9514E-01 (6,44e-4)	8,8929E-01 (1,42e-3) -	8,9016E-01 (8,98e-4) -	8,8513E-01 (1,49e-3) -	8,8575E-01 (2,04e-3) -
	5	8,0073E-01 (1,79e-3)	6,9581E-01 (2,46e-2) -	7,1465E-01 (1,31e-2) -	6,8015E-01 (1,89e-2) -	5,7976E-01 (8,28e-2) -
	7	9,8409E-01 (3,73e-4)	9,5965E-01 (1,37e-2) -	9,4832E-01 (9,08e-2) -	9,5020E-01 (5,38e-3) -	8,1550E-01 (8,85e-2) -
	9	9,9906E-01 (4,77e-5)	9,9167E-01 (3,00e-3) -	9,9774E-01 (5,52e-4) -	9,9580E-01 (1,15e-3) -	8,9575E-01 (8,61e-2) -
WFG3	3	7,4690E-01 (8,21e-4)	7,3796E-01 (2,70e-3) -	7,3600E-01 (2,32e-3) -	7,2594E-01 (5,21e-3) -	7,4582E-01 (1,17e-3) -
	5	7,7039E-01 (8,93e-4)	7,5174E-01 (4,03e-3) -	7,6363E-01 (3,79e-3) -	7,4569E-01 (8,25e-3) -	7,5487E-01 (6,03e-3) -
	7	7,9276E-01 (4,93e-4)	7,6025E-01 (6,72e-3) -	7,8901E-01 (8,00e-4) -	7,1405E-01 (1,89e-2) -	7,5271E-01 (8,58e-3) -
	9	8,4288E-01 (1,16e-3)	7,7123E-01 (3,85e-3) -	8,4642E-01 (3,99e-4) +	8,3004E-01 (2,05e-2) =	7,6674E-01 (7,38e-3) -
WFG4	3	7,0726E-01 (7,92e-4)	6,9041E-01 (3,27e-2) -	7,0692E-01 (7,12e-4) =	7,0457E-01 (9,50e-4) -	7,0334E-01 (7,53e-3) -
	5	3,3303E-01 (4,26e-4)	0,0000E+00 (0,00e+0) -	3,3258E-01 (4,73e-4) -	3,2995E-01 (1,14e-3) -	1,2141E-01 (1,54e-1) -
	7	3,3343E-01 (3,24e-4)	0,0000E+00 (0,00e+0) -	3,3118E-01 (1,08e-3) -	3,2038E-01 (2,76e-3) -	4,9438E-03 (2,21e-2) -
	9	3,3345E-01 (5,61e-4)	0,0000E+00 (0,00e+0) -	3,1992E-01 (1,02e-2) -	3,0483E-01 (8,18e-3) -	0,0000E+00 (0,00e+0) -
WFG5	3	6,7644E-01 (1,33e-3)	6,6698E-01 (2,77e-3) -	6,7682E-01 (5,50e-4) =	6,7680E-01 (3,27e-4) =	6,5747E-01 (2,48e-2) -
	5	1,8348E-01 (4,04e-4)	8,4869E-02 (8,56e-2) -	1,7618E-01 (2,46e-2) -	1,8309E-01 (3,69e-4) -	1,2405E-02 (3,81e-2) -
	7	1,5008E-01 (3,46e-4)	6,8395E-04 (3,06e-3) -	1,1709E-01 (5,57e-2) -	1,4934E-01 (5,09e-4) -	0,0000E+00 (0,00e+0) -
	9	1,1646E-01 (4,15e-4)	0,0000E+00 (0,00e+0) -	2,4648E-02 (3,71e-2) -	1,0930E-01 (2,02e-3) -	0,0000E+00 (0,00e+0) -
WFG6	3	6,6382E-01 (1,16e-2)	6,6269E-01 (1,05e-2) =	6,6839E-01 (1,11e-2) =	6,6957E-01 (1,49e-2) =	6,5778E-01 (1,53e-2) =
	5	1,4323E-01 (5,13e-2)	7,3650E-02 (6,16e-2) -	1,3359E-01 (5,90e-2) =	1,4037E-01 (5,07e-2) =	1,8592E-02 (3,26e-2) -
	7	1,0349E-01 (6,48e-2)	0,0000E+00 (0,00e+0) -	8,5911E-02 (5,43e-2) =	8,3299E-02 (5,82e-2) =	1,3908E-02 (2,90e-2) -
	9	5,2547E-02 (4,39e-2)	0,0000E+00 (0,00e+0) -	3,6747E-02 (4,26e-2) =	5,0880E-02 (5,30e-2) =	0,0000E+00 (0,00e+0) -
WFG7	3	7,0851E-01 (4,57e-4)	7,0795E-01 (9,50e-4) -	7,0831E-01 (4,96e-4) =	7,0832E-01 (4,99e-4) =	6,9485E-01 (1,68e-2) -
	5	3,3350E-01 (4,05e-4)	1,1560E-01 (1,62e-1) -	3,3360E-01 (4,07e-4) =	3,3284E-01 (4,78e-4) -	1,3501E-01 (1,56e-1) -
	7	3,3330E-01 (4,76e-4)	0,0000E+00 (0,00e+0) -	3,3323E-01 (4,16e-4) =	3,2933E-01 (1,49e-3) -	1,5491E-02 (6,93e-2) -
	9	3,3326E-01 (4,26e-4)	0,0000E+00 (0,00e+0) -	3,3143E-01 (9,00e-4) -	3,2271E-01 (2,69e-3) -	0,0000E+00 (0,00e+0) -
WFG8	3	5,7994E-01 (4,29e-3)	5,7049E-01 (1,51e-2) -	5,9381E-01 (3,09e-3) +	5,7857E-01 (2,85e-3) =	5,8787E-01 (1,80e-2) +
	5	0,0000E+00 (0,00e+0)	0,0000E+00 (0,00e+0) =	0,0000E+00 (0,00e+0) =	0,0000E+00 (0,00e+0) =	0,0000E+00 (0,00e+0) =
	7	0,0000E+00 (0,00e+0)	0,0000E+00 (0,00e+0) =	0,0000E+00 (0,00e+0) =	0,0000E+00 (0,00e+0) =	0,0000E+00 (0,00e+0) =
	9	0,0000E+00 (0,00e+0)	0,0000E+00 (0,00e+0) =	0,0000E+00 (0,00e+0) =	6,2314E-02 (1,13e-1) +	0,0000E+00 (0,00e+0) =
WFG9	3	6,9723E-01 (3,46e-3)	6,8554E-01 (5,95e-3) -	6,8867E-01 (8,19e-3) -	6,9465E-01 (2,48e-3) -	6,8966E-01 (1,23e-2) -
	5	2,6783E-01 (6,57e-2)	7,7876E-02 (6,62e-2) -	3,7563E-02 (4,84e-2) -	2,0562E-01 (5,48e-2) -	1,4065E-02 (5,28e-2) -
	7	2,6481E-01 (2,11e-2)	2,4211E-03 (8,02e-3) -	0,0000E+00 (0,00e+0) -	1,8408E-01 (5,40e-2) -	0,0000E+00 (0,00e+0) -
	9	2,4172E-01 (3,18e-2)	0,0000E+00 (0,00e+0) -	0,0000E+00 (0,00e+0) -	1,0097E-01 (4,76e-2) -	0,0000E+00 (0,00e+0) -

'+', '-' and '=' denote that the result is statistically significantly better, worse and comparable to that obtained by wVaEA, respectively.

Table 9
Case 2: mean and standard deviation of wHV values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	m	wVaEA	hpaEA	RPD NSGAII	RVEA	PICEAg
DTLZ1	3	9,4661E-01 (3,42e-4)	9,4668E-01 (5,76e-4) =	9,4497E-01 (4,86e-4) -	9,4672E-01 (4,12e-4) =	9,3407E-01 (3,00e-3) -
	5	9,8936E-01 (2,02e-4)	9,8882E-01 (5,66e-4) -	9,8931E-01 (3,36e-4) =	9,8987E-01 (1,87e-4) +	9,7158E-01 (2,06e-2) -
	7	9,8499E-01 (3,61e-4)	9,8605E-01 (2,91e-4) +	9,8578E-01 (3,59e-4) +	9,8224E-01 (2,74e-3) -	9,5204E-01 (4,02e-2) -
	9	9,8781E-01 (2,50e-4)	2,9768E-01 (3,15e-1) -	9,8792E-01 (3,91e-4) =	9,7994E-01 (2,34e-2) -	9,8391E-01 (5,01e-3) -
DTZL2	3	8,9684E-01 (2,17e-4)	8,9600E-01 (5,66e-4) -	8,9560E-01 (1,71e-3) -	8,9656E-01 (2,56e-4) -	8,9168E-01 (3,51e-3) -
	5	9,8114E-01 (1,54e-4)	9,8097E-01 (1,17e-4) -	9,8111E-01 (1,32e-4) =	9,8118E-01 (1,48e-4) =	9,6563E-01 (6,72e-3) -
	7	9,8112E-01 (1,10e-4)	9,8045E-01 (4,85e-4) -	9,8075E-01 (1,80e-3) =	9,8116E-01 (1,26e-4) =	9,5629E-01 (1,52e-2) -
	9	9,8114E-01 (1,31e-4)	9,7552E-01 (2,50e-3) -	9,8120E-01 (1,65e-4) =	9,8110E-01 (1,20e-4) =	9,3130E-01 (3,88e-2) -
DTZL3	3	8,9642E-01 (4,57e-4)	8,9611E-01 (7,71e-4) =	8,9652E-01 (6,24e-4) =	8,9642E-01 (6,80e-4) =	8,9520E-01 (7,43e-4) -
	5	9,8104E-01 (1,43e-4)	9,7990E-01 (7,74e-4) -	9,8109E-01 (1,24e-4) =	9,8093E-01 (2,57e-4) =	9,2850E-01 (4,74e-2) -
	7	9,8098E-01 (1,77e-4)	6,3962E-01 (3,95e-1) -	9,8118E-01 (1,35e-4) +	9,8101E-01 (1,83e-4) =	8,6645E-01 (1,02e-1) -
	9	9,8106E-01 (1,47e-4)	0,0000E+00 (0,00e+0) -	9,8113E-01 (1,19e-4) =	9,4843E-01 (8,34e-2) -	8,0403E-01 (2,20e-1) -
DTLZ4	3	8,9706E-01 (3,57e-4)	8,4373E-01 (8,64e-2) -	8,9549E-01 (2,22e-3) -	8,9710E-01 (2,55e-4) =	8,4142E-01 (1,06e-1) -
	5	9,8113E-01 (1,31e-4)	9,3300E-01 (1,12e-1) -	9,8115E-01 (1,65e-4) =	9,8121E-01 (1,06e-4) =	9,6419E-01 (1,96e-2) -
	7	9,8115E-01 (1,12e-4)	9,1191E-01 (2,18e-1) -	9,8113E-01 (1,32e-4) =	9,8114E-01 (1,10e-4) =	9,5212E-01 (2,78e-2) -
	9	9,8113E-01 (1,51e-4)	9,6887E-01 (1,90e-2) -	9,8113E-01 (1,47e-4) =	9,8113E-01 (1,65e-4) =	9,3919E-01 (3,01e-2) -
DTLZ5	3	7,5608E-01 (3,72e-4)	7,5587E-01 (4,16e-4) =	7,5621E-01 (3,36e-4) =	7,5362E-01 (9,85e-4) -	7,5585E-01 (5,71e-4) =
	5	3,3334E-01 (5,58e-4)	2,6462E-01 (5,40e-2) -	3,3317E-01 (4,73e-4) =	3,3334E-01 (4,64e-4) =	2,8624E-01 (3,00e-2) -
	7	3,3334E-01 (4,40e-4)	1,1758E-01 (1,17e-1) -	3,0300E-01 (1,51e-2) -	3,2876E-01 (6,16e-3) -	2,4245E-01 (5,96e-2) -
	9	3,3332E-01 (4,69e-4)	4,7367E-02 (6,99e-2) -	3,3240E-01 (7,14e-4) -	3,1017E-01 (1,35e-2) -	2,3203E-01 (7,23e-2) -
DTLZ6	3	7,5616E-01 (4,26e-4)	7,5622E-01 (4,22e-4) =	7,5608E-01 (3,92e-4) =	7,5533E-01 (8,35e-4) -	7,5621E-01 (3,92e-4) =
	5	3,3323E-01 (4,95e-4)	4,1610E-02 (1,04e-1) -	3,3336E-01 (3,85e-4) =	3,3334E-01 (5,37e-4) =	1,3095E-01 (1,18e-1) -
	7	3,3339E-01 (4,68e-4)	0,0000E+00 (0,00e+0) -	3,3219E-01 (5,10e-3) =	3,3321E-01 (3,31e-4) =	1,4999E-01 (1,36e-1) -
	9	3,3336E-01 (3,86e-4)	0,0000E+00 (0,00e+0) -	3,3326E-01 (3,79e-4) =	2,9912E-01 (7,81e-2) -	5,6233E-02 (8,64e-2) -
DTLZ7	3	5,6915E-01 (5,04e-4)	5,5238E-01 (1,39e-2) -	5,6364E-01 (7,21e-3) -	5,5710E-01 (2,54e-3) -	5,2617E-01 (3,65e-2) -
	5	6,3139E-01 (3,22e-3)	5,8166E-01 (2,87e-2) -	6,0320E-01 (7,05e-3) -	6,1659E-01 (4,13e-3) -	5,4435E-01 (2,25e-2) -
	7	5,4215E-01 (3,80e-3)	5,2851E-01 (4,72e-3) -	5,2900E-01 (5,28e-3) -	4,9808E-01 (8,53e-3) -	4,6921E-01 (1,98e-3) -
	9	4,9646E-01 (1,17e-3)	4,5500E-01 (6,10e-3) -	4,5191E-01 (7,98e-3) -	4,4733E-01 (6,78e-3) -	4,3807E-01 (2,36e-3) -
WFG1	3	9,4202E-01 (2,26e-4)	8,7706E-01 (2,53e-2) -	8,9973E-01 (8,80e-2) -	9,3510E-01 (1,56e-3) -	9,3968E-01 (5,01e-4) -
	5	9,9410E-01 (3,19e-4)	9,7796E-01 (7,05e-3) -	9,9014E-01 (6,86e-3) -	9,9408E-01 (2,86e-4) =	9,9222E-01 (8,33e-4) -
	7	9,9977E-01 (2,20e-5)	9,8668E-01 (1,66e-2) -	8,8557E-01 (1,92e-1) -	9,9933E-01 (1,30e-4) -	9,9868E-01 (3,70e-4) -
	9	9,9098E-01 (1,51e-3)	9,0180E-01 (1,50e-1) -	9,9299E-01 (7,16e-3) +	9,9309E-01 (1,24e-3) +	9,8501E-01 (7,40e-3) -
WFG2	3	9,1428E-01 (4,16e-4)	9,0894E-01 (1,03e-3) -	9,1011E-01 (6,98e-4) -	9,0204E-01 (1,85e-3) -	9,0599E-01 (1,77e-3) -
	5	9,9604E-01 (1,15e-4)	9,8778E-01 (3,49e-3) -	9,8858E-01 (2,42e-4) -	9,8963E-01 (9,70e-4) -	9,8830E-01 (1,81e-3) -
	7	9,9952E-01 (4,98e-5)	9,7434E-01 (6,62e-3) -	9,9740E-01 (4,33e-4) -	9,9741E-01 (9,21e-4) -	9,8986E-01 (3,54e-3) -
	9	1,0000E+00 (2,89e-6)	9,9994E-01 (3,01e-5) -	9,9999E-01 (5,27e-6) =	9,9996E-01 (3,14e-5) -	9,9966E-01 (1,10e-4) -
WFG3	3	7,3552E-01 (7,68e-4)	7,2281E-01 (2,19e-3) -	7,1923E-01 (2,88e-3) -	7,0302E-01 (6,92e-3) -	7,3369E-01 (1,05e-3) -
	5	7,2965E-01 (1,24e-3)	7,0201E-01 (5,15e-3) -	7,0174E-01 (9,47e-3) -	7,0060E-01 (5,14e-3) -	7,0526E-01 (5,54e-3) -
	7	7,2932E-01 (5,92e-4)	7,1517E-01 (7,34e-3) -	7,1077E-01 (2,42e-3) -	6,5467E-01 (1,21e-2) -	6,8063E-01 (9,15e-3) -
	9	7,2923E-01 (4,73e-4)	6,9623E-01 (7,42e-3) -	6,6628E-01 (6,86e-3) -	5,7713E-01 (1,37e-2) -	6,8873E-01 (1,02e-2) -
WFG4	3	8,8185E-01 (4,61e-4)	8,7757E-01 (7,36e-3) -	8,8201E-01 (4,55e-4) =	8,8126E-01 (4,79e-4) -	8,8063E-01 (1,12e-3) -
	5	9,9030E-01 (1,30e-4)	9,6883E-01 (4,02e-3) -	9,9033E-01 (1,07e-4) =	9,8062E-01 (2,24e-4) -	9,8684E-01 (1,49e-3) -
	7	9,9965E-01 (1,90e-5)	9,9920E-01 (1,45e-4) -	9,9965E-01 (2,36e-5) =	9,9950E-01 (2,63e-5) -	9,9918E-01 (1,66e-4) -
	9	1,0000E+00 (1,09e-6)	1,0000E+00 (2,21e-6) =	1,0000E+00 (8,13e-7) =	1,0000E+00 (7,86e-7) =	9,9999E-01 (5,42e-6) -
WFG5	3	8,7274E-01 (3,98e-4)	8,6820E-01 (1,11e-3) -	8,7248E-01 (4,18e-4) =	8,7243E-01 (2,91e-4) -	8,6901E-01 (2,88e-3) -
	5	9,8880E-01 (1,30e-4)	9,6862E-01 (1,96e-3) -	9,8871E-01 (1,28e-4) -	9,7854E-01 (1,54e-4) -	9,8488E-01 (1,34e-3) -
	7	9,9956E-01 (1,87e-5)	9,9900E-01 (1,23e-4) -	9,9955E-01 (2,70e-5) =	9,9940E-01 (2,61e-5) -	9,9906E-01 (1,82e-4) -
	9	1,0000E+00 (1,29e-6)	1,0000E+00 (1,50e-6) -	1,0000E+00 (7,88e-7) =	1,0000E+00 (1,05e-6) =	9,9999E-01 (7,37e-6) -
WFG6	3	8,7031E-01 (3,29e-3)	8,6791E-01 (3,22e-3) -	8,6976E-01 (3,51e-3) =	8,7024E-01 (4,73e-3) =	8,6696E-01 (4,27e-3) -
	5	9,8818E-01 (6,01e-4)	9,7089E-01 (2,26e-3) -	9,8803E-01 (4,65e-4) =	9,7756E-01 (1,12e-3) -	9,8456E-01 (1,33e-3) -
	7	9,9954E-01 (3,98e-5)	9,9895E-01 (1,76e-4) -	9,9953E-01 (5,05e-5) =	9,9933E-01 (6,86e-5) -	9,9890E-01 (1,93e-4) -
	9	1,0000E+00 (1,02e-6)	1,0000E+00 (1,41e-6) -	1,0000E+00 (2,06e-6) =	1,0000E+00 (1,28e-6) -	9,9998E-01 (1,17e-5) -
WFG7	3	8,8283E-01 (3,30e-4)	8,8245E-01 (5,96e-4) =	8,8254E-01 (3,00e-4) -	8,8272E-01 (2,37e-4) =	8,8033E-01 (1,97e-3) -
	5	9,9036E-01 (9,55e-5)	9,7527E-01 (1,90e-3) -	9,9033E-01 (1,04e-4) =	9,8077E-01 (1,36e-4) -	9,8778E-01 (1,05e-3) -
	7	9,9966E-01 (1,87e-5)	9,9925E-01 (1,87e-4) -	9,9966E-01 (2,13e-5) =	9,9952E-01 (2,75e-5) -	9,9936E-01 (1,20e-4) -
	9	1,0000E+00 (1,17e-6)	1,0000E+00 (1,45e-6) =	1,0000E+00 (1,10e-6) =	1,0000E+00 (1,28e-6) =	9,9999E-01 (4,14e-6) -
WFG8	3	8,5468E-01 (8,94e-4)	8,4985E-01 (3,11e-3) -	8,5637E-01 (8,86e-4) +	8,5255E-01 (8,14e-4) -	8,5611E-01 (2,02e-3) +
	5	9,8321E-01 (1,53e-4)	9,6984E-01 (2,01e-3) -	9,8208E-01 (6,69e-4) -	9,7853E-01 (1,40e-4) -	9,8142E-01 (1,23e-3) -
	7	9,9950E-01 (3,01e-5)	9,9915E-01 (1,79e-4) -	9,9952E-01 (2,63e-5) +	9,9952E-01 (1,98e-5) =	9,9937E-01 (2,60e-4) -
	9	1,0000E+00 (7,95e-7)	1,0000E+00 (1,29e-6) -	1,0000E+00 (8,87e-7) =	1,0000E+00 (1,12e-6) =	1,0000E+00 (2,73e-6) =
WFG9	3	8,7901E-01 (8,82e-4)	8,7430E-01 (1,54e-3) -	8,7674E-01 (1,65e-3) -	8,7825E-01 (8,57e-4) -	8,7789E-01 (1,67e-3) -
	5	9,8932E-01 (2,25e-4)	9,6880E-01 (2,08e-3) -	9,8209E-01 (4,39e-3) -	9,7854E-01 (4,73e-4) -	9,8574E-01 (1,13e-3) -
	7	9,9960E-01 (2,33e-5)	9,9888E-01 (3,04e-4) -	9,9925E-01 (2,73e-4) -	9,9943E-01 (4,76e-5) -	9,9917E-01 (2,09e-4) -
	9	1,0000E+00 (9,99e-7)	1,0000E+00 (1,37e-6) =	1,0000E+00 (1,29e-6) =	1,0000E+00 (1,36e-6) =	9,9999E-01 (4,70e-6) -

'+', '-' and '=' denote that the result is statistically significantly better, worse and comparable to that obtained by wVaEA, respectively.

convergence of the obtained solutions [33] has been reinforced by w-dominance incorporated in wVaEA, which is reflected by better wGD metric values. At the same time, superior values of wIGD indicate that this good convergence is achieved without sacrificing coverage.

It is particularly interesting that for WFG2 and WFG9 problems wVaEA obtained the best results for all metrics, regardless of the number of objectives (*m*) and the weight settings (Case 1 and 2). Nearly the same is true for DTZ7 problem. Based on this we conclude that wVaEA is a promising algorithm for handling problems with disconnected PFs (DTLZ7) or non-separable fitness landscapes (WFG2 and 9). As for the other test cases, the four reference EMOs only occasionally do better than wVaEA with RPDNSGAII and RVEA being the most successful of them. For some test cases of RPDNSGAII and RVEA evidently do much better than wVaEA (by an order of magnitude) in terms of wGD and wIGD metrics, which indicates significantly better convergence and coverage of those algorithms.

It is noticeable that the best wGD results are not necessarily accompanied by the best wIGD values and, surprisingly, this is also true for RPDNSGAII, which is generally known to have a good coverage. In such cases the inferior wIGD values may result from the phenomena described in section 3.5. Namely, some of the final solutions are w-dominated hence the non w-dominated points in true PF do not have their counterparts in the considered part of the solution set. In comparison, wVaEA is less likely to suffer from this phenomena, because it is oriented on promoting non w-dominated solutions during optimization process. In general, the superior metric values of wVaEA may mostly result from the fact that the algorithm does not use RPs, so its performance is not affected by the imperfect choice of RP coordinates.

4.5. Problematic results of wIGD and wHV metrics for some problems

Of the three proposed metrics, wGD can be considered the most reliable as its values do not depend on the number of w-dominated solutions in the final set. In comparison, wIGD and wHV values are heavily affected by the number of w-dominated (and thus excluded) solutions. This is particularly evident in case of WFG8 problem, where wIGD values are larger by 2 orders of magnitude than wGD values and may suggest at first glance that wVAEA completely failed to solve this benchmark problem. This is not true, in fact, such weak results show that the algorithm could not find solutions, which would approximate the true PF points while being compliant with DM-specified preferences. This can be partially attributed to the already mentioned phenomenon from section 3.5.1 (Fig. 1) and partially to the population size, which is relatively small (150–200 individuals) for 7–9 objective cases. The latter conclusion is supported by the fact that the other algorithms also failed to obtain reasonable wIGD values for WFG8 problem. As for wHV metric, it was particularly sensitive to the size of non-w-dominated part of the solution set, hence largely differing wHV values were obtained for various problems. In general it can be stated that, while it is easy to measure the convergence of the proposed tradeoff-based method, the coverage and diversity are much harder to assess. This is due to the nature of tradeoff-specified preferences: the narrower the weight intervals are, the less DM is actually interested in coverage of the true PF. Nevertheless, improving the population diversity obtained by w-algorithms is one of the goals, which we will pursue in the forthcoming research. Alternatives for wVAEA will be tried for this, especially combinations of w-dominance with most successful recent decomposition-based and decomposition-related algorithms, e.g. OPE-MOEA [61].

4.6. Influence of prescreening PF and prescreening final solutions

In the presented results of wIGD both the true PF and final set were filtered to only include non-w-dominated solutions (as they are of DM's interest). However, in the course of additional simulation considerably better results were obtained for all five algorithms when the final set remained unfiltered (we do not provide detailed results due to limited space). In such case, the points in the filtered true PF had larger chance of having their counterparts in the final set (Fig. 1), which resulted in smaller distances between true PF points and points in the final solution set.

Similarly, in case of wGD, leaving true PF unfiltered resulted in smaller average distances between the final set and true PF. Significantly inferior results have been obtained in case of filtering both true PF and final set, the reason for this being the phenomena illustrated in Fig. 2 (again, we do not provide detailed results).

However, for all combinations of filtered / unfiltered true PF and final set the results registered for wVAEA were considerably better (smaller wIGD and wGD metric values) than those registered for the four RP-based algorithms.

4.7. Influence of diversity encouragement factor

In the course of simulations it has been found that it is best to set the diversity encouragement factor from (14) to 0.5. For values smaller than 0.4 wVAEA tends to lose diversity and converge too fast at the cost of coverage and accuracy of approximating true PF. On the other hand, for values larger than 0.6, the algorithm practically ignores DM's preferences in the initial generations and this lack of focus also translates to a poorer overall performance (once more – we do not provide detailed results due to limited space).

4.8. Additional studies: comparing wVaEA with original VaEA algorithm

Comparing wVaEA with original VaEA is problematic due to different purposes of both algorithms. wVaEA focuses on addressing DM's preferences and thus targets only those parts of the true PF, which comply with those preferences. In contrast to that, original VaEA aims at approximating full true PF. Consequently, applying proposed wGD, wIGD and wHV metrics (which include preference-based prescreening) results in wVaEA outperforming the original VaEA algorithm. This can be seen in Table A1 of Appendix 1, where wVAEA scores better in the vast majority of comparison cases. On the other hand, if we use classic metrics, original VaEA gets significantly better values of IGD, as shown in Table A2 of Appendix 1. The reason for this is that original VaEA tries to approximate full PF and thus returns more diverse solutions. However, in both cases (metrics with and without preference-based prescreening) wVaEA performs better in terms of GD, because good convergence is much easier to achieve for a preference-based algorithm, which deals with a selected subset of objective space.

5. Conclusions

RP-based EMO algorithms remain the most popular means of handling DM preferences in MOPs. However, for some real world optimization problems (especially real time ones) they are inconvenient or even impossible to apply, as they require DM to enter RP coordinates for each particular occurrence of a MOP. In such situations it is much easier for a DM to specify off-time a general policy of tradeoffs, which will then be applied automatically in real time, whenever necessary. In the paper we propose to enter and handle DM preferences as weight intervals assigned to objectives. Such intervals can reflect both DM's assessment of objectives' importance and uncertainty of this assessment. Based on this a new dominance relation – w-dominance – is introduced, which extends the tradeoff coefficient approach [21] from bi-objective to multi-objective. As we show, checking w-dominance can be done linearly with regard to the number of objectives. W-dominance can be incorporated into practically any EMO or MOMH algorithm and we have observed the best results when combining it with VaEA [33] in the form of wVaEA. We have compared the proposed wVaEA algorithm with four state-of-the-art RP-based EMO methods on 3 to 9-objective DTLZ and WFG benchmark problems using specially designed wGD, wIGD and wHV metrics, which take into account DM preferences. wVAEA has outperformed the other four algorithms for the majority of the test cases, thus showing that w-dominance can be an interesting alternative to RP-based approaches, especially for problems with up to five objectives. However, the major difference between w-dominance and RP-based methods remains qualitative rather than quantitative. The main field of application for w-dominance are MOPs where RPs simply cannot be specified because true PF is hard to assess a priori and interaction is impossible due to time pressure or DM being physically engaged in other activities.

Future research into w-dominance will be focused on w-algorithms for constraint problems as well as improving the method's performance for many objective optimization problems. The former will aim at w-enhancing and testing multiple existing MOMHs. Targeted MOEAs will include recent ones based on decomposition (OPE-MOEA and FDEA) [61,62] as well as other emerging algorithms obtaining good convergence and diversity simultaneously, e.g. ensemble MaOEAs [63]. As for improving the method's performance for MAOPs, it will involve developing an efficient non w-dominated sort for many objective optimization and testing the algorithms on more challenging benchmark problems [64]. Also, the topic of w-dominance metrics will be further investigated to address the issue of diversity among the final non-w-dominated solutions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Rafal Szlapczynski: Conceptualization, Methodology, Software, Investigation, Resources, Writing - original draft, Writing - review & editing, Supervision. **Joanna Szlapczynska:** Methodology, Investigation, Resources, Writing - review & editing, Funding acquisition.

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Appendix 1 – Results of Comparing wVaEA with original VaEA algorithm

Table A1 presents comparison of wVaEA and VaEA by means of the proposed w-dominance metrics: wIGD, wGD and wHV. Table A2 presents comparison of wVaEA and VaEA by means of classic IGD and GD metrics. Both tables include data for DTLZ1-7 and WFG1-9 problems with best results shaded.

Table A1
wVaEA vs VaEA comparison: mean of wIGD, wGD and wHV values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	*m*	wIGD		wGD		wHV	
		wVaEA	VaEA	wVaEA	VaEA	wVaEA	VaEA
DTLZ1	3	2,14E-02	2,97E-02	1,92E-04	2,54E-04	8,60E-01	8,53E-01
	5	6,75E-02	1,57E-01	1,55E-03	1,52E-02	9,92E-01	9,59E-01
	7	9,72E-02	3,15E-01	2,85E-03	2,53E-02	1,00E+00	9,15E-01
	9	1,05E-01	3,97E-01	3,31E-03	1,09E-01	1,00E+00	9,73E-01
DTZL2	3	1,07E-02	6,73E-02	6,56E-04	2,22E-03	6,64E-01	6,62E-01
	5	3,30E-06	4,44E-04	1,47E-06	2,27E-04	9,72E-01	9,72E-01
	7	2,97E-06	3,89E-03	1,10E-06	1,78E-03	9,99E-01	9,99E-01
	9	7,07E-06	5,12E-02	2,23E-06	2,09E-02	1,00E+00	1,00E+00
DTZL3	3	9,48E-03	7,12E-02	6,34E-04	1,77E-03	6,63E-01	6,57E-01
	5	1,38E-03	4,28E-01	6,16E-04	1,14E-01	9,72E-01	8,52E-01
	7	1,31E-03	1,41E+00	4,87E-04	4,33E-01	9,99E-01	7,12E-01
	9	9,85E-04	7,54E+00	3,19E-04	5,54E+00	1,00E+00	4,98E-02
DTLZ4	3	8,97E-03	6,33E-02	5,71E-04	2,07E-03	6,65E-01	6,64E-01
	5	2,06E-06	5,10E-04	8,99E-07	2,79E-04	9,72E-01	9,72E-01
	7	2,46E-06	4,39E-03	9,16E-07	2,07E-03	9,99E-01	9,99E-01
	9	7,86E-03	3,59E-02	9,43E-07	1,51E-02	1,00E+00	1,00E+00
DTLZ5	3	8,76E-04	7,11E-03	4,41E-05	7,55E-04	3,48E-01	3,47E-01
	5	3,35E-07	2,94E-01	8,66E-08	9,88E-03	5,70E-01	5,68E-01
	7	2,13E-06	1,65E-03	2,06E-06	1,62E-03	9,09E-02	8,94E-02
	9	3,29E-06	2,33E-02	3,24E-06	2,33E-02	9,10E-02	6,97E-02
DTLZ6	3	6,69E-04	5,70E-03	5,35E-06	1,21E-05	3,48E-01	3,48E-01
	5	1,45E-16	5,20E-01	1,02E-17	2,09E-02	5,69E-01	5,39E-01
	7	6,12E-17	1,43E+00	5,59E-18	9,88E-01	9,09E-02	1,36E-02
	9	6,12E-17	2,55E+00	4,33E-18	2,47E+00	9,10E-02	0,00E+00
DTLZ7	3	2,37E-02	5,17E-02	4,15E-04	3,26E-03	3,35E-01	3,27E-01
	5	2,72E-01	1,16E+00	5,41E-03	1,38E-02	4,31E-01	4,22E-01
	7	2,48E-02	1,19E-01	8,54E-03	1,07E-01	1,17E-05	4,65E-06
	9	3,70E-02	2,45E-01	7,61E-03	2,21E-01	5,50E-07	5,00E-08
WFG1	3	6,82E-02	1,48E-01	1,49E-03	4,07E-03	8,20E-01	8,06E-01
	5	9,94E-02	4,69E-01	1,07E-02	5,83E-02	5,85E-01	1,53E-01
	7	2,66E-01	5,67E-01	2,45E-02	6,44E-02	9,30E-01	6,55E-01
	9	3,66E-01	8,59E-01	2,69E-02	3,42E-01	9,91E-01	1,16E-01
WFG2	3	1,59E-02	1,25E-01	4,40E-03	3,69E-02	7,50E-01	7,28E-01
	5	8,78E-02	3,97E-01	9,51E-03	3,14E-02	5,68E-01	2,45E-01
	7	2,49E-01	5,12E-01	2,35E-02	6,24E-02	9,25E-01	6,40E-01
	9	3,63E-01	5,50E-01	2,71E-02	9,22E-02	9,90E-01	8,21E-01
WFG3	3	2,75E-02	1,46E-01	1,88E-03	2,51E-02	5,25E-01	5,01E-01
	5	1,85E-01	9,31E-01	2,87E-01	5,08E-01	5,44E-01	5,28E-01
	7	1,71E-01	2,03E+00	1,01E-01	8,63E-01	5,71E-01	5,62E-01
	9	8,64E+00	9,03E+00	4,02E-02	1,71E+00	6,52E-01	6,54E-01
WFG4	3	4,07E-03	5,33E-01	1,22E-03	9,67E-03	4,55E-01	4,48E-01
	5	1,20E-05	1,29E-02	1,13E-05	1,16E-02	9,09E-02	6,82E-02
	7	1,93E-05	2,62E-02	1,82E-05	2,59E-02	9,09E-02	4,55E-02
	9	2,77E-05	9,44E-02	2,44E-05	9,32E-02	9,10E-02	0,00E+00
WFG5	3	8,62E-02	5,20E-01	7,17E-03	3,47E-02	3,99E-01	3,99E-01
	5	1,12E-01	1,16E-01	4,57E-02	1,11E-01	0,00E+00	0,00E+00
	7	1,32E-01	1,43E-01	1,21E-02	1,43E-01	0,00E+00	0,00E+00
	9	1,50E-01	2,39E-01	1,01E-01	2,39E-01	0,00E+00	0,00E+00
WFG6	3	1,15E-01	4,62E-01	9,62E-03	4,24E-02	3,79E-01	3,82E-01
	5	1,59E-01	1,58E-01	1,00E-01	1,26E-01	0,00E+00	0,00E+00
	7	1,74E-01	1,69E-01	3,94E-02	1,66E-01	0,00E+00	0,00E+00
	9	1,86E-01	2,02E-01	7,31E-02	2,00E-01	0,00E+00	0,00E+00
WFG7	3	3,29E-04	5,11E-01	8,15E-04	7,29E-04	4,58E-01	4,57E-01
	5	1,50E-05	2,15E-03	6,70E-06	1,70E-03	9,09E-02	8,69E-02
	7	2,04E-05	3,41E-03	1,32E-05	3,30E-03	9,09E-02	8,44E-02
	9	2,52E-05	3,68E-02	2,06E-05	3,68E-02	9,09E-02	2,43E-02
WFG8	3	3,36E-01	5,27E-01	1,47E-02	6,75E-02	2,69E-01	2,75E-01
	5	9,07E-01	9,75E-01	2,09E-01	1,96E-01	0,00E+00	0,00E+00
	7	1,61E+00	1,74E+00	4,04E-01	3,62E-01	0,00E+00	0,00E+00
	9	2,25E+00	2,20E+00	5,01E-01	6,20E-01	0,00E+00	0,00E+00
WFG9	3	2,29E-02	4,83E-01	2,06E-03	1,62E-02	4,41E-01	4,33E-01
	5	4,76E-02	1,41E-01	2,50E-02	1,01E-01	1,81E-02	7,62E-04
	7	5,44E-02	3,26E-01	2,31E-02	3,20E-01	9,29E-03	0,00E+00
	9	5,66E-02	7,06E-01	2,73E-02	5,92E-01	5,72E-03	0,00E+00

Table A2
wVaEA vs VaEA comparison: mean of classic IGD and GD values on DTLZ1–7 and WFG1–9 problems (best results are shaded).

Problem	m	IGD		GD	
		wVaEA	VaEA	wVaEA	VaEA
DTLZ1	3	2,14E-02	2,96E-02	1,92E-04	2,18E-01
	5	6,75E-02	1,56E-01	1,55E-03	5,78E-01
	7	9,72E-02	3,14E-01	2,85E-03	9,89E-01
	9	1,05E-01	3,85E-01	3,31E-03	1,07E+00
DTZL2	3	2,06E-01	5,57E-02	6,32E-04	9,57E-04
	5	6,00E-01	2,07E-01	3,72E-07	5,29E-03
	7	6,54E-01	3,29E-01	3,36E-07	1,00E-02
	9	7,25E-01	4,21E-01	7,69E-07	1,46E-02
DTZL3	3	2,07E-01	5,90E-02	6,30E-04	3,83E-02
	5	6,00E-01	5,54E-01	1,41E-04	2,24E+00
	7	6,54E-01	1,47E+00	1,44E-04	4,51E+00
	9	7,26E-01	7,25E+00	7,24E-05	5,57E+00
DTLZ4	3	2,06E-01	5,50E-02	5,66E-04	7,75E-04
	5	6,00E-01	2,09E-01	2,07E-07	5,33E-03
	7	6,54E-01	3,31E-01	2,25E-07	1,04E-02
	9	7,27E-01	4,22E-01	2,01E-07	1,49E-02
DTLZ5	3	1,50E-01	5,70E-03	4,28E-05	3,77E-04
	5	3,42E-01	1,42E-01	4,25E-06	1,30E-01
	7	7,42E-01	2,73E-01	3,12E-07	1,40E-01
	9	7,42E-01	3,45E-01	4,00E-07	1,41E-01
DTLZ6	3	1,52E-01	4,72E-03	5,33E-06	4,58E-06
	5	3,42E-01	2,98E-01	1,02E-17	3,22E-01
	7	7,42E-01	1,59E+00	5,59E-18	5,21E-01
	9	7,42E-01	2,99E+00	4,33E-18	6,40E-01
DTLZ7	3	2,27E-01	6,24E-02	4,15E-04	2,16E-03
	5	4,21E-01	3,67E-01	4,26E-03	1,52E-02
	7	5,92E+00	6,08E-01	8,54E-03	2,63E-02
	9	7,70E+00	7,90E-01	7,59E-03	1,89E-02
WFG1	3	6,72E-01	1,64E-01	1,49E-03	3,00E-03
	5	1,47E+00	4,96E-01	1,07E-02	2,49E-02
	7	1,87E+00	7,21E-01	2,44E-02	2,95E-02
	9	1,90E+00	1,01E+00	2,69E-02	5,97E-02
WFG2	3	2,66E-01	9,66E-02	4,37E-03	4,17E-02
	5	1,50E+00	5,02E-01	9,50E-03	3,37E-02
	7	1,99E+00	7,80E-01	2,35E-02	5,71E-02
	9	2,09E+00	8,94E-01	2,70E-02	4,62E-02
WFG3	3	2,75E-02	1,32E-01	1,95E-03	8,62E-02
	5	1,85E-01	6,44E-01	2,85E-01	2,49E-01
	7	1,67E-01	1,22E+00	1,00E-01	4,06E-01
	9	8,60E+00	1,64E+00	4,08E-02	4,56E-01
WFG4	3	1,92E+00	2,23E-01	9,06E-04	5,44E-03
	5	6,13E+00	1,18E+00	1,20E-06	3,11E-02
	7	8,71E+00	2,47E+00	1,76E-06	7,96E-02
	9	1,10E+01	3,64E+00	1,95E-06	1,21E-01
WFG5	3	1,81E+00	2,30E-01	7,04E-03	8,22E-03
	5	6,07E+00	1,18E+00	1,12E-02	3,37E-02
	7	8,64E+00	2,44E+00	1,21E-02	8,00E-02
	9	1,09E+01	3,67E+00	1,06E-02	1,23E-01
WFG6	3	1,80E+00	2,46E-01	9,50E-03	1,18E-02
	5	6,04E+00	1,20E+00	1,55E-02	3,65E-02
	7	8,62E+00	2,47E+00	1,59E-02	8,08E-02
	9	1,09E+01	3,78E+00	1,32E-02	1,22E-01
WFG7	3	1,84E+00	2,22E-01	8,03E-04	4,87E-03
	5	6,13E+00	1,18E+00	1,54E-06	3,16E-02
	7	8,71E+00	2,46E+00	1,87E-06	7,77E-02
	9	1,10E+01	3,69E+00	1,83E-06	1,24E-01
WFG8	3	1,72E+00	2,97E-01	1,47E-02	2,08E-02
	5	5,28E+00	1,27E+00	3,94E-02	5,92E-02
	7	7,27E+00	2,60E+00	8,35E-02	1,24E-01
	9	9,22E+00	3,89E+00	8,46E-02	1,58E-01
WFG9	3	2,07E+00	2,20E-01	1,93E-03	5,93E-03
	5	6,40E+00	1,17E+00	4,68E-03	3,82E-02
	7	8,68E+00	2,42E+00	4,99E-03	9,32E-02
	9	1,10E+01	3,58E+00	4,01E-03	1,33E-01

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